Effect of fractons in superconductors with fractal structure

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We present in this paper the first quantitative calculation of critical temperature T_c and gap parameter $\beta(\equiv 2\Delta/k_B T_c)$ of superconductors with fractal structure in the light of electron-fracton coupling. The Kresin's formula which is valid for any arbitrary value of λ is used. The calculations are based on the McMillan function $\alpha^2F(\omega)$ of electron-fracton interaction recently developed by the authors. Variations of T_c and β with the fractal dimension D are plotted. The results exhibit the possibility of obtaining high critical temperature in superconductors where electrons and fractons are strongly coupled. Application of the calculations to amorphous superconductors is discussed.

Currently, the study of physical properties of disordered systems has received considerable attention.[1] Interest has been renewed partly because of the application of fractal theory to disordered materials.[2] From a geometrical point of view, random structures exhibit a dilation symmetry and can be described by using a noninteger Hausdorff (or fractal) dimension D, which is lower than the embedding Euclidean space dimension. The dynamical properties of fractal networks have been accounted for by assuming the existence of localized excitations, namely, fractons.[3] Amorphous materials can be an illustrative example which possess the self-similar property. It was suggested by Alexander *et al.*[4] that one can regard amorphous materials as having a random force-constant structure at short length scales. A fractal connectivity can be assumed for the masses participating in vibrational dynamics at these length scales. A characteristic crossover length ξ then separates this regime from the long length scale (Euclidean) regime. When the phonon wavelength decreases to length scale ξ, crossover occurs to fractons. Existence of this crossover has been confirmed by experiments[5] and was shown by many authors to be responsible for many anomalous physical properties in topologically disordered systems.[6-8]

Among the studies, the electron-fracton interaction has been widely investigated. Entin-Wohlman, Alexander, and Orbach[9] calculated the inelastic scattering rate of the extended electronic state off fractons. They found that the temperature dependence of the inelastic electron scattering rate differs substantially in the fracton regime from that in the phonon regime. Tian, Li, and Zhang[8] derived the Hamiltonian for the interaction between conduction electrons and fractons and obtained the transition matrix element of electrons. They calculated the temperature dependence of resistivity to second order of the interaction between electrons and fractons and demonstrated that resistivity minima at low temperatures in weak-scattering metallic glasses may be due to a competition between negative temperature coefficients of resistivity (TCR's) resulting from the electron-fracton interaction and a positive TCR from the regular electron-phonon interaction. Shortly after the discovery of high-temperature cuprate superconductors, Butter and Blumen[10] suggested a possible explanation for the high-T_c superconducting phase in the Y-Ba-Cu-O system in light of the electron-fracton interaction. When a crystal lattice changes to a fractal one, the cutoff vibrational frequency ω_{FD} can become much greater than the phonon Debye frequency ω_D. This fact exhibits the possibility of obtaining a high critical temperature. The existence of vibrational fractons in a high-T_c cuprate single-crystal sample is questionable, although it is still worthwhile to investigate the possible influence of fractons on superconductivity in superconductors with a fractal nature. Using McMillan's formula,[11] Chakrabarti and Ray[12] studied the behavior of the enhancement ratio T_c'/T_c against the Debye frequency and concluded that T_c' in a fractal structure can be much higher than that of a compact or crystalline structure. Later, Tewari and Gumber[13] pointed out that only under certain conditions will there be a substantial increase in the critical temperature in the fractal superconductor over its value in the corresponding crystalline superconductor. Making use of the interaction matrix element derived by Tian, Li, and Zhang,[8] Xin, Liao, and Yang[14] calculated the function relation between T_c'/T_c and the fractal dimension. They also obtained a scaling law between T_c'/T_c and ρ'/ρ, where ρ' is the resistivity of the fractal structure and ρ is that of the periodic structure in their normal states. These investigations shed light on the importance of the electron-fracton interaction in the superconductivity of the fractal structure. However, there has not been a quantitative calculation presented up to now. More recently, Jiang *et al.*[15] calculated the spectral function $\alpha_{tr}^2F(\omega)$ due to the interaction between the conduction electrons and fractons. Using this spectral function, they studied the temperature dependence of the resistivity arising from the scattering of conduction electrons off fractons. Their results

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showed that this contribution to the resistivity is nearly linear in temperature over a wide temperature range and $\rho(T)$ varies as $T^{3\bar{d}/D-1}$ for fracton scattering at low temperatures. The McMillan function $\alpha^2F(\omega)$ of electron-fracton coupling can also be derived in an analogous way to theirs. This function would permit us to perform quantitative calculations. The purpose of this paper is to present a quantitative calculation of the critical temperature T_c and gap parameter β ($\equiv 2\Delta/k_BT_c$) in light of the electron-fracton interaction.

According to Kresin,[16] the following expression for T_c can be used for any arbitrary coupling constant λ:

$$T_c=\frac{0.25\tilde{\Omega}}{\sqrt{e^{2/\lambda_{\rm eff}}-1}} , \tag{1}$$

where

$$\lambda_{\rm eff}=\frac{\lambda-\mu^*}{1+2\mu^*+\lambda\mu^*t(\lambda)} . \tag{2}$$

Here $\tilde{\Omega}=\langle\omega^2\rangle^{1/2}$, μ^* represent the electron-electron Coulomb interaction, and $t(\lambda)$ is a function which is numerically evaluated and can be approximated closely by[13]

$$t(\lambda)=1.5e^{-0.28\lambda} .$$

Considering the fact that fractons only exist over the frequency range between the crossover frequency ω_c and the fracton cutoff frequency ω_{FD}, the electron-fracton mass enhancement factor λ and $\langle\omega^2\rangle$ can be expressed as

$$\lambda=2\int_{\omega_c}^{\omega_{FD}}\frac{\alpha^2F(\omega)}{\omega}d\omega \tag{3}$$

and

$$\langle\omega^2\rangle=\frac{\int_{\omega_c}^{\omega_{FD}}\alpha^2F(\omega)\omega\, d\omega}{\int_{\omega_c}^{\omega_{FD}}\alpha^2F(\omega)\omega^{-1}d\omega} , \tag{4}$$

respectively. The gap parameter β is given by[16]

$$\beta\equiv 2\Delta/k_BT_c=3.52[1+5.3(T_c/\tilde{\Omega})^2\ln(\tilde{\Omega}/T_c)] . \tag{5}$$

The integral limits in Eqs. (3) and (4) are related to each other by[17]

$$\omega_{FD}=\omega_c(\xi/a)^{D/\bar{d}} , \tag{6}$$

where D is the fractal dimension of the network, $\bar{d}$ the fracton (or spectral) dimension, ξ the correlation length, and a the interatomic spacing. Later $\bar{d}=\frac{4}{3}$ will be used as a good approximation in general.[3]

Following Jiang *et al.*,[15] the function $\alpha^2F(\omega)$ of the electron-fracton interaction is given by

$$\alpha^2F(\omega)=C_{\rm fr}\frac{K_{D-1}}{(2\pi)^D}\frac{\bar{d}}{D}\frac{1}{\xi\omega_c^{\bar{d}/D}}\int_0^{\pi}(\sin\theta)^{D-2}d\theta\,\omega^{\bar{d}/D-2} , \tag{7}$$

with a prefactor which is independent of the fractal dimension D. In order to determine $C_{\rm fr}$, we introduce another quantity λ_0, which is the coupling constant of electrons and fractons for $D=2$. Then we have

$$C_{\rm fr}=4\pi\xi\omega_{FD}^2(\xi/a)^{-3}\frac{\lambda_0}{1-(\xi/a)^{-2}} . \tag{8}$$

Inserting Eqs. (7) and (8) into (3) leads us to

$$\lambda=\frac{2}{\pi}\lambda_0\left[\frac{(\xi/a)^{3/2}}{2\pi}\right]^{D-2}K_{D-1}\times\int_0^{\pi}(\sin\theta)^{D-2}d\theta\frac{1}{\frac{3}{2}D-1}\frac{1-(\xi/a)^{1-3D/2}}{1-(\xi/a)^{-2}} . \tag{9}$$

It is also convenient to obtain

$$\tilde{\Omega}=\omega_{FD}(\xi/a)^{-3D/4}\left[\frac{(\frac{3}{2}D-1)(\xi/a-1)}{1-(\xi/a)^{1-3D/2}}\right]^{1/2} . \tag{10}$$

One can see that the effect of fractals enters both in the prefactor and the exponential term which normally work against each other. In this paper we focus on the effect of fractons and assume that they provide for a contribution to $\alpha^2F(\omega)$. It is straightforward to incorporate other contributions to $\alpha^2F(\omega)$. Using Eq. (1) along with (5), (9), and (10), we have calculated T_c and β for various fractal dimensions D. The parameters used are $a=3$ Å, $\xi=25$ Å, $\omega_{FD}=4000$ K, and $\mu^*=0.1$. Choices of these values are reasonable, in particular that of ω_{FD}; since the density of states in the fracton range varies as $N(\omega)\sim\omega^{1/3}$ for $\omega>\omega_c$, in contrast to $N(\omega)\sim\omega^{d-1}$ for the phonons (d is the Euclidean dimension), the cutoff frequency ω_{FD} should be much higher than the ordinary Debye frequency ω_D for crystals; in fact, Tewari and Gumber[13] estimated that the highest frequency changes from ω_D to $8\omega_D$. The value of ξ is typical in dense amorphous materials.[7] For $D=2.5$, the above values together with Eq. (6) give $\omega_c\approx 1\times 10^{13}$ s^{-1}. This value of ω_c is very close to that used in the study of thermal conductivity of amorphous materials by Orbach and co-workers.[7]

The calculated T_c and β against the fractal dimension D are plotted in Figs. 1 and 2. Curves a, b, c, d, and e correspond to $\lambda_0=0.3$, 0.5, 1.0, 1.5, and 2.0, respectively. One can see from Fig. 1 that, for small values of λ_0, T_c

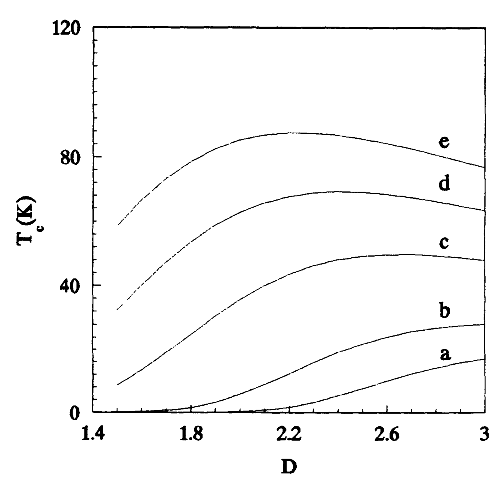

FIG. 1. T_c vs fractal dimension D. Curves a, b, c, d, and e correspond to $\lambda_0=0.3$, 0.5, 1.0, 1.5, and 2.0, respectively.

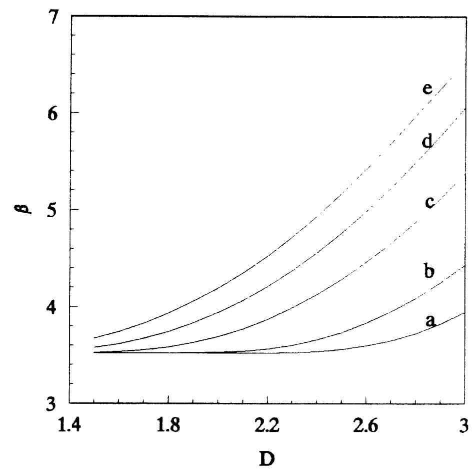

FIG. 2. β vs fractal dimension D. Curves a, b, c, d, and e correspond to $\lambda_0=0.3$, 0.5, 1.0, 1.5, and 2.0, respectively.

increases with D and then tends to saturate. For $D=2.5$ and $\lambda_0=0.5$, T_c is about 20 K, while for larger values of λ_0, T_c presents a maximum on $T_c \sim D$ curves. For $\lambda_0=1.5$, the maximum temperature T_c appears at $D=2.4$ and equals 70 K. In general, T_c increases with λ_0 if D kept constant. As is evident from Fig. 2, the gap parameter β increases monotonically with the coupling strength λ_0 as well as with increasing fracton dimension value D and can be as large as 5–6. As can be seen from Fig. 1, if we take λ_0 to be 2.0, T_c has its maximum at about $D=2.2$ and can even approach 90 K. Therefore there exists the possibility of obtaining high critical temperatures in superconductors with a fractal nature where electrons and fractons are strongly coupled. In materials with a fractal structure, the large value of the coupling constant may arise from the more "open" character of the force-constant network seen by individual atoms or atomic units at short length scales.[7] These weakly constrained constituents would experience a very large amplitude of vibrations,[18] resulting in a large enhancement of the effective coupling constant averaged over the fracton wave function. This effect can be seen from the interaction Hamiltonian derived by Tian, Li, and Zhang,[8] where the coupling strengths between electrons and fractons are closely related to the amplitude of the fracton wave function. It has also been shown that large electron-fracton coupling strength (at least one order of magnitude larger than that of electron-phonon coupling) is essential in explaining the resistivity minima phenomena in weak-scattering metallic glasses at low temperatures.[8] We note with interest that a very large value of the anharmonic fracton-phonon coupling coefficient had been obtained by Orbach and co-workers in the study of the thermal conductivity of amorphous materials.[7]

In the calculations of Tewari and Gumber,[13] variation of T_c with r (which was defined as ω_D/ω_c and represents the extent of fractal region) had been plotted. They found that T_c can increase with r under certain conditions. Using expression (9) of their work and Eq. (7) of this paper, we obtain the relationship between r and D:

$$r=\left[\frac{3}{d}(\xi/a)^D+\left[1-\frac{3}{d}\right]\right]^{1/3} . \tag{11}$$

From this equation, one can see that r increases with the increase of D if we take ξ/a to be a constant. Thus our results presented in Fig. 1 and that shown in Fig. 2(b) of Tewari and Gumber's paper are indeed qualitatively consistent.

There are many interesting experimental observations[19] in amorphous superconductors that have not been well understood. It has been found that most amorphous superconducting metals and alloys possess a higher critical temperature than the corresponding crystalline solids. A prominent example of T_c enhancement due to amorphous disorder is beryllium where T_c in amorphous Be is known to be about 30 times the value in the crystalline state.[13] Another interesting point to be noted is that the values of T_c of many simple metals and alloys are very close in their amorphous states, while T_c differs from material to material in their crystalline states. Also, the amorphous superconducting materials often have larger λ and β. Our results show that the incorporating of electron-fracton coupling may be an alternative explanation of the observed phenomena. The usual way to explain this, however, is concerned with the enhancement of the low-frequency part of $\alpha^2F(\omega)$ when amorphous states appear. The low-frequency part has a greater weight in the calculation of λ, and its enhancement will lead to an increase of λ and thus enhance T_c and β. This explanation is supported by the measurement of $\alpha^2F(\omega)$ from single-particle tunneling experiments of amorphous superconductors. However, it should be pointed out that the tunneling experiment is successful in measuring $\alpha^2F(\omega)$ of crystalline superconductors, although its application to amorphous superconductors is not without dispute.[19] Electrons in amorphous superconductors have small mean free paths, and so the tunneling results are in fact an effect of those electrons rather near the tunneling junction. To what extent the tunneling results can reflect bulk properties is not clear. The electron-fracton coupling, on the other hand, is a more natural and direct explanation of the observed facts since it includes the structural effect and therefore can be more universal.

The fundamental assumption of this paper is that fractons in superconductors with fractal structure take part in electron pairing. In amorphous superconductors, where the fractal geometry is expected to play a role, we have no experimental evidence for the presence of fractons yet. However, most amorphous materials are prepared by use of quenching or vapor-condensation techniques. In those cases, minimum energy configurations are not attained by the solid phase and fractal aggregation seems plausible. On the other hand, experimental evidence for the existence of fractons in different amorphous solids is rich.[5,20,21] The presence of fractons in amorphous superconductors can be detected in several ways. One is to record light scattering or neutron spectroscopy. The other is to measure the superconducting phase boundary $T_c(H)$ to see if there is a crossover between the homogeneous and fractal regimes.[22] Also, measurements of thermal properties can provide some indirect information. Research in this direction will be worthwhile.

Finally, we must emphasize that we use "fractal structure" in the present paper in the sense that was demonstrated by Orbach and co-workers.[7] The mass distribution does not have to be fractal. Rather, it is the vibrational connectivity between the sites participating in the vibrational dynamics that is assumed to be fractal. Therefore "fractal structure" in this paper means "force-constant fractal structure," and one should not confuse this with mass-density fractal structure (such as a site percolation network).

In summary, the effect of fractons on the critical temperature T_c and gap parameter β in superconductors with fractal structure has been studied. The results exhibit the possibility of obtaining a high superconducting transition temperature in light of electron-fracton coupling. Application of the calculations to amorphous superconductors has been discussed.

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