# Quasi-two-dimensional behavior of phonon subsystems and the superconductivity mechanism in perovskitelike compounds

Yu. E. Kitaev, M. F. Limonov, and A. G. Panfilov A. F. Ioffe Physical-Technical Institute, Politekhnicheskaya 26, 194021 St. Petersburg, Russia

R. A. Evarestov

St. Petersburg State University, Universit. pr. 2, Stary Peterhof, 198904 St. Petersburg, Russia

A. P. Mirgorodsky

Institute for Silicate Chemistry, Odoevskogo 24/2, 199155 St. Petersburg, Russia (Received 10 May 1993; revised manuscript received 3 December 1993)

The results of group-theory analysis, lattice dynamical calculations, and experimental data on the vibrational spectra of perovskitelike superconductors have been considered together. The layer description of phonon subsystems in these materials has been shown to be valid. The quasi-two-dimensional nature of the phonon subsystems has been established. The genesis of the bulk modes has been studied. It has been shown that optical vibrations in these crystals can be divided into  $3(N_L - 1)$  interlayer and  $3(N_A - N_L)$  intralayer modes where  $N_L$  and  $N_A$  are the numbers of layers and atoms per primitive unit cell, respectively. The interlayer bulk vibrations originate from acoustic layer modes whereas the intralayer modes are induced by optical vibrations of isolated layers. A sandwich conception of the high- $T_c$  superconductivity mechanism is discussed based on the different roles of different layers.

## I. INTRODUCTION

At the present stage of investigation of high- $T_c$  superconductors, the most intriguing questions concern the common features of all perovskitelike-superconductor phonon spectra, and whether phonons are involved in the pairing mechanism for electrons. In this paper, we try to throw light on these problems.

The perovskitelike compounds have a pronounced layered structure. This gives rise to the quasi-twodimensional (2D) behavior of the electron subsystem in these crystals. However, the question of the influence of the layered structure on the vibrational spectra remains open.

In Ref. 1, within the framework of the single-layer approach, a group-theory analysis of normal vibrations in Bi-based superconductors was performed. The crystal structure was artificially separated into individual layers and the layer mode symmetry was analyzed. However, unlike the case of the well-known layered crystals ( $As_2S_3$ , GaSe, etc.), the validity of such a separation is not obvious. Moreover, the choice of the division of the structure into individual layers is also ambiguous. In this paper, we verify the validity of the single-layer approach to the analysis of vibrational spectra of perovskitelike compounds by comparing the results of group-theory analysis with lattice dynamical calculations, and experimental data.

The paper is organized as follows. In Sec. II, the pronounced layered structure which is a general feature of perovskitelike superconductors is studied. All these compounds are built of a limited number of single layers. The layers are the common elements of the different superconducting compounds (Y, Bi, Tl based). This is used in Secs. III-VII in analyzing the symmetry of phonons, the vibrational spectra, and the role of phonons in the high- $T_c$  superconductivity (HTSC) mechanism.

In the high- $T_c$  superconductors under consideration, the number of atoms per primitive cell is relatively large. Hence, their vibrational spectra contain many lines whose assignment is rather complicated. However, the pronounced layered structure of these materials makes it possible to obtain additional information about lattice dynamics within the framework of symmetry considerations only.<sup>1-4</sup> Within such an approach, for La-, Y-, Bi-, and Tl-based superconductors, the symmetry of normal modes of isolated layers is analyzed; the layer vibrations are classified into acoustic and optical modes; and the bulk vibrations are classified into interlayer and intralayer modes (Sec. III). As a result, the symmetry correspondence between layer and bulk modes is established.

The results of lattice dynamical calculations for  $YBa_2Cu_3O_7$  crystals are presented in Sec. IV. In these calculations the general valence-force-field model is used.

In Sec. V, we analyze the data obtained by Raman spectroscopy, which serves as a tool for obtaining information about lattice vibrations. A reliable interpretation of Raman-scattering spectra has been made for the  $YBa_2Cu_3O_{7-\delta}$  superconductors.<sup>5-7</sup> To interpret with confidence the vibrational spectra of multi-component Bi-based superconductors, we carried out in Ref. 1 a comparative analysis of Raman spectra for a number of compounds formed by consecutive addition of new layers. As a consequence, it was established that the frequencies of Raman-active modes corresponding to the vibrations of atoms in certain layers varied insignificantly in different Bi-based compounds. That is to say, these

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The main results of the group-theory analysis, the lattice dynamical calculations, and the analysis of general characteristics of the Raman spectra are summarized in Sec. VI. The conclusions of this section are used to analyze the "sandwich" model of high- $T_c$  superconductivity, which is discussed in Sec. VII. Within this conception, the different layers are assumed to have different roles: the CuO<sub>2</sub> layers determine the electron-subsystem properties, and the separating layers induce high-frequency  $A_g$  phonons which determine  $T_{c,max}$  within each of the superconductor families.

# II. LAYERED STRUCTURE OF HIGH-T<sub>c</sub> SUPERCONDUCTORS

In this work, we analyze different high- $T_c$  superconductors, namely those that are Y, Bi, and Tl based. To simplify comparison of vibrational spectra of different compounds, we propose an easy notation scheme (Fig. 1). It is based on the general layered structure of these materials. Since, in perovskitelike cuprates, each oxygen atom is in a certain metal-oxygen layer, this atom is labeled O, with a subscript denoting the metal atom in this layer. For example, the oxygen atom in the BiO layer is labeled  $O_{Bi}$  and that in the SrO layer is denoted as  $O_{Sr}$ .

The copper atoms belonging to double  $\text{CuO}_2$  layers are labeled as Cu whereas those in single  $\text{CuO}_2$  layers are denoted as Cu1 (Fig. 1). Similarly, the oxygen atoms in  $\text{CuO}_2$  layers of tetragonal compounds are labeled  $O_{\text{Cu}}$ whereas the nonequivalent oxygen atoms  $O_{\text{Cu}}$  and  $O'_{\text{Cu}}$  in orthorhombic compounds are denoted by a prime. The oxygen atoms in the single layers are denoted as  $O_{\text{Cul}}$ . For further analysis it is essential that the double  $\text{CuO}_2$ layers are corrugated whereas the single  $\text{CuO}_2$  layers are flat.

As an example of materials with layered architecture, let us consider Bi-based superconductors. The nonsuperconducting compounds without copper,  $Bi_{1-x}(Sr_{1-y}Ca_y)_xO_{1.5-x/2}$ ,<sup>1,8</sup> could be chosen as parent structures of Bi superconductors. The other compounds of the Bi family can be built up by a consecutive insertion (layer by layer) of additional layers into the parent structure:  $(Bi,Ca)O_{\delta} \rightarrow (Bi,Sr)O_{\delta} \rightarrow Bi_2Sr_2CuO_6$ 

Y-1:2:3	TI-2:2:2:3	Bi-2:2:2:3
Cu1O <sub>cu1</sub> •••• BaO <sub>Ba</sub> •••• CuO <sub>cu</sub> O <sub>cu</sub> O <sub>cu</sub> •••• Y ••• •••• *	Cu(Õ <sub>cu</sub> )2 Ca	$\begin{array}{c} \bullet \bullet \bullet \bullet & BiO_{Bi} \\ \bullet \bullet \bullet \bullet \bullet & SrO_{Sr} \\ \bullet \bullet \bullet \bullet & Cu(O_{Cu})_2 \\ \bullet & \bullet & \bullet & Ca \\ \bullet \bullet \bullet \bullet & Cu & 1(O_{Cu1})_2 \\ \bullet & \bullet & \bullet \\ \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet$

FIG. 1. Layer composition of crystal structures of high- $T_c$  superconductors. The notations of the atoms are given along-side each layer.

→ Bi<sub>2</sub>Sr<sub>2</sub>CaCu<sub>2</sub>O<sub>8</sub>→ Bi<sub>2</sub>Sr<sub>2</sub>Ca<sub>2</sub>Cu<sub>3</sub>O<sub>10</sub>→ Bi<sub>4</sub>Sr<sub>4</sub>CaCu<sub>3</sub>O<sub>14</sub>. The systems most investigated are the Bi<sub>2</sub>Sr<sub>2</sub>Ca<sub>n-1</sub>Cu<sub>n</sub>O<sub>2n+4</sub> crystal family, including Bi-2:2:0:1 (n=1), Bi-2:2:1:2 (n=2), and Bi-2:2:2:3 (n=3) superconductors. Recently, Bi<sub>4</sub>Sr<sub>4</sub>CaCu<sub>3</sub>O<sub>14</sub> (Bi-4:4:1:3) crystals were synthesized<sup>9</sup> and comprehensively studied.<sup>10</sup> The Bi-4:4:1:3 primitive cell is formed from halves of the Bi-2:2:0:1 and Bi-2:2:1:2 primitive cells.

The structures of Tl-based superconductors are also well known. Note the essential difference of Tl-based superconductors from the Bi-based ones: two Tl superconductor families with one or two TlO layers per unit cell are known at present. The formulas of these families are TlBa<sub>2</sub>Ca<sub>n-1</sub>Cu<sub>n</sub>O<sub>2n+3</sub> and Tl<sub>2</sub>Ba<sub>2</sub>Ca<sub>n-1</sub>Cu<sub>n</sub>O<sub>2n+4</sub>. The representatives of the first family are structurally similar to the 1:2:3 compounds, e.g., the YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> and CaBa<sub>2</sub>TlCu<sub>2</sub>O<sub>7</sub> (n=2) crystals are isostructural. The second family is analogous to the Bi<sub>2</sub>Sr<sub>2</sub>Ca<sub>n-1</sub>Cu<sub>n</sub>O<sub>2n+4</sub> family, except that it is characterized by tetragonal symmetry with no evidence of the superstructure which is inherent in Bi-based superconductors.

The crystals of other HTSC families can be built up in a similar way. According to the block-layer concept,<sup>11</sup> the structure of cuprate superconductors can be viewed as a stack of conducting  $CuO_2$  layers and different insulating ones (the block layers).

## **III. GROUP SYMMETRY ANALYSIS**

In this section, the phonon symmetry of isolated layers is considered, the phonon symmetry of bulk crystals is analyzed, and finally the relationship between layer and bulk vibrational modes is established. For the complete group-theoretical analysis of both layer and bulk phonon symmetries, we use the method of induced band representations of space groups<sup>4,12</sup> which has proved to be much more efficient than the factor-group method,<sup>13</sup> especially for crystals with a large number of atoms per primitive cell and nonzero wave vectors **k** in the Brillouin zone (BZ).

## A. Layer phonon symmetry

The layers are systems with 2D translational and 3D point symmetries. Therefore, the symmetry of a layer can be described by one of the 80 diperiodic groups (DG) in three dimensions. Each of these groups is a subgroup of one of the 230 triperiodic space groups G.<sup>14</sup>

In analyzing the crystal structures of perovskitelike compounds, we have divided them into a set of single layers and established the corresponding symmetries of isolated layers. The results are presented in Table I.

When determining the symmetry of isolated layers constituting the HTSC compounds under consideration, we take into account that the layer space group DG23 is a subgroup of the corresponding triperiodic group  $G = C_{2v}^1(Pmm2),$ DG37 is a subgroup of  $G = D_{2h}^{\overline{1}} (P2/m2/m2/m)$ , DG55 is a subgroup of  $G = C_{4v}^{1}(P4mm)$ , and DG61 is a subgroup of  $G = D_{4h}^{1}(P4/mmm)$ . Induced band representations of a diperiodic group DG can be obtained from induced band representations of a corresponding triperiodic space

<u>+</u> 2			
${ m e}$ symmetry groups (DG) of isolated layers, layer mode symmetries ( $\Gamma_{ m ac}+\Gamma_{ m opt}$ ) and corresponding interlayer ( $\Gamma_{ m inter}$ ) and intralayer			
s, spac		rs	Bi,TI
slike superconductor		Number of layers	Π
perovskit			Y
TABLE I. Layer composition of perovskitelike superconductor	r <sub>intra</sub> ) bulk mode symmetries.	Symmetry of	isolated

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	Symmetry of			ź	umber c	Number of layers	s						
	isolated		Y		Ц			Bi,Tl				I	ſ
Layer	layer	La2CuO4 1:2:3 1:2:0:1 1:2:1:2 1:2:2:3 2:2:0:1 2:2:1:2 2:2:3	1:2:3	1:2:0:1	1:2:1:2	1:2:2:3	2:2:0:1	2:2:1:2	2:2:2:3	Γ <sub>ac</sub>	$\Gamma_{opt}$	$\Gamma_{\text{inter}}$	I intra
Cu(O <sub>Cu</sub> ) <sub>2</sub>	$Cu(O_{Cu})_2 P2mm(DG23)$		2							$A_1 + B_1 + B_2$	$2(A_1+B_1+B_2)$	$A_g + B_{2g} + B_{3g} + B_{1u} + B_{2u} + B_{3u}$	$A_{1} + B_{1} + B_{2} = 2(A_{1} + B_{1} + B_{2}) + A_{g} + B_{2g} + B_{3g} + B_{1u} + B_{2u} + B_{3u} - 2(A_{g} + B_{1u}) + 2(B_{2g} + B_{3u}) + 2(B_{3g} + B_{2u}) + 2(B_{3g} + B_{3u}) + 2(B_{3g} $
	P4mm(DG55)				3	7		7	2	$A_1 + E$	$A_1 + B_1 + 2E$	$A_{1g} + E_g + A_{2u} + E_u$	$(A_{1g} + A_{2u}) + (B_{1g} + B_{2u}) + 2(E_g + E_u)$
	P4/mmm												
Cu1(O <sub>Cu1</sub> ) <sub>2</sub>	(DG61)	1		1		1	1		1	$A_{2u} + E_u$	$A_{2u} + B_{2u} + 2E_u$	$A_{2u} + E_u$	$A_{2u} + B_{2u} + 2E_u$
	P2/m2/m2/m												
Cu10 <sub>Cu1</sub>	(DG37)		1						7	$B_{1u} + B_{2u} + B_{3u}$	$B_{1u} + B_{2u} + B_{3u} B_{1u} + B_{2u} + B_{3u}$	$B_{1u} + B_{2u} + B_{3u}$	$B_{1u} + B_{2u} + B_{3u}$
	P2/m2/m2/m												
Υ	(DG37)		1						7	$B_{1u} + B_{2u} + B_{3u}$		$B_{1u} + B_{2u} + B_{3u}$	
	P4/mmm												
Ca	(DG61)				1	7		1	7	$A_{2u} + E_u$		$A_{2u} + E_u$	
BaO	P2mm(DG23)		2							$A_1 + B_1 + B_2$	$A_1 + B_1 + B_2$	$A_g + B_{2g} + B_{3g} + B_{1u} + B_{2u} + B_{3u}$	$A_g + B_{2g} + B_{3g} + B_{1u} + B_{2u} + B_{3u}  (A_g + B_{1u}) + (B_{2g} + B_{3u}) + (B_{3g} + B_{2u})$
BaO, SrO,													
LaO	P4mm(DG55)	7		7	7	7	7	7	7	$A_1 + E$	$A_1 + E$	$A_{1g} + E_g + A_{2u} + E_u$	$(A_{1g} + A_{2u}) + (E_g + E_u)$
T10,BiO	T10,BiO P4mm(DG55)						7	2	7	$A_1 + E$	$A_1 + E$	$A_{1g} + E_g + A_{2u} + E_u$	$(A_{1g} + A_{2u}) + (E_g + E_u)$
	P4/mmm												
TIO	(DG61)			1	1	-				$A_{2u} + E_u$	$A_{2u} + E_u$	$A_{2u} + E_u$	$A_{2u} + E_u$

group  $G.^4$ 

In this paper, we discuss in detail the symmetry analysis procedure for the YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> compound, whereas only final results are given for Bi- and Tl-based superconductors. The results of the group-theory analysis of phonon symmetry in the Y, BaO<sub>Ba</sub>, Cu1O<sub>Cu1</sub>, and CuO<sub>Cu</sub>O'<sub>Cu</sub> layers constituting the YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> compound are given in Tables II and III for the symmetry points of the primitive rectangular (2D) BZ.

Tables II and III have the following structure. Columns 1-3 contain the atomic arrangements over the Wyckoff positions (sites in direct space). Column 4 contains the Mulliken symbols of those irreducible representations (irreps) of the site symmetry groups for these Wyckoff positions, according to which the components of the vectors of local atomic displacements (x, y, z) transform. The remaining columns give the labels of induced representations in the k basis, with the symbols of k points (wave vectors) in the first row of the tables and the indices of small irreps of little groups in subsequent rows; these determine the symmetries of the normal modes. The labels of the layer group irreps correspond with those of the related triperiodic group G. In these tables and in all the following ones, the labeling of the spacegroup irreps is that of Ref. 15, the labeling of the pointgroup irreps is that of Ref. 16, and the site points q are indexed as Wyckoff positions from Ref. 17.

From Tables II and III, one can easily write down the full vibrational representation at any symmetry point of the BZ. For example, for the  $CulO_{Cul}$  layer, we obtain

$$\Gamma = 2\Gamma_{2}^{-} + 2\Gamma_{3}^{-} + 2\Gamma_{4}^{-} ,$$

$$X = 2X_{2}^{-} + 2X_{3}^{-} + 2X_{4}^{-} ,$$

$$Y = Y_{2}^{-} + Y_{3}^{-} + Y_{4}^{-} + Y_{1}^{+} + Y_{2}^{+} + Y_{3}^{+} ,$$

$$S = S_{2}^{-} + S_{3}^{-} + S_{4}^{-} + S_{1}^{+} + S_{2}^{+} + S_{3}^{+} .$$
(1)

The corresponding vibrational representations at the BZ center for a complete set of layers for YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> together with the results for Bi- and Tl-based compounds are presented in Table I. The layer modes at the BZ

TABLE II. Phonon symmetry in the CulO<sub>Cul</sub> and Y layers. Primitive translations and site symmetry group for the direct lattice are  $\mathbf{a}_1(1,0)$ ,  $\mathbf{a}_2(0,1)$  (in units of ab) and  $D_{2h}(mmm)$   $a(0,0,0), e(0,\frac{1}{2},0), f(\frac{1}{2},\frac{1}{2},0)$  (in units of **a**<sub>i</sub>); for the reciprocal lattice,  $\mathbf{b}_1(1,0)$ ,  $\mathbf{b}_2(0,1)$  (in units of  $2\pi a, 2\pi b$ ) and  $D_{2h}(mmm)$  —  $\Gamma(0,0), X(\frac{1}{2},0), Y(0,\frac{1}{2}), S(\frac{1}{2},\frac{1}{2})$  (in units of **b**<sub>i</sub>).

Cu1O <sub>Cu1</sub>	Y	P2/m2/	/m2/m(DG37)	Г	X	Y	S
1	2	3	4	5	6	7	8
Cu1		1 <i>a</i>	$b_{1u}(z)$	2-	$2^{-}$	2-	2-
			$b_{2u}(y)$	4-	4-	4-	4-
			$b_{3u}(x)$	3-	3-	3-	3-
$O_{Cu1}$		1 <i>e</i>	$b_{1u}(z)$	2-	$2^{-}$	3+	3+
			$b_{2u}(y)$	4-	4-	1+	1+
			$b_{3u}(x)$	3-	3-	2+	2+
	Y	1f	$b_{1u}(z)$	2-	4+	3+	1-
			$b_{2u}(y)$	4-	2+	1+	3-
			$b_{3u}(x)$	3-	1+	2+	4-

TABLE III. Phonon symmetry in the Cu(O<sub>Cu</sub>)<sub>2</sub> and BaO layers. Primitive translations and site symmetry group for the direct lattice are  $\mathbf{a}_1(1,0)$ ,  $\mathbf{a}_2(0,1)$  (in units of ab) and  $C_{2\nu}(2mm) - a(0,0,z)$ ,  $b(0,\frac{1}{2},z)$ ,  $c(\frac{1}{2},0,z)$ ,  $d(\frac{1}{2},\frac{1}{2},0)$  (in units of  $\mathbf{a}_i$ ); and for the reciprocal lattice,  $\mathbf{b}_1(1,0)$ ,  $\mathbf{b}_2(0,1)$  (in units of  $2\pi/a, 2\pi/b$ ) and  $C_{2\nu}(2mm) - \Gamma(0,0)$ ,  $X(\frac{1}{2},0)$ ,  $Y(0,\frac{1}{2})$ ,  $S(\frac{1}{2},\frac{1}{2})$  (in units of  $\mathbf{b}_i$ ).

$Cu(O_{Cu})_2$	BaO	P2mm(DG23)		Г	X	Y	S
1	2	3	4	5	6	7	8
Cu	$\mathbf{O}_{\mathrm{Ba}}$	1 <i>a</i>	$a_1(z)$	1	1	1	1
			$b_2(y)$	3	3	3	3
			$b_1(x)$	4	4	4	4
$O_{Cu}$		1 <i>b</i>	$a_1(z)$	1	1	3	3
			$b_2(y)$	3	3	1	1
			$b_1(x)$	4	4	2	2
$O'_{Cu}$		1 <i>c</i>	$a_1(z)$	1	4	1	4
- Cu			$b_2(y)$	3	2	3	2
			$b_1(x)$	4	1	4	1
_	Ba	1 <i>d</i>	$a_1(z)$	1	4	3	2
			$b_2(y)$	3	2	1	4
			$b_1(x)$	4	1	2	3

center presented in Table I may be divided into "acoustic" and "optical" normal vibrations. Here and in future, we call the vibrations corresponding to a motion of a layer as an entity acoustic, and the others optical. It should be noted that monatomic layers (e.g., Y layers) give acoustic modes only. A layer with  $N_i > 1$  nonequivalent atoms additionally gives  $3(N_i - 1)$  optical modes.

Tables II and III also allow us to establish which atoms and local atomic displacements contribute to the modes

with a given symmetry. This is especially important to interpret first- and second-order vibrational spectra. For example, one can see from Table II that at the S point of the BZ the mode with symmetry  $S_2^-$  is induced by z displacements of Cu1 atoms, whereas the mode with symmetry  $S_1^+$  is induced by y displacements of  $O_{Cu1}$  atoms.

## B. Bulk phonon symmetry

In a similar manner, we have obtained the phonon symmetry of the bulk  $YBa_2Cu_3O_7$  crystals. The results are presented in Table IV. (The phonon symmetry over the entire BZ for Bi- and Tl-based crystals can be found elsewhere.<sup>1,4,18-20</sup>)

From Table IV, one can easily write down the full vibrational representations at the symmetry points of the BZ. For example, at the  $\Gamma$ , Z, and Y points we have

$$\Gamma = 5\Gamma_{1}^{+} + 5\Gamma_{3}^{+} + 5\Gamma_{4}^{+} + 8\Gamma_{2}^{-} + 8\Gamma_{3}^{-} + 8\Gamma_{4}^{-} ,$$
  

$$Y = 7Y_{1}^{+} + 4Y_{2}^{+} + 7Y_{3}^{+} + 3Y_{4}^{+} + 2Y_{1}^{-} + 6Y_{2}^{-}$$
  

$$+ 4Y_{3}^{-} + 6Y_{4}^{-} ,$$
  

$$Z = 7Z_{1}^{+} + 7Z_{3}^{+} + 7Z_{4}^{+} + 6Z_{2}^{-} + 6Z_{3}^{-} + 6Z_{4}^{-} ,$$
 (2)

where, at the  $\Gamma$  point, the correspondence between two traditionally used sets of notations is the following:  $\Gamma_1^+ = A_g, \ \Gamma_3^+ = B_{3g}, \ \Gamma_4^+ = B_{2g}, \ \Gamma_2^- = B_{1u}, \ \Gamma_3^- = B_{3u}$ , and  $\Gamma_4^- = B_{2u}$ .

#### C. The correspondence between layer and bulk modes

We now establish the correspondence between layer and bulk modes. Since, for the  $YBa_2Cu_3O_7$  lattice, the

TABLE IV. Phonon symmetry in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> compounds. Primitive translations and site symmetry groups for the direct lattice are  $\mathbf{a}_1(1,0,0)$ ,  $\mathbf{a}_2(0,1,0)$ ,  $\mathbf{a}_3(0,0,1)$  (in units of *abc*) and  $D_{2h}(mmm) - c(0,0,\frac{1}{2})$ ,  $f(\frac{1}{2},\frac{1}{2},0)$ ,  $g(0,\frac{1}{2},\frac{1}{2})$ ;  $C_{2v}(2mm) - q(0,0,z)$ ,  $r(0,\frac{1}{2},z)$ ,  $s(\frac{1}{2},0,z)$ ,  $t(\frac{1}{2},\frac{1}{2},z)$  (in units of  $\mathbf{a}_i$ ); and for the reciprocal lattice,  $\mathbf{b}_1(1,0,0)$ ,  $\mathbf{b}_2(0,1,0)$ ,  $\mathbf{b}_3(0,0,1)$  (in units of  $\mathbf{b}_i$ ) and  $D_{2h}(mmm) - \Gamma(0,0,0)$ ,  $X(\frac{1}{2},0,0)$ ,  $Y(0,\frac{1}{2},0)$ ,  $Z(0,0,\frac{1}{2})$ ,  $S(\frac{1}{2},\frac{1}{2},0)$ ,  $T(0,\frac{1}{2},\frac{1}{2})$ ,  $U(\frac{1}{2},0,\frac{1}{2})$ ,  $R(\frac{1}{2},\frac{1}{2},\frac{1}{2})$  (in units of  $\mathbf{b}_i$ ).

<b>D</b>	$\frac{1}{2h}(\mathbf{Pm})$	mm)	Г	X	Y	Ζ	S	Т	U	R
Cu1	1 <i>c</i>	$b_{1u}(z)$	2-	2-	2-	1+	2-	1+	1+	1+
		$b_{2u}(y)$	4	4-	4-	3+	4-	3+	3+	3+
		$b_{3u}(x)$	3-	3-	3-	4+	3-	4+	4+	4+
Y	1f	$b_{1u}(z)$	2-	<b>4</b> <sup>+</sup>	3+	2-	1-	3+	4+	1-
	·	$b_{2u}(y)$	4-	2+	1+	4-	3-	1+	2+	3-
		$b_{3u}(x)$	3-	1+	2+	3-	4-	2+	1+	4-
$O_{Cu1}$	1g	$b_{1u}(z)$	2-	2-	3+	1+	3+	4-	1+	4-
cui	0	$b_{2u}(y)$	4-	4-	1+	3+	1+	2-	3+	2-
		$b_{3u}(x)$	3-	3-	2+	4+	2+	1 -	4+	$1^{-}$
$O_{Ba}$	2q	$a_1(z)$	1+,2-	1+,2-	1+,2-	$1^+, 2^-$	1+,2-	$1^+, 2^-$	1+,2-	$1^+, 2^-$
Cu	-	$b_1(x)$	3-,4+	3-,4+	3-,4+	3-,4+	3-,4+	3-,4+	3-,4+	3-,4+
		$b_2(y)$	3+,4-	3+,4-	3+,4-	3+,4-	3+,4-	3+,4-	3+,4-	3+,4-
$\mathbf{O}_{\mathbf{Cu}}$	2 <i>r</i>	$a_1(z)$	1+,2-	1+,2-	3+,4-	1+,2-	3+,4-	3+,4-	$1^+, 2^-$	3+,4-
		$b_1(x)$	3-,4+	3-,4+	1-,2+	3-,4+	$1^{-},2^{+}$	1-,2+	3-,4+	1-,2+
		$b_2(y)$	3+,4-	3+,4-	1+,2-	3+,4-	1+,2-	1+,2-	3+,4-	1+,2-
O' <sub>Cu</sub> 2s	$a_1(z)$	1+,2-	3 <sup>-</sup> ,4 <sup>+</sup>	1+,2-	1+,2-	3-,4+	1+,2-	3-,4+	3-,4+	
		$b_1(x)$	3-,4+	1+,2-	3-,4+	3-,4+	1+,2-	3-,4+	$1^+, 2^-$	1+,2-
		$b_2(y)$	$3^+, 4^-$	1-,2+	3+,4-	3+,4-	$1^{-},2^{+}$	3+,4-	1-,2+	1-,2+
Ba	2 <i>t</i>	$a_1(z)$	$1^+, 2^-$	3-,4+	3+,4-	1+,2-	1-,2+	3+,4-	3-,4+	1-,2+
		$b_{1}(x)$	3-,4+	$1^+, 2^-$	1-,2+	3-,4+	3+,4-	1-,2+	1+,2-	3+,4-
		$b_2(y)$	3+,4-	1-,2+	1+,2-	3+,4-	3-,4+	1+,2-	1-,2+	3-,4+

layer groups DG23 and DG37 are subgroups of the space group of the bulk crystal  $(D_{2h}^{1})$ , the mode correspondence can be obtained by the direct induction procedure.<sup>4</sup>

For the Y layer, the layer modes transform into the following bulk modes

$$\Gamma_2^- \to \Gamma_2^- + Z_2^-; \quad \Gamma_3^- \to \Gamma_3^- + Z_3^-; \quad \Gamma_4^- \to \Gamma_4^- + Z_4^- ,$$
(3)

whereas those of the  $CulO_{Cul}$  layer transform into

$$\Gamma_2^- \to \Gamma_2^- + Z_1^+ ; \quad \Gamma_3^- \to \Gamma_3^- + Z_4^+ ; \quad \Gamma_4^- \to \Gamma_4^- + Z_3^+ .$$
(4)

The layer vibrations of  $Cu(O_{Cu})_2$  and BaO layers transform into the following bulk modes

$$\Gamma_{1} \rightarrow \Gamma_{1}^{+} + \Gamma_{2}^{-} + Z_{1}^{+} + Z_{2}^{-};$$
  

$$\Gamma_{3} \rightarrow \Gamma_{3}^{+} + \Gamma_{4}^{-} + Z_{3}^{+} + Z_{4}^{-};$$
  

$$\Gamma_{4} \rightarrow \Gamma_{4}^{+} + \Gamma_{3}^{-} + Z_{4}^{+} + Z_{3}^{-}.$$
(5)

Taking into account that the perovskitelike crystals have pronounced layered structures and supposing that intralayer interactions are stronger than interlayer ones, it is natural to assume that the normal-mode spectra consist of intralayer and interlayer vibrations. The latter are superpositions of displacements of different layers relative to each other.

Since acoustic layer modes at the BZ center correspond to the translations of a layer as a whole, they transform into interlayer bulk modes. The optical layer modes transform into intralayer bulk modes.

Thus, for the Y layer, the general relationship between layer ( $\Gamma_L$ ) and bulk crystals ( $\Gamma_C$ ) modes is the following

$$\Gamma_L = \Gamma_{ac} + \Gamma_{opt} = (B_{1u} + B_{2u} + B_{3u}) + (O)$$
  

$$\rightarrow \Gamma_C = \Gamma_{inter} + \Gamma_{intra}$$
  

$$= (B_{1u} + B_{2u} + B_{3u}) + (O) , \qquad (6)$$

whereas for the Cu1O<sub>Cu1</sub> layer we have

$$\Gamma_{L} = \Gamma_{ac} + \Gamma_{opt}$$
  
=  $(B_{1u} + B_{2u} + B_{3u}) + (B_{1u} + B_{2u} + B_{3u})$   
 $\rightarrow \Gamma_{C} = \Gamma_{inter} + \Gamma_{intra}$   
=  $(B_{1u} + B_{2u} + B_{3u}) + (B_{1u} + B_{2u} + B_{3u})$ . (7)

Similarly, for the  $Cu(O_{Cu})_2$  layer, the mode relationship is

$$\Gamma_{L} = \Gamma_{ac} + \Gamma_{opt} = (A_{1} + B_{1} + B_{2}) + (2A_{1} + 2B_{1} + 2B_{2})$$
  
$$\rightarrow \Gamma_{C} = \Gamma_{inter} + \Gamma_{intra} = (A_{g} + B_{1u} + B_{2g} + B_{3u} + B_{3g} + B_{2u}) + 2(A_{g} + B_{1u} + B_{2g} + B_{3u} + B_{3g} + B_{2u}), \quad (8)$$

whereas that for the  $BaO_{Ba}$  layer is

$$\Gamma_{L} = \Gamma_{ac} + \Gamma_{opt} = (A_{1} + B_{1} + B_{2}) + (A_{1} + B_{1} + B_{2})$$
  

$$\rightarrow \Gamma_{C} = \Gamma_{inter} + \Gamma_{intra} = (A_{g} + B_{1u} + B_{2g} + B_{3u} + B_{3g} + B_{2u}) + (A_{g} + B_{1u} + B_{2g} + B_{3u} + B_{3g} + B_{2u}) .$$
(9)

Summing the contributions of the layers, we obtain the result that, for the YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> crystal, the full vibrational representation at the  $\Gamma$  point of the BZ can be divided into acoustic, interlayer, and intralayer optical modes as follows:

$$\Gamma_{c} = \Gamma_{ac} + \Gamma_{inter} + \Gamma_{intra} = (B_{1u} + B_{2u} + B_{3u}) + (2A_{g} + 3B_{1u} + 2B_{2g} + 3B_{3u} + 2B_{3g} + 3B_{2u}) + (3A_{g} + 4B_{1u} + 3B_{2g} + 4B_{3u} + 3B_{3g} + 4B_{2u}).$$
(10)

(Similar results for a complete set of layers in La-, Bi-, and Tl- based compounds are listed in Table I.)

So, the main results of the group-theory analysis are as follows.

(i) In YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>, the bulk modes can be classified into three acoustic modes, 15 interlayer modes (originating from layer acoustic vibrations), and 21 intralayer modes (originating from layer optical vibrations). It is easy to obtain the general case that the number of interlayer modes is equal to  $3(N_L - 1)$ , where  $N_L$  is the number of layers per unit cell in the layered compounds under consideration. The number of intralayer modes is equal to  $3(N_A - N_L)$  with  $N_A$  being the total number of atoms per unit cell.

Thus, the vibrational representation at the  $\Gamma$  point of the BZ can be written in any one of the layered crystals as follows,

$$\Gamma = 3\Gamma_{\rm ac} + 3(N_L - 1)\Gamma_{\rm inter} + 3(N_A - N_L)\Gamma_{\rm intra} .$$
(11)

(ii) It follows from Eq. (5) that, for a weak interlayer interaction, there is a small frequency splitting of intralayer modes. In this case, the frequencies of the intralayer bulk phonons with symmetries  $\Gamma_1^+$  ( $A_g$ ) and  $\Gamma_2^-$  ( $B_{1u}$ ) as well as  $\Gamma_3^+$  ( $B_{3g}$ ) and  $\Gamma_4^-$  ( $B_{2u}$ ) or  $\Gamma_4^+$  ( $B_{2g}$ ) and  $\Gamma_3^-$  ( $B_{3u}$ ) modes should be close. Since the space group of Cu(O<sub>Cu</sub>)<sub>2</sub> and BaO<sub>Ba</sub> layers does not contain an inversion, the layer modes  $\Gamma_1$  ( $A_1$ ),  $\Gamma_3$  ( $B_2$ ), and  $\Gamma_4$  ( $B_1$ ) are both ir and Raman active. At the same time, the space group of the bulk crystal contains an inversion. Therefore, in the corresponding Davydov doublets originating from the layer optical modes, the even components  $\Gamma_1^+$  ( $A_g$ ),  $\Gamma_3^+$ ( $B_{3g}$ ), and  $\Gamma_4^+$  ( $B_{2g}$ ) are Raman active whereas the odd components  $\Gamma_2^-$  ( $B_{1u}$ ),  $\Gamma_4^-$  ( $B_{2u}$ ) and  $\Gamma_3^-$  ( $B_{3u}$ ) are infrared active. It should be stressed that the magnitudes of the splittings are determined by the interlayer interaction. Thus, the intralayer modes of noncentrosymmetrical layers induce Davydov doublets in centrosymmetrical bulk crystals. In YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>, there are three  $(A_g, B_{1u})$ , three  $(B_{3g}, B_{2u})$ , and three  $(B_{2g}, B_{3u})$  doublets.

(iii) One can also see from Eqs. (4), (5) that the frequencies of the corresponding intralayer bulk modes at the  $\Gamma$  and Z points should be close. In other words, the intralayer modes are almost dispersionless along the  $\Gamma$ -Z symmetry line. This correlates with the results of theory<sup>21</sup> and has been proved to be correct by neutronscattering experiments. It will be discussed below.

## **IV. DYNAMICAL CALCULATIONS**

To comprehend better the properties of phonon subsystems of high- $T_c$  superconductors, we have performed a lattice dynamical treatment of the YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> lattice within a unified description of its potential function. The results of similar computations of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6</sub> and GdBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> will be presented in detail elsewhere.<sup>22</sup>

We used the general valence-force-field model (GVFF), program code CRYME,<sup>23</sup> and the structural data determined at ambient conditions in Ref. 24. The forceconstant magnitudes have been derived from a fitting procedure using the experimentally measured frequencies of ir- and Raman-active modes,<sup>5-7,25</sup> and the linear compressibilities given in Ref. 26. The set of GVFF parameters adopted (see Table V) consists of diagonal stretching constants  $S_1-S_{12}$ , diagonal bending constants  $B_1-B_3$ , and two off-diagonal constants  $H_1, H_2$  describing stretch-stretch interactions. The  $H_2$  constant has been found to be of extreme importance in reproducing some

TABLE V. Force constants for YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>.

	Atoms	Distance (Å)	
	involved	or angle (deg)	Value <sup>a</sup>
$\boldsymbol{S}_1$	Cu1-O <sub>Ba</sub>	1.837	1.8
$S_2$	Cu-O <sub>Cu</sub>	1.931	1.35
$S_3$	Cu1-O <sub>Cu1</sub>	1.943	1.3
$S_4$	Cu-O' <sub>Cu</sub>	1.961	1.25
$S_5$	Cu-O <sub>Ba</sub>	2.315	0.7
$S_6$	Y-O' <sub>Cu</sub>	2.384	0.55
	Y-O <sub>Cu</sub>	2.400	0.55
$S_7$	Ba-O <sub>Ba</sub>	2.743	0.27
$S_8$	Ba-O <sub>Cu1</sub>	2.880	0.23
$S_9$	Ba-O' <sub>Cu</sub>	2.960	0.2
	Ba-O <sub>Cu</sub>	2.996	0.2
$S_{10}$	$O_{Cu}-O_{Cu}$	2.817	0.2
	$O'_{Cu}-O'_{Cu}$	2.854	0.2
<i>S</i> <sub>11</sub>	Ba-Cu1	3.474	0.2
$S_{12}$	Y-Cu	3.207	0.1
$\boldsymbol{B}_1$	$O_{Cu}$ -Cu- $O'_{Cu}$	88.87	0.45
$\boldsymbol{B}_2$	O <sub>Ba</sub> -Cu1-O <sub>Cu1</sub>	90.00	0.45
<b>B</b> <sub>3</sub>	$O_{Ba}$ -Cu- $O'_{Cu}$	97.74	0.15
	$O_{Ba}$ -Cu- $O_{Cu}$	98.42	0.15
$H_1$	O <sub>Ba</sub> -Cu1, Cu1-O <sub>Ba</sub>		-0.2
$H_2$	O <sub>Cu</sub> -Y, Y-O <sub>Cu</sub>		0.13

<sup>&</sup>lt;sup>a</sup>Force constant units are:  $S_i$ , mdyn/Å;  $B_i$ , mdyn Å; and  $H_i$ , mdyn/Å.

of the principal peculiarities of the properties considered.<sup>22</sup>

The above force constants lead to the set of optical frequencies given in Table VI for the BZ center  $\Gamma(0,0,0)$ and for the symmetry points  $Z(0,0,\pi/c)$  and  $Y(0,\pi/b,0)$ . The calculated displacement patterns corresponding to each of these modes in the BZ center are also presented in Table VI. When analyzing this table, one can come to some important conclusions, which we discuss below.

#### A. Interlayer modes

Let us scrutinize the low-frequency vibrations. These modes are usually attributed to the heavy-metal-atom vibrations, each of the vibrations being connected with atoms of a certain kind. However, according to our computations, another approach is more correct. It can be seen from Table VI that most of the low-frequency modes are formed by displacements of certain layers relative to each other. What is more, for different atoms in one layer, displacement amplitudes are close in some cases. So, the displacement patterns resemble translations of layers as a whole. These modes can be regarded as interlayer vibrations.

In particular, low-frequency vibrations of  $B_{2g}$  and  $B_{3g}$  symmetries appear to be in-phase (with frequencies 78 and 79 cm<sup>-1</sup>) and out-of-phase (103 and 104 cm<sup>-1</sup>) displacements of the BaO<sub>Ba</sub> and CuO<sub>Cu</sub>O'<sub>Cu</sub> layers relative to immobile Cu1O<sub>Cu1</sub> and Y layers. By the way, Y, Cu1, and O<sub>Cu1</sub> atoms do not participate in even vibrations at all.

Unlike  $B_{2g}$  and  $B_{3g}$  modes, the interlayer character of low-frequency  $A_g$  modes does not manifest itself so strongly. The amplitudes of z displacements of Ba (110 cm<sup>-1</sup>) and Cu (151 cm<sup>-1</sup>) atoms are larger than those of oxygen atoms.

In the case of odd vibrations of low frequency, all four layers are displaced. For instance, the out-of-phase shift of the Y and  $CuO_{Cu}O'_{Cu}$  layers relative to the  $BaO_{Ba}$  and  $Cu1O_{Cu1}$  layers corresponds to the 64-cm<sup>-1</sup> vibration of the  $B_{3u}$  symmetry.

Thus, the number of modes which could be attributed to interlayer ones according to lattice dynamical calculations is 15. These are low-frequency modes  $(2A_g+3B_{1u}+2B_{2g}+3B_{3u}+2B_{3g}+3B_{2u})$ —see Table VI. Hence, the results obtained by lattice dynamical calculations agree with those of the group-theory analysis.

#### **B.** Intralayer modes

Quite different is the case of higher-frequency vibrations. It is seen from Table VI that each of these modes is formed by the displacements of oxygen atoms belonging to one layer. The other atoms are displaced an order of magnitude less. Therefore, these modes can be attributed to intralayer vibrations, with each mode being induced by an optical vibration of an isolated layer. From Table VI, it follows that there are 21 high-frequency intralayer vibrations, namely  $(3A_g + 4B_{1u} + 3B_{2g} + 4B_{3u} + 3B_{3g} + 4B_{2u})$ , which coincides with the results of symmetry analysis. Let us consider three high-frequency  $A_g$  modes. The 335-cm<sup>-1</sup> vibration originates from out-of-phase displacements along the z axis of atoms  $O_{Cu}$  and  $O'_{Cu}$ ; the 439-cm<sup>-1</sup> vibration is formed on the whole by in-phase z displacements of the same atoms; the 502-cm<sup>-1</sup> vibration is determined by z displacements of the  $O_{Ba}$  atoms.

# C. The splitting of layer vibrations

The above symmetry analysis predicts that every layer mode should split into two vibrations when passing from an isolated layer without an inversion center ( $CuO_2$  and BaO in  $YBa_2Cu_3O_7$ ) to the bulk centrisymmetrical crystal. One of the vibrations should be Raman active and its counterpart ir active. For layers with an inversion center, each layer mode transforms into one bulk mode.

The model computations confirm this. It is seen from Table VI that every high-frequency (intralayer) even vibration of atoms belonging to  $Cu(O_{Cu})_2$  and  $BaO_{Ba}$  layers has as a counterpart a corresponding odd vibration with similar displacements of atoms. At the same time, the vibrations of atoms located in  $Cu1O_{Cu1}$  and Y layers do not give such doublets. Note, the frequencies of components differ slightly both within the three doublets  $(B_{2g}, B_{3u})$ 

TABLE VI. Calculated frequencies at the  $\Gamma$ , Z, and Y symmetry points and displacements of atoms at the  $\Gamma$  point in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>. The amplitudes of atomic displacements are given within an accuracy of 0.5 a.u.; the atomic displacements with amplitudes less than 0.5 a.u. are not presented. The displacements of atoms of the upper half of the primitive unit cell are given. The corresponding atoms of the lower part move in phase for odd modes and out of phase for even modes.

Г	Z	Y	Atomic displacements at the $\Gamma$ point of the BZ
110	74	123	$6^{\mathbf{Z}}\mathbf{Ba} + (1^{\mathbf{Z}}\mathbf{Cu} + 1^{\mathbf{Z}}\mathbf{O}_{\mathbf{Cu}} + 1^{\mathbf{Z}}\mathbf{O}_{\mathbf{Cu}}') \qquad \qquad \mathbf{A}_{g}$
151	164	164	$3^{\mathbf{Z}}\mathbf{O}_{\mathbf{Ba}} + (9^{\mathbf{Z}}\mathbf{Cu} + 1^{\mathbf{Z}}\mathbf{O}_{\mathbf{Cu}} + 1^{\mathbf{Z}}\mathbf{O}_{\mathbf{Cu}})$
335	335	343	$13^{2}O_{Cu}^{2} - 12^{2}O_{Cu}^{\prime}$
439	439	496	$12^{Z}O_{Cu} + 13^{Z}O_{Cu} - 1^{Z}Cu - 1^{Z}O_{Ba}$
502	502	501	$17^2 O_{Ba} - 1^2 Cu$
			$B_{2g}$
78	90	116	$(5^{A}Ba + 5^{A}O_{Ba}) + (4^{A}Cu + 4^{A}O_{Cu} + 2^{A}O_{Cu})$
103	99	117	$(3^{X}Ba + 2^{X}O_{Ba}) - (7^{X}Cu + 7^{X}O_{Cu} + 3^{X}O_{Cu})$
261	261	248	$17^{X}O_{Ba} - 2^{X}Ba - 1^{X}O_{Cu}$
357	357	352	$17^{X}O_{Cu}' - 1^{X}Cu - 2^{X}O_{Cu}$
596	596	596	$16^{X}O_{Cu} - 4^{X}Cu + 1^{X}O_{Cu}'$
			$B_{3g}$
79	89	95	$(5^{\nu}Ba + 4^{\nu}O_{Ba}) + (3^{\nu}Cu + 2^{\nu}O_{Cu} + 3^{\nu}O_{Cu}')$
104	101	113	$(2^{\nu}Ba + 1^{\nu}O_{Ba}) - (7^{\nu}Cu + 3^{\nu}O_{Cu} + 7^{\nu}O_{Cu}')$
310	310	290	$17^{\nu}O_{Ba} - 1^{\nu}Ba - 1^{\nu}O_{Cu}$
361	361	382	$17^{\nu}O_{Cu} - 1^{\nu}Cu + 1^{\nu}O_{Ba} - 2^{\nu}O_{Cu}$
575	575	512	$16^{\nu}O_{Cu}' - 4^{\nu}Cu + 1^{\nu}O_{Cu}$
			$B_{1u}$
121	124	120	$6^{Z}Y - (4^{Z}Ba - 1^{Z}O_{Ba}) - 2^{Z}O_{Cu1} + (4^{Z}Cu + 2^{Z}O_{Cu} + 2^{Z}O_{Cu})$
139	129	237	$7^{Z}Y + (1^{Z}Ba - 5^{Z}O_{Ba}) + (5^{Z}Cu1 + 2^{Z}O_{Cu1}) + (5^{Z}Cu - 3^{Z}O_{Cu} - 3^{Z}O_{Cu})$
227	228	239	$2^{2}Y - (1^{2}Ba - 6^{2}O_{Ba}) + (8^{2}Cu1 + 7^{2}O_{Cu1}) - (5^{2}Cu + 2^{2}O_{Cu} + 1^{2}O_{Cu})$
264	264	265	$13^{2}O_{Cu} - 12^{2}O_{Cu}$
295	294	270	$11^{z}O_{Cu} + 11^{z}O_{Cu}' - 4^{z}Y - 1^{z}Ba + 1^{z}Cu1 - 2^{z}Cu + 1^{z}O_{Ba}$
336	336	372	$24^{Z}O_{Cu1} - 1^{Z}Ba - 3^{Z}Cu1 + 1^{Z}Cu - 3^{Z}O_{Ba}$
620	620	619	$15^{Z}O_{Ba} - 7^{Z}Cu1 - 1^{Z}Cu + 1^{Z}O_{Cu1}$
			B <sub>3u</sub>
64	38	114	$5^{X}Y - (3^{X}Ba + 3^{X}O_{Ba}) - (4^{X}Cu1 + 3^{X}O_{Cu1}) + (5^{X}Cu + 5^{X}O_{Cu} + 4^{X}O_{Cu})$
126	126	117	$(2^{X}Ba + 3^{X}O_{Ba}) - (11^{X}Cu1 - 4^{X}O_{Cu1}) - (1^{X}Cu + 1^{X}O_{Cu})$
153	153	125	$8^{X}Y - (5^{X}Cu + 5^{X}O_{Cu} - 2^{X}O_{Cu}') - 1^{X}Ba$
212	212	215	$24^{X}O_{Cu1}-1^{X}Ba-4^{X}O_{Ba}$
262	262	248	$16^{X}O_{Ba} - 2^{X}Ba - 1^{X}O_{Cu}' + 4^{X}O_{Cu1}$
371	371	367	$17^{X}O_{Cu}' - 3^{X}Y - 1^{X}Cu - 2^{X}O_{Cu}$
596	596	596	$16^{X}O_{Cu} - 4^{X}Cu + 1^{X}O_{Cu}$
			$B_{2u}$
65	40	104	$5^{y}$ Y - $(3^{y}$ Ba + $3^{y}$ O <sub>Ba</sub> ) - $(4^{y}$ Cu1 + $4^{y}$ O <sub>Cu1</sub> ) + $(5^{y}$ Cu + $4^{y}$ O <sub>Cu</sub> + $5^{y}$ O' <sub>Cu</sub> )
153	154	270	$4^{y}Y + (3^{y}Ba - 1^{y}O_{Ba}) - (8^{y}Cu1 + 8^{y}O_{Cu1}) - (3^{y}Cu - 2^{y}O_{Cu} + 3^{y}O_{Cu})$
156	155	283	$7^{y}Y - 2^{y}Ba + (5^{y}Cu1 + 6^{y}O_{Cu1}) - (4^{y}Cu - 2^{y}O_{Cu} + 4^{y}O_{Cu}')$
315	315	331	$17^{y}O_{Ba} - 1^{y}Ba - 2^{y}Cu1 - 1^{y}O_{Cu} - 3^{y}O_{Cu1}$
375	375	348	$17^{y}O_{Cu} - 3^{y}Y + 1^{y}Cu - 2^{y}O_{Cu}'$
575	575	512	$16^{y}O_{Cu}^{\prime}-4^{y}Cu+1^{y}O_{Cu}$
593	593	525	$22^{y}O_{Cu1} - 6^{y}Cu1 + 1^{y}O_{Ba}$

and within the three doublets  $(B_{3g}, B_{2u})$ . At the same time, the splitting magnitude is rather large for the three doublets  $(A_g, B_{1u})$ . This gives evidence in favor of a different influence of interlayer interactions on modes with different symmetries.

When comparing the above theory with experiments, we emphasize the following results. The anomalous temperature behaviors at  $T < T_c$  of the vibrations at 335 and 435 cm<sup>-1</sup> in Raman spectra<sup>27,28</sup> as well as the vibrations at 275 and 315 cm<sup>-1</sup> in ir spectra of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7-8</sub>  $(\delta \rightarrow 0)^{29}$  confirm that all these modes are induced by vibrations of the same oxygen atoms, namely of the O<sub>Cu</sub> and O'<sub>Cu</sub> atoms [corresponding frequencies calculated are 335 and 439 cm<sup>-1</sup> ( $A_g$ ), 264 and 295 cm<sup>-1</sup> ( $B_{1u}$ )]. So, these even and odd vibrations do form the doublets.

#### D. The dispersion of vibrational branches

Within the framework of the dynamical model used, the dispersion curves of vibrational branches have also been calculated along symmetry lines in the BZ. Those for directions  $(0,0,\xi)$  and  $(0,\xi,0)$  are given in Fig. 2. The vibration symmetries at the  $\Gamma$ , Z, and Y points of the BZ were obtained in Sec. III.

It follows from Fig. 2 and from Table VI that all highfrequency branches corresponding to intralayer vibrations appear to be practically flat for the  $(0,0,\xi)$  direction. This agrees with the predictions of the group-theory analysis above. At the same time, most of these phonon branches are essentially curved for the  $(0,\xi,0)$  direction. Most vibrations from the low-frequency region (interlayer modes) manifest significant wave-vector dependence for all directions.

Note, the described behavior of the dispersion curves is an inherent property of the different perovskitelike compounds. It has been observed in inelastic neutronscattering experiments for YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>, <sup>30</sup> La<sub>2</sub>CuO<sub>4</sub>, <sup>31</sup> and Nd<sub>2</sub>CuO<sub>4</sub>. <sup>32</sup> Thus, when classifying normal modes into intralayer and interlayer modes, one should take into ac-

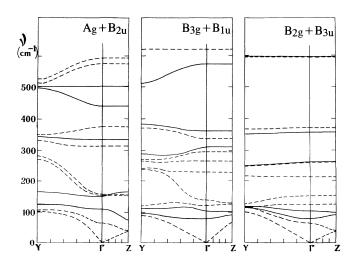


FIG. 2. Calculated dispersion curves corresponding at the BZ center to even (solid lines) and odd (dashed lines) modes in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> along the  $(0, \xi, 0)$ , and  $(0, 0, \xi)$  symmetry lines.

count the following criteria: the character of mode displacement patterns, the dispersion of phonon branches, and the splitting of some normal modes.

#### V. RAMAN SPECTRA OF HTSC-COMPOUNDS

In this paper, we bring together the data on Raman spectra of Y-, Bi-, and Tl-based compounds. According to the interpretation of polarized Raman spectra of various perovskitelike superconductors, the most intense lines are induced by the phonons with  $A_{1g}$   $(A_g)$  or  $B_{1g}$ symmetries. These modes are formed by displacements of atoms in the z direction. Table VII and Fig. 3 show the experimental frequency values of these vibrations in Y-, Tl-, and Bi-based superconductors.<sup>1,6-8,10,33-39</sup>

Raman-active  $E_g$  modes (tetragonal structures) as well as the  $B_{2g}$  and  $B_{3g}$  modes (orthorhombic structures) are formed by displacements of atoms in the x and y directions. The corresponding lines in the Raman spectra are weak. For example, in 1:2:3- compounds these lines are 100 times weaker than the  $A_g$  lines.<sup>40</sup> In the Tl- and some Bi-based compounds, these modes have not been observed.

One can see that it is possible to divide the Raman spectra into low- and high-frequency regions. The lines from the low-frequency region ( $v \le 200 \text{ cm}^{-1}$ ) can be attributed to interlayer vibrations participated in by both the heavy atoms (Bi, Tl, Cu, Ba, Sr) and the O atoms.

The intralayer vibrations are observed in the highfrequency region. The latter region may also be subdivided into a set of spectral subregions. The vibrations of  $O_{Cu}$  atoms lie in the intermediate subregion (250-450 cm<sup>-1</sup>). The out-of-phase  $B_{1g}$  ( $A_g$ ) vibrations have a lower frequency and the in-phase  $A_{1g}$  ( $A_g$ ) vibrations

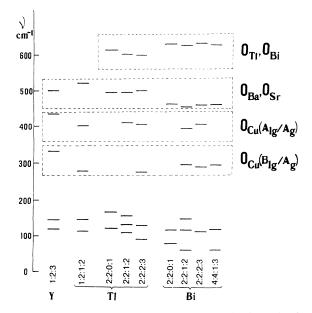


FIG. 3. Frequencies of  $A_{1g}$  ( $A_g$ ) and  $B_{1g}$  ( $A_g$ ) modes in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> (Ref. 33), Tl-1:2:1:2 (Refs. 34,35), Tl-2:2:0:1 (Ref. 36), Tl-2:2:1:2 (Ref. 39), Tl-2:2:2:3 (Ref. 37), Bi-2:2:0:1 (Ref. 20), Bi-2:2:1:2 (Ref. 38), Bi-2:2:2:3 (Ref. 1), Bi-4:4:1:3 (Ref. 10).

				Frequenc	ies $(cm^{-1})$	, assignme	nt, symmet	ry	
		Bi	<b>T1</b>	Ba,Sr	Cu	O <sub>Cu</sub>	$O_{Cu}$	$O_{Ba,Sr}$	O <sub>T1, Bi</sub>
Compound	Ref.	$A_{1g}, A_g$	$A_{1g}$	$A_{1g}, A_g$	$A_{1g}, A_g$	$B_{1g}, A_g$	$A_{1g}, A_g$	$A_{1g}, A_g$	$A_{1g}, A_g$
Y-1:2:3	6			117	148	335	438	502	
Y-1:2:3	7			119	145	336	435	499	
Y-1:2:3	33			118	145	335	440	500	
T1-1:2:1:2	34			116	146		403	520	
Tl-1:2:1:2	35			120	148	278		525	
T1-2:2:0:1	36		167	122				495	612
Tl-2:2:1:2	39		130	108	158	316	407	494	<b>59</b> 9
Tl-2:2:1:2	34		132	109	158		410	495	602
Tl-2:2:2:3	37		129	92		275	405	498	599
Bi-2:1:0:0	8	83							651
Bi-2:2:0:1	20	80		118				461	629
Bi-2:2:1:2	38	62		118	148	296	395	456	626
Bi-2:2:2:3	1			115		290	406	460	633
Bi-4:4:1:3	10	63		120		295		461	627

TABLE VII. Raman line frequencies and their assignments.

have a higher frequency.

In the higher-frequency subregion of the Raman spectra  $(450-520 \text{ cm}^{-1})$ , there are  $A_{1g}$   $(A_g)$  vibrations of  $O_{Ba}$  atoms in Y- and Tl-based superconductors, and of  $O_{Sr}$  atoms in Bi-based compounds. These are so-called bridge oxygen atoms connecting two Cu atoms, or Cu and Tl, or Cu and Bi in the corresponding systems. Finally, the highest-frequency  $A_{1g}$   $(A_g)$  modes  $(600-650 \text{ cm}^{-1})$  involve the vibrations of  $O_{Tl}$  and  $O_{Bi}$  atoms.

It is clearly seen that the frequencies of the intralayer vibrations of oxygen atoms belonging to a particular layer vary insignificantly for crystals not only within the Bibased superconductor family<sup>1</sup> but also within the entire group of compounds under consideration (i.e., for Y-, Tl-, and Bi-based families). Therefore, these frequencies can be considered to be the signatures of each layer.

# VI. QUASI-2D BEHAVIOR OF THE PHONON SUBSYSTEMS OF PEROVSKITELIKE SUPERCONDUCTORS

Now, let us summarize the main characteristics of the phonon subsystems of perovskitelike superconductors, following from the layered structure of these materials. To perform a group-theory analysis of normal vibrations (Sec. III), we considered isolated layers, determined the symmetry of acoustic and optical layer modes, and established a correspondence between layer and bulk vibrations. As a result, two important conclusions have been made. First, some intralayer modes split into Ramanactive and ir-active components and, second, the intralayer modes are nearly dispersionless along the  $\Gamma$ -Z symmetry line in the BZ.

On the other hand, the lattice dynamical model of  $YBa_2Cu_3O_7$  was chosen without an *a priori* assumption of a layered character of the structure. Nevertheless, the results of lattice dynamical calculations (Sec. IV) completely confirmed the group-theory predictions. What is more important, both the conclusions of the group-theory

analysis and the results of lattice dynamical calculations are confirmed by experiments on inelastic neutron scattering as well as by optical measurements. Thus, the validity of both the layer approach and the chosen separation of perovskitelike crystal lattices into single layers has been justified.

Another significant result is the small variation of intralayer phonon frequencies when passing from one compound to another within the Y-, Tl-, and Bi-based compounds considered (Sec. V).

Summing up, the conclusion can be made that the layered structure of the perovskitelike compounds is responsible for the principal characteristics of their phonon subsystems, and that the phonon subsystems manifest quasi-2D behavior.

## VII. SANDWICH CONCEPTION OF HIGH-TEMPERATURE SUPERCONDUCTIVITY

The strong anisotropy of the high- $T_c$  superconductors manifests itself in their transport, elastic, and optical properties. To comprehend better the special features of these compounds, one should consider the idea of perovskitelike superconductors as compounds with quasi-2D properties. The structure of cuprate superconductors can be considered as a set of conducting CuO<sub>2</sub> layers "sandwiched" by block layers.<sup>11</sup>

The above analysis shows the phonon subsystem to have quasi-2D character. Weak coupling between the conducting  $CuO_2$  layers leads to quasi-2D behavior of the electron subsystem too. This manifests itself particularly in the 2D character of the electron bands found by band numerical calculations (see, e.g., Refs. 19,41).

Since the thickness of the blocking-layer stacks separating the adjacent  $CuO_2$  layers does not exceed 10-20 Å, the sandwich conception of Ref. 42 is applicable. This conception has been widely discussed with respect to layered superconductors.<sup>43</sup> It supposes the conducting layers to be separated by insulating regions.

In this case, phonons and electron excitations in the blocking-layer stack may play the role of coupling bosons.

The correlation<sup>44,45</sup> between the critical temperature  $T_c$  and frequencies of intense Raman lines in perovskitelike superconductors seems to be a pronounced manifestation of the importance of nonpolar phonons in the high- $T_c$  superconductivity mechanism. The essence of this correlation is the regular increases both in  $T_c$  and in the characteristic Raman frequencies within the row La- $\rightarrow Y - \rightarrow Bi - \rightarrow Tl$ -based superconductors. We emphasize that the highest-frequency intense lines in the Raman spectra are caused by the  $A_{1g}$  ( $A_g$ ) vibrations of oxygen atoms in the layers separating copper oxide planes (see Sec. V). Indeed, this vibration is related to oxygen atoms in the LaO layers in La-based compounds, in the BaO layers in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>, and in the BiO (TlO) layers in Bi (Tl) compounds.

We believe that the correlation is not accidental and that it reflects a common proportionality between  $T_c$  and an average frequency describing the whole vibrational spectrum.<sup>43,46,47</sup> The correlation was explained in Refs. 48,49. In Ref. 48, the transition temperature  $T_c$  was calculated within the framework of the Eliashberg theory,<sup>50</sup> the frequencies of nonpolar (Raman-active) phonons being taken as a basis. The main result of Ref. 48 consists in a good agreement between experimental and calculated  $T_c$  for all superconductor families at the same  $\lambda \approx 1.5-2$  and  $\mu^* \approx 0.1-0.2$ .

Such a value of the coupling constant  $\lambda$  is reasonable. Indeed, there are a number of estimations of  $\lambda$ . The analysis of results of very different experiments gives  $\lambda \simeq 1-3$ . According to tunneling experiments,  $\lambda \simeq 2.04$ ,<sup>51</sup>  $\lambda \simeq 3.5$ .<sup>52</sup> Measurements on the electronic specific heat give  $\lambda \simeq 2$ .<sup>53</sup> To describe the ir properties of high- $T_c$  superconductors, the electron-boson coupling constant  $\lambda$ was supposed to be close to 2 in Ref. 54. According to the results of electron-spin resonance investigations, the parameter  $\lambda$  is 1.65.<sup>55</sup> From the resistivity data on single crystals together with results of optical investigations,  $\lambda$ was estimated to be about 1.5–2 in Ref. 56.

The correlation between  $T_c$  and Raman frequencies<sup>44,45</sup>

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seems to be a result of a common superconductivity mechanism for all high- $T_c$  superconductors. This arises from the presence of the same CuO<sub>2</sub> layers (which determine the properties of the quasi-2D electron subsystem) and from the dissimilarity of the separating layers (which determine important properties of the quasi-2D phonon subsystem).

An advantage of sandwich systems is the possibility of the selection of layers with the most favorable properties. Thus, the electron and phonon subsystems can be optimized practically independently.<sup>43</sup> The idea of such an optimization relevant to high- $T_c$  superconductors was formulated in Ref. 45. According to Ref. 45, the phonon subsystem determines the maximum value of the critical temperature within each high- $T_c$  superconductor family. Within each of the families,  $T_{c,max}$  is realized in a compound with an optimized electron subsystem, in particular with the carrier density  $n \simeq 10^{21}$  cm<sup>-3</sup>.<sup>45,57</sup> The existence of such a maximum in the  $T_c$  vs *n* dependence can be explained within the framework of the phonon mechanism of superconductivity, if quasi-two-dimensionality of the electron subsystem is assumed.<sup>58</sup>

## **VIII. CONCLUSIONS**

Paying special attention to the layered structure of the crystal lattices of high- $T_c$  superconductors, we have analyzed step by step their crystal structures, phonon symmetry properties, vibrational spectra, and the role of phonons in the superconductivity mechanism. As a result, the quasi-2D nature of the phonon subsystem of perovskitelike superconductors has been established. The differentiation of roles of the various layers in creating the high- $T_c$  superconductivity phenomenon has been discussed.

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