Universal conductivity in the boson Hubbard model in a magnetic field

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The universal conductivity at the zero-temperature superconductor-insulator transition of the two-dimensional boson Hubbard model is studied for cases both with and without magnetic field by Monte Carlo simulations of the (2+1)-dimensional classical XY model with disorder represented by random bonds correlated along the imaginary time dimension. The effect of the magnetic field is characterized by the frustration f. From the scaling behavior of the stiffness, we determine the quantum dynamical exponent z, the correlation length exponent ν, and the universal conductivity σ^*. For the disorder-free model with $f = 1/2$, we obtain $z \approx 1$, $1/\nu \approx 1.5$, and $\sigma^*/\sigma_Q = 0.52 \pm 0.03$, where σ_Q is the quantum conductance. We also study the case with $f = 1/3$, in which we find $\sigma^*/\sigma_Q = 0.83 \pm 0.06$. The value of σ^* is consistent with a theoretical estimate based on the Gaussian model. For the model with random interactions, we find $z = 1.07 \pm 0.03$, $\nu \approx 1$, and $\sigma^*/\sigma_Q = 0.27 \pm 0.04$ for the case $f = 0$, and $z = 1.14 \pm 0.03$, $\nu \approx 1$, and $\sigma^*/\sigma_Q = 0.49 \pm 0.04$ for the case $f = 1/2$.

I. INTRODUCTION

Through recent experimental studies in various two-dimensional systems,[1–9] the presence of a zero-temperature phase transition between the superconducting phase and the insulating phase has been convincingly suggested.[10,11] The transition has been tuned by changing parameters, such as the thickness of homogeneous films,[1–3] the strength of charging energy in Josephson junction arrays,[4] the strength of disorder,[5] or the magnitude of the external magnetic field applied perpendicular to the two-dimensional systems.[6–9] We view this transition as a boson localization problem, in which electron pairs are treated as point bosons small on the scale of the diverging phase correlation length near the transition. This picture is certainly suitable in granular superconductors and Josephson junction arrays where the electron-pair wave functions are well defined in each grain and the superconductivity is obtained by the establishment of a long-range phase correlation among them. Recently it has been proposed[12,13] that even in homogeneous films, the zero-temperature superconductor-insulator transition will be controlled by phase fluctuations of the electron-pair wave function. The picture is based on the assumption that below a mean-field temperature, electrons form pairs but the true superconducting temperature is lowered by the enhanced phase fluctuations in low-dimensional systems. The size of the electron pairs is finite at the transition. Hebard and Paalanen[7,14] reported some evidence that in homogeneous films, such as composite InO_x films, the magnitude of the superconducting order parameter is finite at the transition, supporting the idea that the relevant fluctuations at the transition are the long wavelength twists in the phase of the pair wave function. We cannot rule out the possibility, however, that in these systems the transition might possibly be induced by the vanishing of the magnitude of the order parameter.[15] We will make the assumption throughout this paper that, below some characteristic mean-field temperature, a fermionic gap forms freezing out the fermion degrees of freedom. Below this temperature a bosonic (i.e., phase fluctuation) model is presumed to be valid. Our simulations study the zero-temperature quantum critical point. Real experiments are of course carried out at finite temperature, but scaling of the data can be used to demonstrate that one is probing the zero-temperature quantum critical regime.[12,13,10]

Based on the assumption that the transition is a continuous phase transition and that the phase fluctuations of the superconducting order parameter give the primary contribution to the singular part of the free energy, scaling theories have predicted some universal properties of the transition such as critical exponents[16,17] and dimensionless combinations of critical amplitudes.[18] In particular, the conductivity at the zero-temperature superconductor-insulator transition has been predicted[19] to be a universal number, which is analogous to the universal jump of the superfluid density at Kosterlitz-Thouless transition. Experimentally measured values of resistivity (or resistance per square) at very low temperature in the vicinity of the superconductor-insulator transition are close to the quantum resistance,[1,4–6] $R_Q \equiv h/(2e)^2$, in various systems, even though no consensus concerning whether the temperature is low enough to reach the critical regime and whether the measured values are truly universal (i.e., independent of material parameters) has emerged yet. Theoretically a model of interacting bosons moving in the presence of disorder has been proposed to capture the appropriate univer-

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sality class.[16,19] Considerable theoretical effort has been devoted to the study of the phase transition in this model by Monte Carlo simulations[20,21] and by renormalization group calculations.[22,23] Also the universal conductivity was calculated in the various versions of the model through different techniques.[11,13,24–27] Even though those calculations support the existence of a *finite* critical conductivity at the transition, the calculated values are either inconsistent among different studies or a factor of 2–3 away from experimental values. So far, theoretical studies mostly concentrated on the case without magnetic field. Here we want to study the phase transition and the universal conductivity in an interacting boson model in the presence of an external magnetic field. We also consider the effect of disorder given in the form of a short-range repulsive random interaction.

This zero-temperature phase transition can be understood by adopting the charge-vortex duality picture.[12] Near the transition, vortices and antivortices are induced by quantum fluctuations. In the superconducting state, delocalized bosons condense whereas vortices are localized, yielding no dissipation. On the other hand, in the insulating phase, bosons are localized and vortices condense. According to this scenario, at the superconductor-insulator transition both bosons and vortices are expected to be mobile,[13] yielding a finite conductivity because the motion of charged bosons carries current while that of vortices produces voltage. The external magnetic field changes the density of vortices and breaks time reversal symmetry of the vortex distribution thereby modifying the universality class.

In this work, we study the transition in a "phase-only" model. Here we argue that fluctuations of the magnitude of the superconducting order parameter are irrelevant near the transition because they cost more energy than the fluctuations of the phase. This picture implies that there are coarse-grained boson islands in which lots of bosons condense to have well-defined phase angles locally. These islands are connected through interisland hopping. In this picture the phase angle is the only dynamical variable and the number operator and the phase operator are conjugate variables. The transition is determined by the competition of the density fluctuations and the phase angle fluctuations. Strong disorder and interactions in general suppress the density fluctuations locally and cause more fluctuations in phase angle. The external magnetic field causes a frustration in the phase couplings in this model.

As we show below, one can avoid the problems of complex phase factors associated with the magnetic field by adopting a phase representation. However we also want to consider the effect of random disorder. Unfortunately disorder generates complex weights in the phase representation and is normally treated in a boson world-line representation.[10,11,20,21] To circumvent this difficulty we introduce a new universality class having disorder in the form of a random Hubbard U on each site. This class of models maintains particle-hole symmetry on each site and can be treated in the phase representation.

More explicitly, we consider a lattice boson Hubbard model with random on-site interaction:

$$\mathcal{H} = \sum_i U_i(n_i - n_0 - \mu)^2 - \sum_{\langle ij \rangle}(t_{ij} b_i^\dagger b_j + t_{ji} b_j^\dagger b_i), \quad (1)$$

where b_i ($b_i^\dagger$) is the boson annihilation (creation) operator at site i, n_i is the boson number operator at site i, U_i is the interaction between bosons at site i, t_{ij} is the nearest-neighbor hopping matrix element, and $n_0 + \mu$ (n_0 is an integer and $-1/2 < \mu < 1/2$) is the parameter which determines the average background boson density. In the presence of an external magnetic field, we have

$$t_{ij} = t_0 e^{iA_{ij}}, \quad (2)$$

$$A_{ij} = \frac{2\pi}{\Phi_0} \int_{x_i}^{x_j} \mathbf{A}(\mathbf{x}) \cdot d\mathbf{x}, \quad (3)$$

where Φ_0 is the flux quantum and $\mathbf{A}(\mathbf{x})$ represents the vector potential. When there is a well-defined phase angle θ_i at each site i,

$$n_i = \frac{1}{i}\frac{\partial}{\partial \theta_i} \quad (4)$$

and

$$b_i \approx \sqrt{n_0} e^{i\theta_i} \quad (5)$$

if $n_0 \gg 1$. The model then reduces to the quantum rotor model

$$\mathcal{H} = \sum_i U_i(n_i - \mu)^2 - \sum_{\langle ij \rangle} J \cos(\theta_i - \theta_j - A_{ij}), \quad (6)$$

where $J = 2n_0 t_0$. Here we have shifted $n_i - n_0 \to n_i$. When $\mu = 0$, we have the corresponding (2+1)-dimensional classical action, $S[\theta]$, defined by

$$Z = \mathrm{Tr}_{\{\theta\}} e^{-\beta\mathcal{H}} = \mathrm{Tr}_{\{\theta\}} e^{-S[\theta]}, \quad (7)$$

which is obtained by the path integral transformation[28]

$$S[\theta] = -\sum_r [K_\tau(i)\cos(\Delta_\tau \theta_r) + K\cos(\Delta_x \theta_r) + K\cos(\Delta_y \theta_r)], \quad (8)$$

where i and τ denote position on the spatial plane and the (imaginary) temporal axis, respectively, $r \in (i, \tau)$ is a point in the (2+1)-dimensional lattice, and $\Delta_\tau \theta_r = \theta_{r+\hat{\tau}} - \theta_r$, $\Delta_x \theta_r = \theta_{r+\hat{x}} - \theta_r - A_{r+\hat{x},r}$, etc. $K_\tau(i)$ is the coupling parameter along the (imaginary) temporal axis, depending only on the spatial position. Here we restrict the model to the particle-hole symmetric case $\mu = 0$, since otherwise the action is complex. The randomness of U_i in Eq. (6) is transformed into the randomness of K_τ. The frustration f of the model is defined by the summation of the phase change around each plaquette so that

$$2\pi f = A_{ij} + A_{jk} + A_{kl} + A_{li}. \quad (9)$$

This model is equivalent to a model high-T_c superconductor with columnar defects along the magnetic field recently studied by Lee, Stroud, and Girvin.[29]

In Sec. II, we describe the properties of the model, including scaling hypotheses necessary to analyze the numerical data. We then study the pure model, in which $K_\tau(i) = K$, with frustration $f = 1/2$ and $f = 1/3$ in Sec. III. A random-U model where K_τ are random numbers bounded by $0 < K_\tau/K < 2$, with frustration $f = 0$ and $f = 1/2$ is also studied. Section IV is reserved for the conclusions.

II. PROCEDURE

The phase transition of the model defined in Eq. (8) is studied by Monte Carlo simulations. The two-dimensional $T = 0$ *quantum* path integral has been mapped onto the (2+1)-dimensional *classical* frustrated XY model with random bonds in the third direction. We use the standard Metropolis algorithm with local moves of single-angle variables. The angles were discretized into 4000 steps so that Boltzman factors for moves could be precomputed. The number of Monte Carlo sweeps used in each case is discussed further below.

We take simple cubic (2+1)-dimensional lattices whose sizes and aspect ratios are systematically changed to fit data using finite-size scaling. Periodic boundary conditions are imposed and the ordinary Landau gauge is used for the phase change due to the magnetic field. We change the coupling strength parameter K to tune the transition of the model.

The stiffness to a twist at the boundaries is measured to find the transition point and the critical exponents. The stiffness along a spatial direction is proportional to the superfluid density, and has a finite value in the superfluid phase but vanishes in the insulating phase. In a finite-size system, however, it vanishes smoothly and there is a residual superfluid density at the transition point, whose magnitude depends on the size of the system. From the size dependency, therefore, we can use standard finite-size scaling analysis[13,30] to find the scaling behavior by changing the system size while keep the same geometry of the system. Since the model is anisotropic in the presence of the correlated disorder along the imaginary time dimension or the frustration in the spatial planes, the diverging behaviors of correlation lengths near critical point are not isotropic in general, but they are related through the dynamical scaling hypothesis

$$\xi_\tau \sim \xi^z, \tag{10}$$

where z is the quantum dynamical exponent. This implies that in order to simulate the model to find the asymptotic scaling in a finite-size system in which correlation lengths are limited by the size of the system, we need to take lattices with

$$L_\tau = A_s L^z, \tag{11}$$

where A_s is the aspect ratio, and L and L_τ are the sizes in the spatial directions and in the imaginary time direction, respectively. In this work, we use lattices with $L_x = L_y = L$ and L_τ which is determined by Eq. (11).

In order to determine the different sizes of lattice with the same aspect ratio, therefore, information about the value of z is prerequisite. For the pure boson Hubbard model with the particle-hole symmetry, it is known that $z = 1$.[16] Quenched disorder or long-range interactions, which alter the diverging behavior of correlation lengths, will change z.[16,19] For the model we are considering, we expect that impurities hinder the correlation along the spatial directions so that the correlation length along the spatial directions will diverge more slowly than that along the imaginary time direction. This certainly implies that $z > 1$. The microscopic particle-hole symmetry property of the model $[\partial/i\partial\theta_j \leftrightarrow -\partial/i\partial\theta_j$ in Eq. (6)] implies that the occupation of the lowest delocalized eigenstate (i.e., the mobility edge state) does not change the total number of particles at the brink of the superconductor-insulator transition. Since the compressibility has a form

$$\kappa \sim \xi^{z-d}, \tag{12}$$

this condition implies that $z < d$ where d is the spatial dimension. Thus the model is different from that with random on-site chemical potential with a short-range interaction, where it is expected the compressibility of the system is finite even at the transition and $z = d$. For the model we consider, therefore, we expect

$$1 < z < 2. \tag{13}$$

Due to the lack of the information about an exact value of z for the model we are studying, we have tried several numbers for z. We also find z by directly measuring the correlation functions. The correlation functions along one spatial direction, say x, and along the imaginary time direction have asymptotic behavior[10,33]

$$\begin{aligned} G_x(r) &\sim r^{-y_x}, \\ G_\tau(r) &\sim r^{-y_\tau}, \end{aligned} \tag{14}$$

at the transition point, respectively, where $y_x = d - 2 + z + \eta$ and $y_\tau = (d - 2 + z + \eta)/z$. The value of z is obtained from $z = y_x/y_\tau$. However in finite-size systems with periodic boundary conditions, instead of Eq. (14) we use the form[33]

$$\begin{aligned} G_x(r) &= C_x[r^{-y_x} + (L-r)^{-y_x}], \\ G_\tau(r) &= C_\tau[r^{-y_\tau} + (L_\tau - r)^{-y_\tau}], \end{aligned} \tag{15}$$

where C_x and C_τ are fitting constants. The results are discussed in the next section. Also one might expect the magnetic field perpendicular to the two-dimensional system to change the exponent z. We find that the external magnetic field has little effect on z for the pure model and only weakly changes z in the random-U model. We have no rigorous argument for why this should be so in the presence of disorder. However, for the disorder-free case we present in the next section a simple argument motivating this result.

The frequency-dependent stiffness is given by[13]

$\rho_{\mu\nu}(i\omega_n)$

$$= \frac{1}{L_x L_y L_\tau}[K_\nu \langle \epsilon_\nu \rangle \delta_{\mu\nu} - K_\mu K_\nu \langle J_\mu^*(i\omega_n) J_\nu(i\omega_n) \rangle + K_\mu K_\nu \langle J_\mu^*(i\omega_n) \rangle \langle J_\nu(i\omega_n) \rangle]_{\rm avg} \quad (16)$$

with

$$\epsilon_\nu = \sum_r \cos(\Delta_\nu \theta_r) \quad (17)$$

and

$$J_\nu(i\omega_n) = \sum_r \sin(\Delta_\nu \theta_r) e^{i\omega_n \tau}, \quad (18)$$

where the frequency ω_n is the wave number in the τ direction. In Eq. (16), $\langle \cdots \rangle$ represents the thermal average and $[\cdots]_{\rm avg}$ represents the ensemble average over the impurity configurations. In this work, the contribution of the last term in Eq. (16) drops out because Lf is an integer in the pure model or because there is vortex-antivortex symmetry (i.e., $f = 0$ or $f = 1/2$) in the disordered model.

The finite-size scaling ansatz for $\rho_{xx}(0)$ is given by[13]

$$\rho_{xx}(0) = \frac{1}{L_\tau} \tilde{\rho}_{xx}(tL^{1/\nu}, L_\tau / L^z), \quad (19)$$

where $t = K - K^*$ (K^* is the critical coupling), ν is the correlation length critical exponent ($\xi \sim t^{-\nu}$), and $\tilde{\rho}_{xx}$ is a universal scaling function. Near the transition,

$$L_\tau \rho_{xx}(0) = \tilde{\rho}_{xx}(0, L_\tau / L^z) + AtL^{1/\nu} + \cdots, \quad (20)$$

where $\tilde{\rho}_{xx}(0, L_\tau / L^z)$ is a universal number, and A is a nonuniversal constant. Similar behavior holds for $(L^2/L_\tau)\rho_{\tau\tau}(0)$. We use these properties to find the transition point, the quantum dynamical exponent, z, and the critical exponent, ν, in the next section.

The conductivity is obtained through the Kubo formula[13]

$$\sigma_{\mu\nu} = 2\pi \sigma_Q \lim_{\omega_n \to 0} \frac{\rho_{\mu\nu}(i\omega_n)}{\omega_n}, \quad (21)$$

where $\sigma_Q = (2e)^2/h$. It is a (scaling) dimensionless quantity and has a universal value at the transition. In practice, however, the finite-size contribution, which gives the residual superfluid density, must be subtracted to find the universal behavior of the conductivity at the transition. Because of the frequency dependence and the finite-size contribution, we expect the conductivity has a scaling form[13]

$$\frac{\sigma(i\omega_n)}{\sigma_Q} = \frac{\sigma^*}{\sigma_Q} - c\left(\frac{\omega_n}{\omega_0} - \alpha \frac{\omega_0}{\omega_n L_\tau}\right) + \cdots, \quad (22)$$

where ω_0 is a given frequency such as the ultraviolet cutoff frequency. We set $\omega_0 = 2\pi/a$ in this work, where a is the lattice constant. We take the parameter α which gives the smallest deviation among the interpolated scaling curves for the different size systems.

III. RESULTS

A. Pure model with $f = 1/2$ and $f = 1/3$

First, we consider the case of the pure boson model with frustration. We first have to check if the transition is first order or continuous in this case, because the scaling argument which supports the universal conductivity is based on the assumption of the continuous transition. A recent Monte Carlo calculation of the case $f = 1/6$ on a triangular lattice suggests that the transition is first order.[31] We use the Lee and Kosterlitz method[32] to check the nature of the transition at larger values of f. Figure 1 is the probability distribution of energy, $p(e)$, for $f = 1/2$ and $f = 1/3$, where $p(e) = e^{-KE}/{\rm Tr}_{\{E\}} e^{-KE}$ and e is the energy per volume. If the transition is first order, we expect a double-well-type curve, in which an energy barrier between the two wells separates two phases. In finite systems, the energy barrier might be disguised by a finite-size effect that will be reduced as the size of system increases so that the energy barrier increases as the size of system increases. We do not have any energy barrier increasing as the size of the system increases, and the result supports a continuous phase transition for both $f = 1/2$ and $f = 1/3$.

Figure 2 shows the scaling behavior of $L_\tau \rho_{xx}(0)$ and $(L^2/L_\tau)\rho_{\tau\tau}(0)$ for the pure (2+1)-dimensional XY model with frustration $f = 1/2$ for systems of different sizes. We have taken averaging steps over 400 – 600 blocks which contains 1000 measurement sweeps after 2000 thermalization sweeps. We assume that $z = 1$, which is supported by the correlation function calculation (see Fig. 3). The curves cross around $K^* = 0.707 \pm 0.001$. The slopes of the curves give us information about the correlation length critical exponent ν. The inset of Fig. 2 is the finite-size scaling of $L_\tau \rho_{xx}(0)$ with respect to the single variable $(K - K^*)L^{1/\nu}$. With $1/\nu = 1.5 \pm 0.3$, good scaling is obtained. This value of ν is very close to ν in the unfrustrated three-dimensional XY model.[34] The

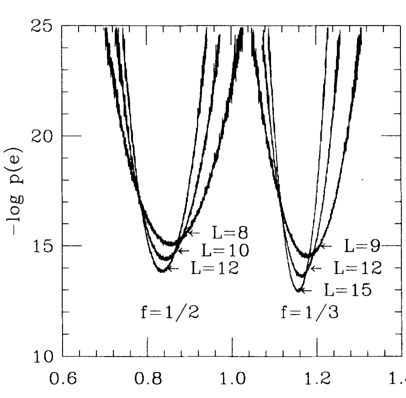

FIG. 1. Probability distribution of energy in Monte Carlo simulation of the pure boson model for the case with $f = 1/2$ and $f = 1/3$ for different size systems indicated in the figure. The absence of an energy barrier separating two phases shows that the transition is continuous.

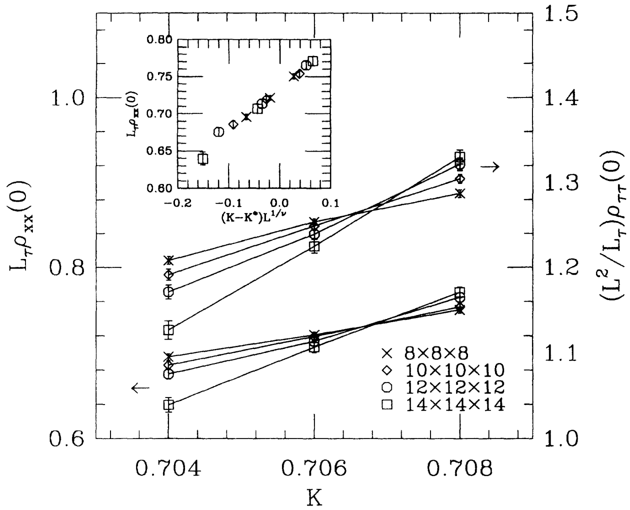

FIG. 2. Finite-size scaling behavior of $L_\tau \rho_{xx}(0)$ and $(L^2/L_\tau)\rho_{\tau\tau}(0)$ for the pure (2+1)-dimensional XY model with $f = 1/2$. The transition point determined from these crossing curves is $K^* = 0.707 \pm 0.001$. Inset: The scaling behavior of $L_\tau \rho_{xx}(0)$ with respect to the single variable $(K - K^*)L^{1/\nu}$. Here we use $K^* = 0.7068$ and $1/\nu = 1.5$.

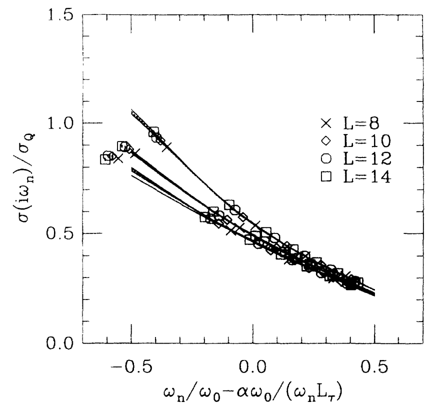

FIG. 4. Finite-size scaling curves of $\sigma(i\omega_n)/\sigma_Q$ for the pure (2+1)-dimensional XY model. From the top, the sets of curves represent the interpolated curves for $K = 0.704, 0.706$, and 0.708, respectively, obtained for different size systems. Here we choose $\alpha = 0.68, 0.61$, and 0.48, respectively. From these curves, the dc conductivity is determined by the value at $\omega_n/\omega_0 - \alpha\omega_0/(\omega_n L_\tau) = 0$. The universal conductivity obtained at the transition point is $\sigma^*/\sigma_Q = 0.52 \pm 0.03$ in this case.

scaling behavior of the conductivity is shown in Fig. 4. From these curves, we obtain $\sigma^*/\sigma_Q = 0.52 \pm 0.03$. The error range is mostly due to the uncertainty in the critical coupling, K^*. Compared to the unfrustrated case where $\sigma^*/\sigma_Q = 0.285 \pm 0.02$,[13] the universal conductivity for the case $f = 1/2$ is almost doubled. This result was predicted by Granato and Kosterlitz[35] based on a picture of coupled XY models.[36] For the fully frustrated case ($f = 1/2$), there are two degenerate ground state configurations. Thus when we construct an effective Ginzburg-Landau action, we need two complex fields (or two species of bosons) fluctuating around each ground state configuration. If we neglect the couplings between fields, we have the Gaussian model. The universal conductivity is proportional to the number of boson species in the Gaussian model. Since the $1/N$-expansion calculation of the universal conductivity[13] supports the notion that the Gaussian model is good as a first approximation, the universal conductivity of the fully frustrated case will be estimated to be doubled compared to the unfrustrated case. Our calculation supports this conjecture. Furthermore one sees that at the Gaussian level, while the conductivity is doubled, the exponent z is unchanged. This is also consistent with our observation.

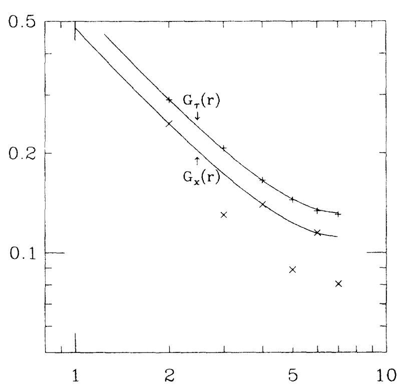

FIG. 3. Spin-spin correlation function of the pure (2+1)-dimensional XY model along one spatial axis, say x, and along the imaginary time axis for frustration $f = 1/2$. (×) and (+) represent actual data points and the solid lines represent the interpolated curves. Because of the frustration the spin-spin correlation function along a spatial axis oscillates slightly. The interpolated curve of $G_x(r)$ is the correlation function for spins separated by even numbers of lattice constants. The parameters used in the interpolation are $C_x = 0.4495$, $y_x = 1.071$, $C_\tau = 0.5341$, and $y_\tau = 1.075$. From the ratio of y_x and y_τ, the dynamical exponent is determined to be $z \approx 1$.

We also study the case with $f = 1/3$, obtaining that $K^* = 0.841 \pm 0.03$ and $\sigma^*/\sigma_Q = 0.83 \pm 0.06$. Again z is assumed to be 1, which is consistent with a calculation of the correlation function, and we find $1/\nu \approx 1.5$. The universal conductivity in this case is approximately tripled compared to the unfrustrated case, which is consistent with the existence of three different ground state configurations.

B. Random-U model

For the (2+1)-dimensional XY model with random correlated bonds in the imaginary time direction, we have tried several values for the quantum dynamical exponent, z. Since the lattice sizes along the imaginary time direction determined by Eq. (11) are in general noninteger numbers, we adjust the aspect ratio to make the lattice sizes as close to integers as possible. More than 1000 dis-

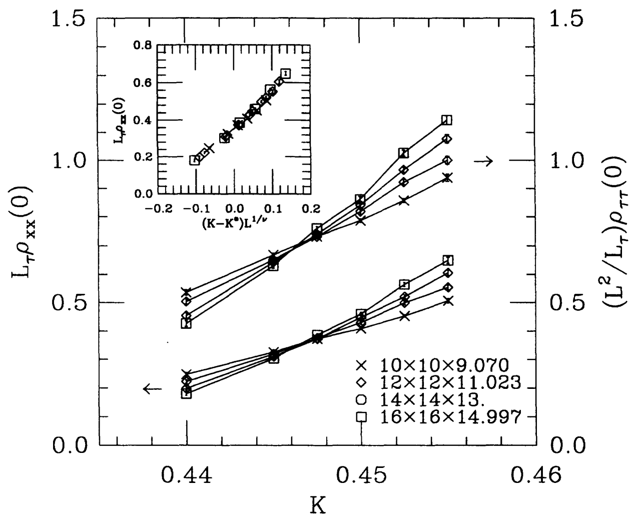

FIG. 5. Finite-size scaling behavior of $L_\tau\rho_{xx}(0)$ and $(L^2/L_\tau)\rho_{\tau\tau}(0)$ for the (2+1)-dimensional XY model with correlated random bonds along the imaginary time axis. The data points for the systems with nonintegral size are obtained by the linear interpolation of the data obtained for systems which have nearby integer sizes. The transition point determined from these crossing curves is $K^* = 0.4465 \pm 0.025$. Inset: The scaling behavior of $L_\tau\rho_{xx}(0)$ with respect to the single variable $(K - K^*)L^{1/\nu}$. $K^* = 0.4465$ and $1/\nu = 1.0$ are used.

order configurations are taken near the transition. For each disorder realization, 2000 thermalization sweeps and 1000 measurement sweeps were used. Statistical errors are dominated by fluctuations between different disorder realizations rather than by thermal fluctuations for a given disorder. Hence we found it useful to increase the number of disorder realization at the expense of shorter thermal averaging times.

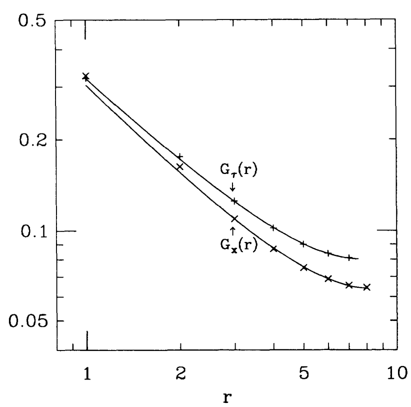

FIG. 6. Spin-spin correlation function of the (2+1)-dimensional XY model along one spatial axis, say x, and along the imaginary time axis. (×) and (+) represent actual data points and the solid lines represent the interpolated curves. The parameters used in the interpolation are $C_x = 0.2865$, $y_x = 1.054$, $C_\tau = 0.2975$, and $y_\tau = 0.994$. From the ratio of y_x and y_τ, the dynamical exponent is determined to be $z \approx 1.06$.

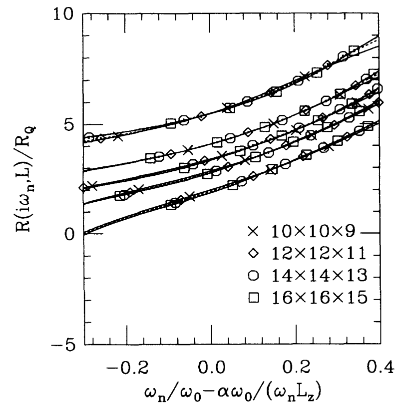

FIG. 7. Finite-size scaling curves for the resistivity, $R(i\omega_n)/R_Q$, for the (2+1)-dimensional XY model with random bonds correlated along the imaginary time axis. From the top, the sets of curves represent the interpolated curves for $K = 0.440$, 0.445, 0.4475, 0.450, and 0.455, respectively. We choose $\alpha = 0.88$, 0.55, 0.39, 0.28, and 0.16, respectively. The universal conductivity determined from these curves is $\sigma^*/\sigma_Q = 0.27 \pm 0.04$.

When $f = 0$, we have the best scaling behavior near $z \approx 1.07$. Figure 5 shows that, in this case, the scaling curves of $L_\tau\rho_{xx}(0)$ and $(L^2/L_\tau)\rho_{\tau\tau}(0)$ for systems with the sizes of $10 \times 10 \times 9.070, 12 \times 12 \times 11.023, 14 \times 14 \times 13,$ and $16 \times 16 \times 14.997$ cross around a transition point $K^* = 0.4465 \pm 0.025$. Since the stiffness depends weakly on the aspect ratio, these curves were obtained by linear interpolation of the data from the systems which have sizes corresponding to the nearby integers. Here the aspect ratio is $A_s = 0.772$. For significantly dif-

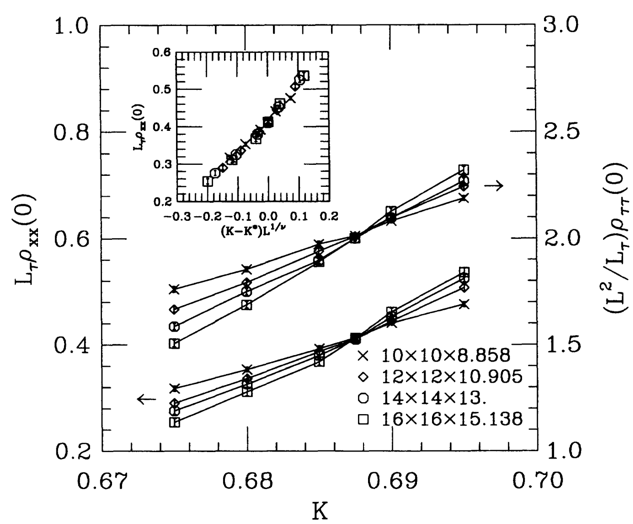

FIG. 8. Finite-size scaling behavior of $L_\tau\rho_{xx}(0)$ and $(L^2/L_\tau)\rho_{\tau\tau}(0)$ for the (2+1)-dimensional XY model with correlated random bonds along the imaginary time axis with frustration $f = 1/2$ in the spatial plane. The transition point determined from these crossing curves is $K^* = 0.6875 \pm 0.025$. Inset: The scaling behavior of $L_\tau\rho_{xx}(0)$ with respect to the single variable $(K - K^*)L^{1/\nu}$. $K^* = 0.6875$ and $1/\nu = 1.0$ are used.

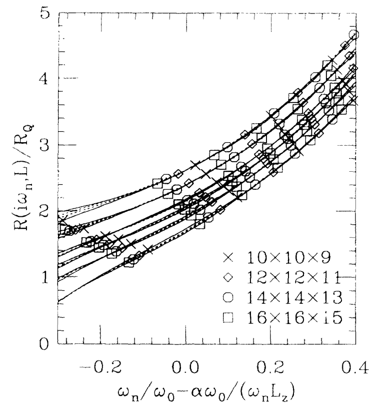

FIG. 9. Finite-size scaling curves of the resistivity, $R(i\omega_n)/R_Q$, for the (2+1)-dimensional XY model with random bonds correlated along the imaginary time axis with $f=1/2$. From the top, the sets of curves represent the interpolated curves for $K=0.670, 0.680, 0.685, 0.6875, 0.690$, and 0.695. We choose $\alpha=0.40, 0.35, 0.30, 0.27, 0.24$, and 0.20, respectively. The universal conductivity determined from these curves is $\sigma^*/\sigma_Q=0.49\pm0.04$.

ferent z, either we cannot obtain the crossing behavior at all or $L_\tau\rho_{xx}(0)$ and $(L^2/L_\tau)\rho_{\tau\tau}(0)$ cross at different points. The error bound for the number z, which gives good scaling, is roughly $z=1.07\pm0.03$. A slightly larger number was obtained by Lee *et al.*[29] in a similar model at $f=1/4$, however their error bound is too big to meaningfully compare the numbers. The value of z is also consistent with the direct calculation of correlation functions as shown in Fig. 6, where we find $z\approx1.06$. The value of $\nu=1.0\pm0.3$ is obtained from the scaling curve shown in the inset of Fig. 5, where the error bound is the rough range for good scalings. In Fig. 5, we used $1/\nu=1.0$ and $K^*=0.4465$. This number for ν is consistent with the simple extension of the Chayes lower bound condition[37] for the model with correlated disorder, which tells us that $\nu\geq2/d$ where $d=2$ is the number of dimensions in which the disorder occurs. The error bound on ν obtained in this fitting, however, is too big to confirm the condition for this model. Figure 7 shows the curve of the resistivity for different couplings. Here we plot the resistivity instead of the conductivity. In a disordered boson model, Sørensen *et al.*[11] could fit their resistivity data in a Drude form. Even though our data expressed in the form of resistivity are not fit in a Drude form, better scalings are obtained from them than those obtained from the corresponding conductivity data. The curves are obtained from system sizes $10\times10\times9, 12\times12\times11, 14\times14\times13$, and $16\times16\times15$. Since the errors caused by the uncertainty of the critical coupling are dominant compared with those caused by the uncertainty of z, we use systems with nearby integer sizes. From those curves, we determine the universal conductivity of the model: $\sigma^*/\sigma_Q=0.27\pm0.04$.

With frustration $f=1/2$ in the spatial planes of the model, we have $z\approx1.14\pm0.03$, $K^*=0.6875\pm0.025$, $\nu=1.0\pm0.3$ (see Fig. 8), and $\sigma^*/\sigma_Q=0.49\pm0.04$ (see Fig. 9). Again, the error bound on the values of z and ν are the rough range for good fitting. The correlation function calculation gives $z\approx1.17$. The universal conductivity in this case is also approximately doubled compared with the unfrustrated model. This means that the type of disorder we consider does not wash out the magnetic length. As for the critical coupling, even though the shape of the lattices used are slightly different in the simulations with and without randomness along the imaginary time axis, the critical coupling is consistently suppressed in the disordered case. This result is consistent with the recent simulation[29] of a model high-T_c superconductor with columnar defects along the c axis, where the T_c is increased because of the vortex pinning effect due to the random disorder.

IV. CONCLUSIONS

We have calculated the universal conductivity for a random-U boson Hubbard model. For the fully frustrated ($f=1/2$) pure system, we obtain $\sigma/\sigma_Q=0.52\pm0.03$. The conductivity is almost doubled, as predicted by Granato and Kosterlitz,[35] compared with the unfrustrated system. For $f=1/3$, we find $\sigma/\sigma_Q=0.83\pm0.06$ so that the conductivity is approximately tripled. The phase transition in the above two cases is continuous, and we find $z\approx1$ and $1/\nu\approx1.5$. For the disordered case, we study the model for $f=0$ and $f=1/2$. We obtain the quantum dynamical exponents $z=1.07\pm0.03$ and $z=1.14\pm0.03$, respectively. The critical exponent $1/\nu\approx1$ is obtained, the value of which is consistent with Chayes lower bound condition.[37] The universal conductivity is $\sigma/\sigma_Q=0.27\pm0.04$ and $\sigma/\sigma_Q=0.49\pm0.04$ for $f=0$ and $f=1/2$, respectively. Disorder in the model has little effect on the value of the universal conductivity (at least at $f=1/2$). Also it does not wash out the length scale set by the frustration, although the critical exponents are changed.

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