Comments

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Comment on "Effect of fractons and magnons on the resistivity of dilute ferromagnets"

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The double-time Green's-function decoupling scheme used by Li, Tian, and Jiang [Phys. Rev. B **47**, 11 905 (1993)] is non-self-consistent and non-moment-preserving. When replaced by a self-consistent, moment-preserving decoupling scheme, important differences arise. In particular, the results presented by Li *et al.* for a system of spin S and spin coupling constant J_{ij} are, instead, appropriate for a system with spin $S/2$ and spin coupling constant $2J_{ij}$.

In their recent treatments of the *s-d* exchange model, Li and co-workers[1,2] performed a Dyson-Maleev transformation on the Heisenberg component and obtained results to second order in the *s-d* exchange parameter by decoupling double-time Green's functions (DTGF) of the form $\langle\langle BC|D\rangle\rangle$ where B is a product of Dyson-Maleev boson operators and C and D are operators which are unaffected by the decoupling. The decouplings only involve B and are in that sense independent of C and D. Since the decouplings are essentially independent of C and D, particular choices of C and D can be used to establish the fundamental validity of the decouplings.

In particular, Li and co-workers use the decoupling

$$\langle\langle b_j^+ b_j b_j C|D\rangle\rangle = \langle b_j^+ b_j\rangle\langle\langle b_j C|D\rangle\rangle \ . \tag{1}$$

If we choose $C=1$, $D=b_j^+$ we obtain one of the DTGF which must be decoupled to determine the magnetic properties of the system. In this case Eq. (1) becomes

$$\langle\langle b_j^+ b_j b_j|b_j^+\rangle\rangle_E = \langle b_j^+ b_j\rangle\langle\langle b_j|b_j^+\rangle\rangle_E \ . \tag{2}$$

Equating the first moment of each side of Eq. (2) gives

$$\langle [b_j^+ b_j b_j, b_j^+]\rangle = \langle b_j^+ b_j\rangle\langle [b_j, b_j^+]\rangle \tag{3}$$

which becomes, upon evaluating the commutators,

$$2\langle b_j^+ b_j\rangle = \langle b_j^+ b_j\rangle \ . \tag{4}$$

The decoupling of Eq. (1) is therefore non-moment-preserving. The same difficulty arises in the other decouplings used in Ref. 1. In addition, if the spectral theorem[3] is applied to both sides of Eq. (2) we obtain the correlation relation

$$\langle b_j^+ b_j^+ b_j b_j\rangle = \langle b_j^+ b_j\rangle^2 \ , \tag{5}$$

i.e.,

$$\langle (b_j^+ b_j)^2\rangle = \langle b_j^+ b_j\rangle(1+\langle b_j^+ b_j\rangle) \ . \tag{6}$$

However, if the decouplings of Ref. 1 (with $C=1, D=b_j^+$) are used for the $I=0$ limit of the Hamiltonian of Ref. 1 [cf. Eq. (11) therein] one readily obtains correlation relations including

$$\langle (b_j^+ b_j)^2\rangle = \langle b_j^+ b_j\rangle(1+2\langle b_j^+ b_j\rangle) \tag{7}$$

which is inconsistent with the result [Eq. (6)] obtained directly from the decouplings. Similar difficulties arise with the other decouplings of Ref. 1. In this sense, the decouplings of Ref. 1 are non-self-consistent.

A decoupling scheme can be developed which does not suffer from the above deficiencies. Based on the concepts of cumulant averages,[3] for the commutator DTGF of Ref. 1 it produces the symmetric decouplings

$$\langle\langle b_j^+ b_j b_j C|D\rangle\rangle_E = 2\langle b_j^+ b_j\rangle\langle\langle b_j C|D\rangle\rangle_E \ , \tag{8}$$

$$\langle\langle b_j^+ b_j a_i^+ C|D\rangle\rangle_E = \langle b_j^+ b_j\rangle\langle\langle a_i^+ C|D\rangle\rangle_E + \langle b_j^+ a_i^+\rangle\langle\langle b_j C|D\rangle\rangle_E \ , \tag{9}$$

$$\langle\langle b_j^+ a_i^+ a_i^+ C|D\rangle\rangle_E = 2\langle b_j^+ a_i^+\rangle\langle\langle a_i^+ C|D\rangle\rangle_E \ , \tag{10}$$

$$\langle\langle a_i^+ b_j^+ C|D\rangle\rangle_E = \langle a_i^+ b_j^+\rangle\langle\langle C|D\rangle\rangle_E \ , \tag{11}$$

$$\langle\langle a_i b_j C|D\rangle\rangle_E = \langle a_i b_j\rangle\langle\langle C|D\rangle\rangle_E \ , \tag{12}$$

$$\langle\langle a_i^+ b_j^+ b_j^+ b_j C|D\rangle\rangle_E = 2\langle a_i^+ b_j^+\rangle\langle\langle b_j^+ b_j C|D\rangle\rangle_E \ , \tag{13}$$

$$\langle\langle a_i^+ a_i a_i b_j C|D\rangle\rangle_E = 2\langle a_i^+ a_i\rangle\langle\langle a_i b_j C|D\rangle\rangle_E \ . \tag{14}$$

It is readily shown that the results of Ref. 1 reduce to those obtained by the moment-preserving, self-consistent

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decouplings of Eqs. (8)–(14) under the transformations

$$S \rightarrow S/2 \,, \tag{15}$$

$$J_{ij} \rightarrow 2J_{ij} \,. \tag{16}$$

That is, the results reported by Li and co-workers for a system of spin S and coupling constant J_{ij} are, instead, appropriate for a system of spin $S/2$ and coupling constant $2J_{ij}$.

[1] J-X Li, D-C Tian, and Q. Jiang, Phys. Rev. B **47**, 11 905 (1993).
[2] J-X Li, Q. Jiang, Z-H. Zhang, and D-C Tian, Phys. Rev. B **46**, 14 095 (1992).
[3] E. B. Brown, Phys. Rev. A **42**, 7107 (1990).