Quantum dynamics of a fluxon in a long circular Josephson junction

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(Received 19 August 1993)

The dynamics of the fluxon in a long circular Josephson junction is studied. A bias charge across the junction is shown to act like a gauge potential on the fluxon. The system is then quantized, and the energy levels of the fluxon are found to resemble those of a small capacity-dominated Josephson junction. The quantized fluxon is shown to exhibit a persistent motion in analogy with the persistent current of electrons in a metal ring threaded by a magnetic flux. This motion manifests itself in a voltage across the junction. At finite temperatures the voltage is reduced by the thermal distribution of the fluxon's energy. The dependence of the voltage on the ratio of the junction's circumference to the Josephson penetration depth is examined.

I. INTRODUCTION

Following an extensive and fruitful study of manifestations of the Aharonov-Bohm (AB) effect in condensed-matter electronic transport, some attention was recently focused on a variation of that effect, sometimes known as the Aharonov-Casher (AC) effect.[1-3] While in the AB effect an electron is split into wave packets that go around flux tubes, in this variation[2] flux tubes are split into wave packets that go around charges. This variation is considerably more complicated than the original one, since the interfering object (the flux tube) is a many-body excitation, whose mass, dynamics, and phase-breaking mechanisms are far less understood than those of the electron.

While these recent works have examined the AC effect of vortices in a two-dimensional (2D) array of Josephson junctions, in this paper we study the quantum dynamics of a fluxon trapped in a single, long, effectively one-dimensional *circular* Josephson junction (see Fig. 1). In particular, we examine the effect of an externally imposed bias charge on that dynamics. We regard this bias charge, created by a driven continuous external current, as a controlled parameter. The phenomena we discuss are manifested in the dependence of the junction's energy, the voltage across the junction, and the fluxon's velocity on this bias charge.

The classical dynamics of a fluxon in a Josephson junction is well understood.[4-8] As long as the junction is ideal (no dissipation or spatial disorder) the main effect of the fluxon is to suppress dc Josephson effect, i.e., to disable the transfer of net current without an accompanying voltage. Dynamically, the equation of motion describing the junction is the sine-Gordon equation. A trapped fluxon is described by the one-soliton solution to that equation. In the absence of an externally driven current the fluxon moves at a constant velocity along the junction. An externally driven uniform current accelerates the fluxon due to an exchange of momentum mediated by the Lorentz force. This Lorentz force exerted by the current on the fluxon is described, in Hamiltonian description, as resulting from a time-dependent gauge potential. The classical aspects of the fluxon's dynamics in a circular junction have been studied experimentally.[9]

In the work we report here, we consider the effect a time-independent gauge potential has on the quantum dynamics of the fluxon. We find that the Hamiltonian eigenstates and eigenvalues depend on the total "flux" associated with this gauge potential, i.e., the line integral of this gauge potential around the junction. This "flux" is proportional to the externally imposed bias charge across the junction. The spectrum of the fluxon is then periodic

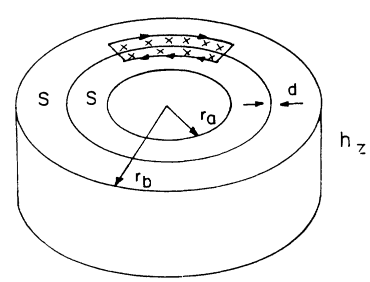

FIG. 1. The Josephson junction considered. Two concentric superconducting cylinders are separated by a thin insulating layer. The junction encloses a fluxon of magnetic field, formed by a "solenoid" of current.

with respect of the bias charge, with a period of $2e$ (e being the electron charge). The voltage across the junction, given by the derivative of the energy with respect to the bias charge, is $2e$ periodic too. The energy and voltage periodicity with respect to the charge are similar to those found for two-dimensional annulus-shaped arrays of Josephson junctions.[2] However, many of the details are different.

The kinetic mass of the fluxon plays an important role in the problem we discuss. We calculate this mass by examining the dependence of the fluxon's energy on its velocity. We focus on the dependence of the kinetic mass on the ratio of the junction's length L to the Josephson penetration depth Λ_J. We find that the mass is L independent when $L \gg \Lambda_J$. For typical values, it is much smaller than the electron mass. When $L \ll \Lambda_J$ the mass becomes approximately proportional to L^{-1} and independent of Λ_J. A short review of the fluxon's classical dynamics, a derivation of its Hamiltonian, and a discussion of its mass are given in Sec. II.

Our study of the quantum mechanics of the fluxon is detailed in Sec. III. Quantizing the Hamiltonian of the fluxon, we observe that for an ideal junction the energy levels and momentum values of the fluxon are quantized. We show that this quantization is a quantization of the number of Cooper pairs charging the junction. Then we analyze the dependence of the Hamiltonian on the bias charge, and interpret this dependence as a persistent motion of the fluxon around the ring. We draw the analogy between this motion and the persistence current of electrons in normal and superconducting rings. The voltage response to an infinitesimal bias charge defines the capacitance of the junction, which is closely related to the mass of the fluxon. We examine the dependence of this capacitance on the ratio L/Λ_J. We find that in the limit $L \gg \Lambda_J$ the effective capacitance of the function is different from its geometric capacitance. In the limit $L/\Lambda_J \to 0$ the effective capacitance approaches the geometric capacitance. Then, the sole effect of the fluxon is to shut off the Josephson dc effect.

Section IV deals with the smearing of the energy dependence on the bias charge due to the thermal distribution of the fluxon energy at finite temperatures. We find that the effects we discuss are exponentially suppressed by temperature, and that the characteristic temperature is of the order of the spacing between consecutive energy levels. For experimentally accessible values, this temperature is of the order of 10–100 mK. We conclude in Sec. V by a few remarks about nonideal junctions.

II. CLASSICAL DYNAMICS AND MASS OF A FLUXON IN A CIRCULAR JOSEPHSON JUNCTION

Consider a circular Josephson junction (see Fig. 1) made of two superconducting concentric cylinders separated by a thin insulating layer. The width of the cylinders is much larger than both λ_L, the London penetration depth, and d, the width of the insulating layer. We assume that the dielectric constant and the permeability of this layer are both equal to 1 (in cgs units). Nevertheless, since the electric field goes to zero at the edges of the insulating layer, while the magnetic field penetrates inside the superconductors to a distance λ_L, the speed of light in the junction, $\bar{c}$, is different from the speed of light in vacuum c, $\bar{c} \equiv c\sqrt{d/2\lambda_L}$ (we approximate everywhere $2\lambda_L + d \approx 2\lambda_L$). The strength of the Josephson coupling between the two superconductors can be characterized by any one of the following two parameters: J_J, the critical Josephson current density, and Λ_J, the Josephson penetration depth. Below we remind the reader of the interrelations between the two. The height of the junction h_z is much smaller than Λ_J, but much larger than λ_L. Thus, the ring is effectively one-dimensional, i.e., the phase difference between the two superconductors is approximated to be independent of z. We denote the polar coordinate (the only relevant dimension) by x. The circumference of the ring is denoted by L.

In Hamiltonian description the junction is characterized by the field $\theta(x)$, $\theta(x) \equiv \phi_2(x) - \phi_1(x) - (2e/\hbar c)\int_1^2 \mathbf{A}\cdot d\mathbf{e}$ (where $\phi_{1,2}$ are the phases of the two superconductors, and $\mathbf{A}$ is the electromagnetic vector potential), describing the gauge-invariant phase difference between the two superconductors, and its conjugate field $\hbar h_z n(x)$. The field $n(x)$ describes the two-dimensional number density (number per unit area) of Cooper pairs on the two sides of the junction. The magnetic field in the junction is directed in the z direction (neglecting edge effects), and is given by $(\phi_0/4\pi\lambda_L)\theta_x$. In the presence of a uniform bias-charge density, denoted by $2e\sigma$, the electric field is $8\pi e[n(x)+\sigma]$. The Hamiltonian of the junction is then given by (unless otherwise stated, all integrals are one dimensional, taken from $x=0$ to $x=L$),

$$H \equiv \int \mathcal{H}\, dx = \hbar\bar{c} \int \left\{ \beta^2 \frac{h_z^2}{2}(n-\sigma)^2 + \frac{1}{\beta^2}\left[\frac{1}{2}\theta_x^2 + \frac{1}{\Lambda_J^2}(1-\cos\theta) \right] \right\} dx , \tag{1}$$

where β is the dimensionless parameter

$$\beta^2 \equiv 16\pi \frac{e^2}{\hbar c} \frac{\sqrt{2\lambda_L d}}{h_z} . \tag{2}$$

For the clarity of the expressions, the x dependence of both σ and n is suppressed. The first term in the Hamiltonian is the electrostatic, i.e., capacitive, energy, the second term is the magnetic, i.e., inductive, energy, and the third term is the Josephson energy. The equation of motion derived from the Hamiltonian (1) is the sine-Gordon equation,

$$\frac{1}{4\pi}\frac{\hbar}{2ed}\ddot{\theta} - \frac{1}{4\pi}\frac{\hbar c^2}{4e\lambda_L}\theta_{xx} + J_J \sin\theta = 2e\dot{\sigma} , \tag{3}$$

where $J_J \equiv (1/4\pi)(\hbar c^2/4e\lambda_L)(1/\Lambda_J^2)$. This equation of motion is nothing but a continuity equation for the electric current. The external bias-current density $2e\dot{\sigma}$ is split into there parts. $J_J\sin\theta$ is the Josephson current flowing across the junction. $(1/4\pi)(\hbar\ddot{\theta}/2ed)$ is the displacement current charging the junction.

$(1/4\pi)(\hbar c^2/4e\lambda_L)\theta_{xx}$ is the difference between the inductance current (flowing along the junction) to the right and to the left of the point x (See Fig. 2). Since the junction is circular, the phase difference satisfies $\theta(x)=\theta(x+L)+2\pi m$, with m being an integer. A single fluxon enclosed in the junction is described by the solution to Eq. (3), with $m=1$.

In the absence of a bias current, i.e., when $\dot{\sigma}=0$, the solution of Eq. (3) describing a fluxon trapped in the junction moving in a velocity $v \ll \bar{c}$ is,[4]

$$\sin\{\tfrac{1}{2}[\theta(x,t)-\pi]\}=\mathrm{sn}\left[\frac{x-X_0-vt}{k\Lambda_J}\right], \tag{4}$$

where $\mathrm{sn}(z)$ is the Jacobi elliptic function and k is implicitly given by the complete elliptic integral of the first kind (see, for example, Ref. 10)

$$L/\Lambda_J=2kK(k). \tag{5}$$

The parameters X_0 and v describe the initial position and velocity of the fluxon, respectively. In the limit $L \gg \Lambda_J$ (long junction), the phase changes from 0 to 2π along a narrow strip of length Λ_J, and is approximately constant everywhere else. The magnetic and electric fields are nonzero only within that strip. In that limit Eq. (4) can be approximated by the solution for an infinite junction, namely,

$$\theta \approx 4\tan^{-1}\left[\exp\left[\frac{x-X_0-vt}{\Lambda_J}\right]\right] \quad \text{for } L \gg \Lambda_J. \tag{6}$$

In the limit of $L \ll \Lambda_J$ (short junction), the phase changes almost linearly along the junction, i.e.,

$$\theta \approx 2\pi\left[\frac{x-X_0-vt}{L}\right] \quad \text{for } L \ll \Lambda_J. \tag{7}$$

The magnetic and electric fields are then spread uniformly along the junction. The solutions of Eq. (3) for three values of L/Λ_J are presented in Fig. 3.

The equation of motion (3) is not solvable for a general $\dot{\sigma}$. However, for the purpose of studying the dynamics of the fluxon, this solution is not essential. Rather, it is sufficient to derive and study the equations of motion (and, consequently, the Hamiltonian) for X, a collective coordinate describing the position of the fluxon, and for P, its conjugate coordinate, describing the fluxon's

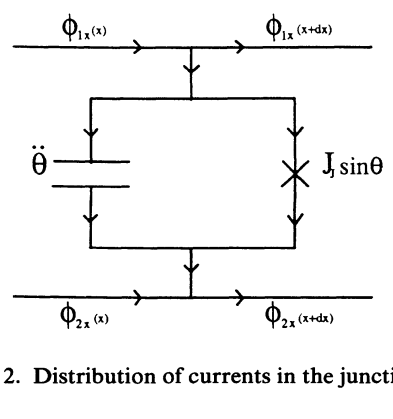

FIG. 2. Distribution of currents in the junction.

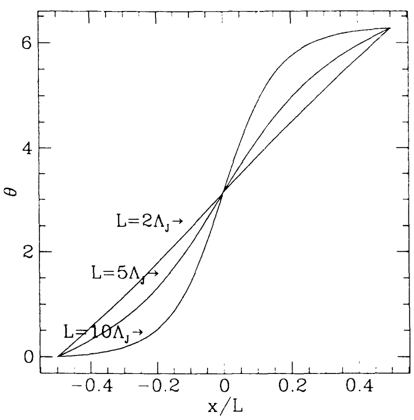

FIG. 3. One-fluxon solutions of circular Josephons junctions of lengths $10\Lambda_J$, $5\Lambda_J$, and $2\Lambda_J$.

momentum. The introduction of collective coordinates to describe the motion of topological solutions of the sine-Gordon equation has been studied extensively in the past, and we now review it shortly.[7,11–13] The main assumption taken when the description of the junction by the fields n,θ is replaced by a description in terms of two variables P,X is that the time evaluation of the fluxon's field configuration is of the form $\theta(x-X(t))$. Within this approximation, the fluxon is a rigid body, whose shape does not change when it moves. A Newtonian equation of motion for $X(t)$ can be obtained by substituting this assumed form for the field configuration in the sine-Gordon equation [Eq. (3)], multiplying the sine-Gordon equation by θ_x, and integrating over x. The resulting equation is

$$M\ddot{X}=-\frac{2\pi\hbar}{L}\frac{\dot{Q}}{2e}, \tag{8}$$

where the kinetic mass of the fluxon, M, is given by

$$M=\frac{\hbar^2 h_z}{4\pi(2e)^2 d}\int \theta_x^2\,dx, \tag{9}$$

and $Q\equiv 2e\sigma L h_z$ is the *total* bias charge on the junction. The right-hand side of the equation of motion (8) is the force acting on the fluxon. This force is independent of the detailed structure of the fluxon. It depends only on the total external current $\dot{Q}$ and on the topological charge of the fluxon. The microscopic origin of this force is the Lorentz force. The mass of the fluxon, appearing on the left-hand side of Eq. (8), is, obviously, independent of $X(t)$. As seen in Eq. (9), it is proportional to the energy stored in the magnetic field of the fluxon. As such, the mass depends on the details of the field configuration, and in particular, on the ratio L/Λ_J. For a long junction ($L \gg \Lambda_J$) the magnetic field is independent on L, and so is the mass. Substituting the field configuration for that case, Eq. (6), in the expression for the mass (9), we find

$$M=\frac{2}{\pi}\frac{\hbar^2 h_z}{(2e)^2 d\Lambda_J} \quad \text{for } L \gg \Lambda_J. \tag{10}$$

For typical values, this mass is much smaller than the mass of the electron. For a short junction the magnetic field is L dependent, and so is the mass, given by

$$M=\pi\frac{\hbar^2 h_z}{(2e)^2 dL} \quad \text{for } L \ll \Lambda_J , \tag{11}$$

i.e., it diverges as L^{-1}.

As seen in Eq. (8), the uniform (x-independent) external current exerts a uniform force on the fluxon. As such, this force has to result from a time-dependent gauge potential. The Hamiltonian giving rise to this equation of motion is therefore

$$H_{\text{fluxon}}=\frac{1}{2M}\left[P-\frac{2\pi\hbar}{L}\frac{Q}{2e}\right]^2 . \tag{12}$$

This Hamiltonian deserves a few comments. First, it includes only the part of the energy relevant for the fluxon's dynamics. This part is the electric, capacitive energy. The rest energy of the fluxon, composed of the magnetic and Josephson contributions, does not affect the dynamics, and therefore is not included. Second, the equation of motion $\dot{P}=0$, derivable from this Hamiltonian, is a consequence of the translational invariance of the problem, and is, of course, exact. Third, the collective coordinates X and P can be represented in terms of the fields θ, n. Their representations are given by[7]

$$X=\frac{1}{2\pi}\int x\theta_x dx \tag{13}$$

and

$$P=-\hbar h_z \int n\theta_x dx . \tag{14}$$

As seen from Eq. (13), X is the "center of mass" of the fluxon, and P is the total momentum of the field. The nature of P is better understood by noting that the electric field $E=8\pi e(n-\sigma)$ and the magnetic field $B=(\phi_0/4\pi\lambda_L)\theta_x$. Substituting these expressions into Eq. (14), we observe that the field momentum P is a sum of a contribution associated with the electromagnetic field,

$$P=\frac{c}{4\pi c^2}\int \mathbf{E}\times\mathbf{B}\, d^3x , \tag{15}$$

and a contribution associated with the bias charge, $(2\pi\hbar/L)(Q/2e)$. Thus, the conservation of the fluxon's momentum implies the suppression of the Josephson dc effect: any change in the bias charge, i.e., any charging current, has to be accompanied by a change in the electric field. The position X and momentum P given by Eqs. (13) and (14) do satisfy the canonical commutation relations.

III. QUANTIZATION OF THE FLUXON'S DYNAMICS

In this section we approximate the quantization of the field theory defined by the Hamiltonian (1) by a quantization of the effective one-particle fluxon Hamiltonian (12). We first discuss the results originating from this approximation, and later discuss its limits of validity.

The quantization of the fluxon Hamiltonian (12) is, of course, straightforward. The Hamiltonian commutes with the momentum operator. The eigenstates of the momentum operator are plane waves, with a discrete set of eigenvalues. The energy spectrum is discrete, too, and is given by

$$E_N=\frac{1}{2M}\left[\frac{2\pi\hbar}{2eL}\right]^2 (2eN-Q)^2 , \tag{16}$$

where N is an integer. This form of the energy suggests an interpretation of the integer N as the number of Cooper pairs changing the junction, and of the quantity $C_{\text{eff}}\equiv(2eL/2\pi\hbar)^2 M$ as the junction's effective capacitance. The *quantization* of momentum eigenvalues is, therefore, the statement that only an integral number of Cooper pairs can tunnel across the junction and charge it. The *conservation* of momentum, valid only for an ideal junction, is the well-known statement that in an ideal junction enclosing a fluxon there is no matrix element for tunneling of Cooper pairs, i.e., the Josephson effect is suppressed. Below we comment on the way this statement is invalidated in the presence of weak disorder. The spectrum (16) of the fluxon's Hamiltonian is manifestly periodic with respect to the bias charge Q, with the period being $2e$. The voltage across the junction is $\partial E_N/\partial Q=(1/C_{\text{eff}})(2eN-Q)$, and is proportional to the expectation value of the velocity of the fluxon, given by $(L/2\pi\hbar)(\partial E_N/\partial N)$. The bias charge then induces a motion of the fluxon around the junction, and this motion manifests itself in a voltage across the junction.

The effective capacitance of the junction, being proportional to the mass of the fluxon, depends on the ratio L/Λ_J. Using the expressions for the mass [Eqs. (10) and (11)] we find that the effective junction capacitance is identical to the geometric capacitance $Lh_z/4\pi d$ for a short junction ($L \ll \Lambda_J$), but differs from it for a long junction ($L \gg \Lambda_J$). In the latter case the effective capacitance is given by $L^2 h_z/2\pi^3 d\Lambda_J$. This difference deserves an elaboration: for short junctions, the sole effect of the fluxon is to turn off the Josephson coupling. The junction then becomes a capacitor. The charging is uniform along the junction, and so are the electric and magnetic fields. Therefore, the effective capacitance is the geometric one. The long-junction case is more interesting. The presence of the fluxon shuts off the Josephson coupling, and makes the junction a capacitor. However, as described in Sec. II, the charging occurs in a region of order Λ_J around the moving center of the fluxon, so, electrostatically, a moving fluxon is a moving capacitor. Since the charging of a quantized fluxon is done by an integral number of Cooper pairs, we conclude that a quantized fluxon behaves electrostatically like a small tunnel junction with a unit charge of $2e$. The quantized Josephson junction can be thought of, therefore, as a small tunnel junction moving in a circle with a momentum proportional to the number of unit charges that have tunneled across it.

In the presence of weak spatial disorder the fluxon's momentum is not conserved, i.e., Cooper pairs can tunnel across the junction. The energy levels given by Eq. (16) are a set of parabolas, each centered at an integer multiple of $2e$. The parabolas intersect one another at $(2j+1)e$ (j being an integer). The spatial disorder opens gaps in the energy levels at the intersections of the parabolas. Then, if an unbiased junction is adiabatically charged by a bias charge Q, the voltage across the junction oscillates as a function of Q. Whenever the value of Q becomes $(2j+1)e$, a Cooper pair tunnels across the junction, and the momentum of the fluxon is changed by $2\pi\hbar/L$.

The picture emerging from the analysis of the fluxon's Hamiltonian (12) is similar to that emerging in a few other problems in related fields. Most notably, it is similar to the picture emerging from the discussion of persistent currents in normal and superconducting rings threaded by a magnetic flux.[14] In the latter case a unit charge is being driven by a flux and exhibits a persistent current with a periodicity of a flux quantum. In the Josephson junction case a flux quantum is being driven by a charge and exhibits a persistent voltage, with a periodicity of $2e$. For typical junction parameters $d=20$ Å, $h_z=5000$ Å, $\Lambda_J=30$ μm, $L=100$ μm the amplitude of the voltage oscillations is of the order of 10^{-7} V.

To conclude this section, we turn to justify our approximation of the dynamics of the field Hamiltonian (1) by the single-particle fluxon Hamiltonian (12). The quantization of theories with topologically nontrivial solutions, especially the sine-Gordon theory, has been studied extensively in the context of high-energy physics. Several quantizations methods have been used, among them a semiclassical, WKB-like quantization (the Dashen-Hasslacher-Neveu formula),[15] and a Born-Oppenhemier approximation.[11,16] In the latter method, the classical soliton solution is regarded as the ground state ("vacuum") of a Fock space called the one-soliton sector. This sector is completely disjoint from the sector containing no soliton, thus reflecting the topological stability of the soliton. The states of the one-soliton sector are constructed by a perturbative expansion in the coupling constant of the theory, β^2 [Eq. (2)]. This expansion is valid as long as β^2 is small, and blows up when $\beta^2=4\pi$. Using the typical junction parameters values giving above and $\lambda_L\sim 1000$ Å, we find that in our case $\beta^2\sim 10^{-2}$, and thus the Born-Oppenhemier approximation is well justified. The ground state of the one-soliton sector is the classical fluxon solution, and the higher states consist of the fluxon and a collection of the small-amplitude plasma oscillations of the junction (plasmons). However, the plasmon spectrum is separated from the vacuum state by a gap, given by $\hbar\bar{c}/\Lambda_J$. Thus, as long as the frequency at which the fluxon encircles the ring is lower than $\bar{c}/\Lambda_J$, and the temperature is lower than $\hbar\bar{c}/\Lambda_J$ (~ 1 K), this interaction can be disregarded.

IV. THE EFFECT OF THE THERMAL DISTRIBUTION OF THE FLUXON'S ENERGY

At finite temperatures, the fluxon's energy is thermally distributed. Since for a given value of the bias charge Q the voltage across the junction depends on the momentum of the fluxon, and the momentum depends on the energy level, the voltage across the junction is thermally averaged. The relevant energy to compare with the temperature is the energy difference between consecutive levels. The energy levels are given by Eq. (16). For low levels in the long-junction limit ($L\gg\Lambda_J$), the energy difference is of the order of $(2e)^2/2C_{\text{eff}}$. For the junction parameters given in Sec. III, this difference is of the order of 10^{-1}–10^{-2} K.

For the calculation of the thermal average of the voltage induced by the motion of the fluxon, it is convenient to define the scaled bias charge $q\equiv Q/2e$, and the scaled temperature $t\equiv[2C_{\text{eff}}/(2e)^2]k_BT$. We are interested in $\langle\partial E/\partial Q\rangle$. Our first step is to calculate the partition function, given by

$$Z(q,t)=\sum_{l=-\infty}^{\infty}\exp-\frac{(l-q)^2}{t}\;. \tag{17}$$

Using Poisson's summation formula we find that

$$Z(q,t)=\sqrt{\pi t}\left[1+2\sum_{m=1}^{\infty}\exp[-(\pi m)^2 t]\cos 2\pi m q\right]. \tag{18}$$

While Eq. (17) expressed the partition function as a sum over eigenstates, Eq. (18) decomposes it to a sum over harmonics.[17] As is evident from this decomposition, the charge-dependent terms of the partition function are exponentially suppressed by thermal averaging. The characteristic (scaled) temperature for this suppression is $1/\pi^2$. Consequently, the induced voltage,

$$\langle V\rangle=\left\langle\frac{\partial E}{\partial Q}\right\rangle=-\left[\frac{2e}{2C_{\text{eff}}}\right]t\frac{1}{Z}\frac{\partial Z}{\partial q} \tag{19}$$

decreases exponentially with temperature, too (see Fig. 4).

V. CONCLUDING REMARKS

The discussion given in the previous section has brought us to the conclusion that a bias charge induces a voltage across an ideal circular Josephson junction enclosing a fluxon. In this section we make a few remarks

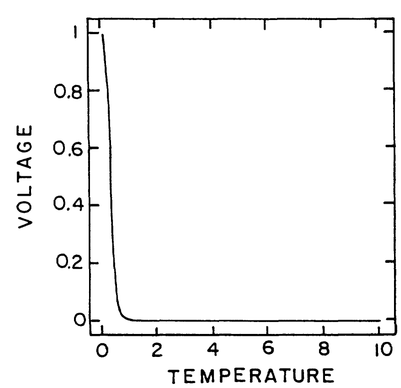

FIG. 4. Induced voltage as a function of temperature, for $q=0.05$. The voltage is drawn relative to its value at zero temperature. The dimensionless units for the temperature are defined in the text.

about nonideal junctions.

We have already commented on the effect of weak disorder on the voltage across the junction. If the disorder is strong, one might expect a localization of the fluxon's eigenstates, and thus a suppression of the voltage across the junction. Since the system is one dimensional, localization should take place when the mean free path of the fluxon is smaller than the junction's circumference.[18,19]

As discussed in Sec. III, the voltage across the junction results from a quantum interference effect. As such, it is expected to be suppressed by interactions of the fluxon with external degrees of freedom. In particular, interactions with plasmons [which are included in the ideal-junction field Hamiltonian (1) but are neglected in the fluxon Hamiltonian (12)] and with quasiparticles (which are neglected in both) are expected to dephase the interference, and introduce a phase-braking length.[20] Both mechanisms for dephasing will be examined in future works.

ACKNOWLEDGMENTS

We thank Y. Imry and Y. Aharonov for stimulating discussions. This research was partially supported by a grant from the Wulfson foundation through the Israeli Academy of Science. A.S. is supported by the Harvard Society of Fellows.

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