Quantum transport of electrons scattered inelastically from disordered media

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The effect of the multiple elastic scattering of electrons in disordered media on the features of a type of weak localization is studied. This type of weak localization necessarily involves inelastic scattering and is the localization of electrons scattered inelastically with a fixed value of the lost energy. Earlier the possibility of such a localization was established in a model when electrons underwent multiple scattering through small angles and single scattering through a large angle. In this paper, an attempt is made to show that multiple elastic scattering of electrons through arbitrary angles does not destroy this type of weak localization. Also, the influence of the surface on the angular distribution of the scattering of electrons from a semi-infinite disordered system under conditions of quantum transport is treated. The results obtained can be applied to the theory of Raman scattering of light by randomly distributed scatterers.

I. INTRODUCTION

Quantum transport is a type of motion of particles in disordered media in which particles undergo collisions, with the next act of scattering beginning before the previous one has ended. The motion of particles may result either from the influence of an applied electric field or due to the initial kinetic energy. In this paper we shall deal with the case in which it is due to the initial kinetic energy.

The classical kinetic equation for the distribution function of electrons does not account for the phenomenon of quantum transport. The discussion about the nature of the corrections to the kinetic equation started in the 1930's.[1] These corrections were more thoroughly evaluated after the kinetic equation had been derived from the Liouville-von Neumann equation for the density matrix in the framework of quantum statistical physics.[2,3]

Corrections of the same nature also exist in the theory of radiation transfer. They result from the interference of a field of a classical type. The steady wave equation of the electromagnetic theory and the stationary Schrödinger equation have the same mathematical form, both containing the Laplacian as their only differential operator. The kinetic equation for electrons in solids and the equation of radiation transfer in the electromagnetic theory can be derived from the Schrödinger equation and Maxwell's equations, respectively, and are approximations to them. Naturally, certain results obtained from the approximative and precise approaches turned out to be different. Backscattering of radiation from a turbulent atmosphere is an example of a classical problem in which the two approaches yield different results.[4,5]

A theory describing the quantum corrections to the usual kinetic equation began to develop rapidly as soon as it was realized that disordered media is a fertile ground for the investigation of the phenomenon of quantum transport. The concept of Anderson localization has appeared during the investigation of electron motion in a random medium.[6–9] Weak localization of electrons is a manifestation of quantum corrections to the classical kinetic equation in the problem of electron conductance in disordered media. This localization also exhibits itself in the transport of photons in classical wave transfer.

In recent years, quantum transport in general and weak localization in particular have been studied on a wide scale, both experimentally and theoretically. Work is being done in searching for new systems and situations in which weak localization can exist; there has been discussion of the effects involving weak localization of waves in artificially created incommensurate layer systems[10] and surface waves.[11] A treatment of weak localization in both disordered and nonlinear media was given in Ref. 12. Intensity correlations under conditions of weak localization are also worthy of attention.[13] Negative anomalous magnetoresistance in solid-state physics and weak localization of light in classical electrodynamics give strong evidence for the existence of peculiar interference phenomena.

The usual weak localization of electrons is due to the elastic collisions of electrons with scatterers. There is widely spread prejudice that any inelasticity should destroy interference and make the value of the quantum corrections negligible, which holds, of course, for ordinary weak localization. However, inelastic processes proved to be the source of new quantum interference phenomena. One of them is the new type of weak localization, in which an electron suffers an event of inelastic collision and undergoes the usual elastic scattering, escaping the medium with a fixed energy loss.[14]

The phenomenon of the usual weak localization has been verified in experiments on electron conductance, viz., with slow electrons. Negative magnetoresistance may be regarded as evidence for such a localization. In the case of fast electrons or light there is only one possibility to observe the phenomenon by the means of registering the current of electrons or the intensity of photons, which undergo scattering in a random medium and escape the medium. Experiments with light confirm the existence of weak localization and are easier to conduct

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than those with fast electrons, due to the comparatively large value of the ratio of λ/l in the case of light. Here λ is the wavelength of the light wave and l is mean free path of the photons. The usual weak localization of fast electrons is to be manifested in an increase of the elastic backward scattering in an extremely narrow range of solid angles of the order of λ/l, where λ is the wavelength of electron wave and l is mean free path of the electron. In this case the ratio $\lambda/l \ll 1$, and there is little chance of registering the current of fast electrons that have been scattered during the usual weak localization process, and escaped the target within such a small solid angle.

In contrast to the ordinary weak localization the new type of localization takes place at other scattering angles, the scattering being greatest in the range of angles close to $\pi/2$. This range is of the order of $\gamma/\omega=(\lambda/l)(E/\hbar\omega)$, where γ is the electron-collision frequency, E is the electron energy, and $\hbar\omega$ is the energy loss. The range of angles is considerably wider than in the case of the usual localization. Apart from its purely theoretical interest, the latter circumstance is also important in that it makes direct experimental observation of the localization of comparatively fast electrons more realistic than that of ordinary localization.

The possibility of the existence of this different type of weak localization was shown in Ref. 14 on the assumption that the electron trajectory is determined by a single elastic scattering of the electron through a large angle and multiple scattering through small angles. The event of inelastic scattering was also taken into account. Thus, the question arises as to whether the multiple elastic scattering of the electrons through arbitrary angles destroys this type of weak localization. Besides, the presence of a surface can alter the angular distribution of scattered electrons.

In Ref. 14 we were dealing solely with weak localization. Quantum transport, however, is not reduced to weak localization only; it is possible that weak localization is negligible under conditions of strong quantum interference. To this circumstance we shall also pay attention.

In this paper we shall demonstrate that multiple elastic scattering of electrons through arbitrary angles does not destroy this type of weak localization, with the maximum of the angular distribution of inelastically scattered electrons lying in the same range of the electron-scattering angles as was found in Ref. 14. We shall also show how the presence of a surface affects the probability of electron scattering under quantum transport.

The paper is structured as follows. In Sec. II we shall treat the structure of density matrix and its connection with the angular distribution of electrons scattered inelastically with a fixed energy loss. In Sec. III the density matrix of the coherent part of the field is presented. In Sec. IV we shall consider the kinetic description of quantum transport. Section V deals with the case of an infinite disordered medium. In Sec. VI we describe the influence of a surface on the angular distribution of scattered electrons when the medium occupies a semi-infinite space. Section VII contains summary and conclusions.

II. DENSITY-MATRIX DESCRIPTION OF QUANTUM TRANSPORT

We shall designate by $\psi(\mathbf{r},\mathbf{R})$ the wave function of a system that consists of a particle undergoing scattering and a disordered medium. Here $\mathbf{r}$ is the position of the particle, and $\mathbf{R}$ is the set of the position vectors of the electrons of the medium. The wave function $\psi(\mathbf{r},\mathbf{R})$ is governed by the Schrödinger equation

$$\Delta\psi(\mathbf{r},\mathbf{R})+\frac{2m}{\hbar^2}[E-U(\mathbf{r})-U_e(\mathbf{R})-U_{ee}(\mathbf{r},\mathbf{R})]\times\psi(\mathbf{r},\mathbf{R})=0 . \quad (1)$$

Here the Laplace operator $\Delta=\Delta_{\mathbf{r}}+\Delta_{\mathbf{R}}$ contains second derivation with respect to all components of the variables $\mathbf{r}$ and $\mathbf{R}$. The quantity $U(\mathbf{r})$ is the potential of the force centers distributed in the medium randomly, the particle being scattered by the centers elastically. The operator $U_e(\mathbf{R})$ describes the interaction of the electrons of the medium. The quantity $U_{ee}(\mathbf{r},\mathbf{R})$ is the energy of the interaction of the particle undergoing scattering with the electrons of the medium.

Let us compare Eq. (1) with the inhomogeneous equation

$$\Delta\psi(\mathbf{r},\mathbf{R})+\frac{2m}{\hbar^2}[E-U(\mathbf{r})-U_e(\mathbf{R})-U_{ee}(\mathbf{r},\mathbf{R})]\times\psi(\mathbf{r},\mathbf{R})=g(\mathbf{r},\mathbf{R}) , \quad (2)$$

where $g(\mathbf{r},\mathbf{R})$ is an arbitrary function. The wave function $\psi(\mathbf{r},\mathbf{R})$ can be expanded in terms of orthogonal functions $\Phi_n(\mathbf{R})$ as follows

$$\psi(\mathbf{r},\mathbf{R})=\sum_n \psi_n(\mathbf{r})\Phi_n(\mathbf{R}) , \quad (3)$$

where a complete set of orthogonal functions $\Phi_n(\mathbf{R})$ describes the various states of the medium. The function $\Phi_n(\mathbf{R})$ of the electrons of the medium obeys the equation

$$\Delta_{\mathbf{R}}\Phi_n(\mathbf{R})+\frac{2m}{\hbar^2}[\varepsilon_n-U_e(\mathbf{R})]\Phi_n(\mathbf{R})=0 , \quad (4)$$

where $\varepsilon_n=\hbar\omega$ is the energy lost by the particle.

Substituting (3) into (2), we then multiply this relationship by $\Phi_m^*(\mathbf{R})$, the complex conjugate wave function of the mth state of the medium and integrate with respect to $\mathbf{R}$, thus obtaining

$$\Delta_{\mathbf{r}}\psi_m(\mathbf{r})+\frac{2m}{\hbar^2}[E-\varepsilon_n-U(\mathbf{r})]\psi_m(\mathbf{r}) -\frac{2m}{\hbar^2}\sum_n \psi_n(\mathbf{r})\int \Phi_m^*(\mathbf{R})U_{ee}(\mathbf{r},\mathbf{R})\Phi_n(\mathbf{R})d\mathbf{R}=g_m(\mathbf{r}) , \quad (5)$$

where we designate $g_m(\mathbf{r})$ the quantity

$$g_m(\mathbf{r})=\int \Phi_m^*(\mathbf{R})g(\mathbf{r},\mathbf{R})d\mathbf{R} . \quad (6)$$

Let us introduce the notation

$$T(\mathbf{r},m\to n)=\int \Phi_n^*(\mathbf{R})U_{ee}(\mathbf{r},\mathbf{R})\Phi_m(\mathbf{R})d\mathbf{R} . \quad (7)$$

Then the equation for the function $\psi_n(\mathbf{r})$ can be written in the form

$$\Delta_{\mathbf{r}}\psi_n(\mathbf{r})+\frac{2m}{\hbar^2}[E-\varepsilon_n-U(\mathbf{r})]\psi_n(\mathbf{r}) =\frac{2m}{\hbar^2}\psi_i(\mathbf{r})T(\mathbf{r},i\rightarrow n)+g_m(\mathbf{r}) . \quad (8)$$

Here we assumed that the initial state i is the ground state, and the state m the only excited one.

The solution of Eq. (8) has the form

$$\psi_n(\mathbf{r})=\int G_n(\mathbf{r},\mathbf{r}_1)\frac{2m}{\hbar^2}\psi_1(\mathbf{r}_1)T(\mathbf{r}_1,i\rightarrow n)d\mathbf{r}_1 . \quad (9)$$

From this point we shall omit the arbitrary function $g_m(\mathbf{r})$ in the right-hand part of Eq. (9). The Green's function $G_n(\mathbf{r},\mathbf{r}_1)$ in Eq. (9) describes the particle propagation under conditions of elastic scattering, the energy of the particle being $E-\varepsilon_n$:

$$\Delta G_n(\mathbf{r},\mathbf{r}_1)+\frac{2m}{\hbar^2}[E-\varepsilon_n-U(\mathbf{r})]G_n(\mathbf{r},\mathbf{r}_1)=\delta(\mathbf{r}-\mathbf{r}_1) . \quad (10)$$

The current of electrons scattered inelastically is defined by the density matrix

$$\rho_{nn}(\mathbf{r},\mathbf{r}')=\langle\psi_n(\mathbf{r})\psi_n^*(\mathbf{r}')\rangle=\left[\frac{2m}{\hbar^2}\right]^2\int\langle G_n(\mathbf{r},\mathbf{r}_1)G_n^*(\mathbf{r}',\mathbf{r}_2)\psi_i(\mathbf{r}_1)\psi_i^*(\mathbf{r}_2)T(\mathbf{r}_1,i\rightarrow n)T^*(\mathbf{r}_2,i\rightarrow n)\rangle d\mathbf{r}_1d\mathbf{r}_2 . \quad (11)$$

The angular brackets $\langle\cdots\rangle$ denote the statistical average with respect to the ensemble of atomic potentials. TT^* will be factored out of the brackets if the inelasticity is not connected with the excitation of the force centers.

Strictly speaking, the density matrix depends on both the variables $\mathbf{r}$ and $\mathbf{r}'$, the positions of the particles, and the state of the medium. We shall bear in mind that Eq. (11) is summed over the final states of the medium. Further we shall go over from the summation to integration with respect to momentum transferred to the medium during the event of inelastic scattering for a fixed value of the transferred energy.

The wave function

$$\psi_i(\mathbf{r})=\int G_i(\mathbf{r},\mathbf{r}_1)g_i(\mathbf{r}_1)d\mathbf{r}_1 , \quad (12)$$

where the Green's function corresponds to the particle with the energy E, undergoing elastic scattering. Thus,

$$\rho_{nn}(\mathbf{r},\mathbf{r}')=\left[\frac{2m}{\hbar^2}\right]^2\int\Gamma(\mathbf{r},\mathbf{r}';\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)T(\mathbf{r}_1,i\rightarrow n)T^*(\mathbf{r}_2,i\rightarrow n)g_i(\mathbf{r}_3)g_i^*(\mathbf{r}_4)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3d\mathbf{r}_4 , \quad (13)$$

where the function

$$\Gamma(\mathbf{r},\mathbf{r}';\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)=\langle G_n(\mathbf{r},\mathbf{r}_1)G_n^*(\mathbf{r}',\mathbf{r}_2)G_i(\mathbf{r}_1,\mathbf{r}_3)G_i^*(\mathbf{r}_2,\mathbf{r}_4)\rangle . \quad (14)$$

The simplest density matrix refers to the simplest function Γ shown in Fig. 1. This density matrix has the form

$$\rho_{nn}^0(\mathbf{r},\mathbf{r}')=\left[\frac{2m}{\hbar^2}\right]^2\int\langle G_n(\mathbf{r},\mathbf{r}_1)\rangle\langle G_i(\mathbf{r}_1,\mathbf{r}_3)\rangle\langle G_n^*(\mathbf{r}',\mathbf{r}_2)\rangle\langle G_i^*(\mathbf{r}_2,\mathbf{r}_4)\rangle T(\mathbf{r}_1,i\rightarrow n) \times T^*(\mathbf{r}_2,i\rightarrow n)g_i(\mathbf{r}_3)g_i^*(\mathbf{r}_4)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3d\mathbf{r}_4=\langle\psi_n(\mathbf{r})\rangle\langle\psi_n^*(\mathbf{r}')\rangle . \quad (15)$$

Thus in this approximation the nnth element of the density matrix is the product of two multipliers, $\langle\psi_n(\mathbf{r})\rangle$ being the coherent component of wave field. This approximation takes into account the attenuation of the wave field during penetration into matter. However, no information is available about how the particle is scattered through a large angle.

The diagrams for the force-center-average density matrix containing such information have the form shown in Figs. 2 and 3. The upper and lower solid lines in these figures represent the retarded-particle and advanced-particle Green's functions, respectively. The shaded blocks correspond to all possible connections of crosses on the upper and lower solid lines (a cross designates a force-center potential). The connections represent the physical realization of the statistical average in which the positions $\mathbf{r}_j$ of the force centers in the matter are assumed to be randomly distributed. A wavy line joins the two points that correspond to the factors $T(i\rightarrow n)$ and $T^*(i\rightarrow n)$ and describe the event of inelastic scattering with a fixed energy loss. In their nature, the rules for dealing with such diagrams are much the same as in the theory of the electron conductivity of impurity systems.

The contribution of diagram 2(a) to the density matrix $\rho_{nn}(\mathbf{r},\mathbf{r}')$ equals

$$\left[\frac{2m}{\hbar^2}\right]^2\int G_n(\mathbf{r},\mathbf{r}_1)G_i(\mathbf{r}_1,\mathbf{r}_3)G_n^*(\mathbf{r}',\mathbf{r}_2)G_i^*(\mathbf{r}_2,\mathbf{r}_4)M_{ii}(\mathbf{r}_3,\mathbf{r}_4;\mathbf{r}_5,\mathbf{r}_6)T(\mathbf{r}_1,i\rightarrow n)T^*(\mathbf{r}_2,i\rightarrow n)\rho_{ii}^0(\mathbf{r}_5,\mathbf{r}_6)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3d\mathbf{r}_4d\mathbf{r}_5d\mathbf{r}_6 . \quad (16)$$

In writing Eq. (16) we made use of Eq. (12). The result of averaging is clear from the structure of the diagram. From here and what follows we shall omit the symbols of averaging over the centers of elastic scattering. In particular, it is easily seen from diagram 2(a), that in Eq. (16) every Green's function and the shaded block are the force-center averages, i.e., we have the product of average values. The shaded block will be called the M block.

In analogy to this consideration one can write analytical formulas corresponding to the contribution of other diagrams in Fig. 2 to the density matrix. For example, the diagram 2(b) contributes to the matrix $\rho_{nn}(\mathbf{r},\mathbf{r}')$

$$\int G_n(\mathbf{r},\mathbf{r}_1)G_n^*(\mathbf{r}',\mathbf{r}_2)M_{nn}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)\rho_{nn}^0(\mathbf{r}_3,\mathbf{r}_4)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3d\mathbf{r}_4 \ . \tag{17}$$

In contrast to diagrams 2(a) and 2(b), diagrams 2(d)–2(g) contain the M-block with $i\neq n$. The diagrams in Fig. 2 do not contribute to the phenomenon of the different type of weak localization.

Let us take, for example, diagram 2(a) in the momentum representation. The contribution of the diagram is proportional to

$$\int d\mathbf{p}_1 d\mathbf{q}|G_i(\mathbf{p})|^2M_{in}(\mathbf{p}-\mathbf{p}_1)w(\mathbf{q})G_i(\mathbf{p}-\mathbf{p}_1)|G_n(\mathbf{p}-\mathbf{q}-\mathbf{p}_1)|^2G_n^*(\mathbf{p}-\mathbf{q})M_{nn}(\mathbf{Q}-\mathbf{q}-\mathbf{p}_1)|G_n(\mathbf{p}-\mathbf{Q})|^2 \ . \tag{18}$$

Here $\mathbf{p}$ is initial momentum of the particle, $\mathbf{p}-\mathbf{Q}$ is the momentum in the final state, i.e., $\mathbf{Q}$ is the total transferred momentum. The momentum $\mathbf{q}$ is transferred during the event of inelastic collision, $w(\mathbf{q})$ being the probability of such collision.

The quantum interference will give rise to this different type of weak localization,[14] if the dependence of the product of the electron Green's functions corresponding to the virtual states on the upper and lower solid lines in a diagram, on the total transferred momentum $\mathbf{Q}$ remains after the integration with respect to $\mathbf{q}$ and $\mathbf{p}_1$. This dependence holds in the diagrams 3(a) and 3(b) only. The contribution of the diagrams 3(a) and 3(b) to the density matrix is

$$\left[\frac{2m}{\hbar^2}\right]\left\{\int G_n(\mathbf{r},\mathbf{r}_5)G_n^*(\mathbf{r}',\mathbf{r}_2)M_{in}(\mathbf{r}_5,\mathbf{r}_6,\mathbf{r}_7,\mathbf{r}_8)G_i^*(\mathbf{r}_2,\mathbf{r}_6)T^*(\mathbf{r}_2,i\to n)\rho_{ni}^0(\mathbf{r}_7,\mathbf{r}_8)d\mathbf{r}_2d\mathbf{r}_5d\mathbf{r}_6d\mathbf{r}_7d\mathbf{r}_8 \right.$$
$$\left. +\int G_n(\mathbf{r},\mathbf{r}_1)G_i(\mathbf{r}_1,\mathbf{r}_5)G_n^*(\mathbf{r}',\mathbf{r}_6)M_{in}(\mathbf{r}_5,\mathbf{r}_6,\mathbf{r}_7,\mathbf{r}_8)T(\mathbf{r}_1,i\to n)\rho_{in}^0(\mathbf{r}_7,\mathbf{r}_8)d\mathbf{r}_1d\mathbf{r}_5d\mathbf{r}_6d\mathbf{r}_7d\mathbf{r}_8\right\} . \tag{19}$$

All the contributions to the density matrix are independent of the type of the function $g(\mathbf{r},\mathbf{R})$. Therefore our results will be valid if the function $g(\mathbf{r},\mathbf{R})$ in the bulk under consideration is zero.

After finding the density matrix the current density may be calculated from the formula

$$\mathbf{j}=\frac{\hbar}{2m}\lim_{\substack{t'\to t+0\\ \mathbf{r}'\to\mathbf{r}}}(\nabla_{\mathbf{r}'}-\nabla_{\mathbf{r}})G(x,x') \ ,$$

the Green's function $G(x,x')$ being associated with the density matrix as follows

$$\rho(\mathbf{r},\mathbf{r}')=-iG(\mathbf{r},t;\mathbf{r}',t'\to t+0) \ .$$

Here $x=(\mathbf{r},t)$.

However, on dealing with a monoenergetic flow of electrons, we can write the angular distribution $S(\mu)$ of the escape probability of the electrons from a surface at an angle whose value is $\cos^{-1}\mu$, to the surface normal, in the form

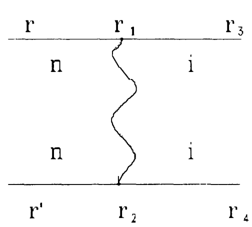

FIG. 1. The simplest diagram (lowest order).

$$S(\mu)=(2\pi)^{-1}A^{-1}k_n^2\mu\rho_{nn}(\mathbf{q},z;\mathbf{q},z)|_{z=z_0} \ . \tag{20}$$

Here $\mathbf{q}$ is the component of the particle momentum parallel to the surface, and A is the area of the surface. k is the absolute value of the wave vector of the particle. $z=z_0$ corresponds to the plane of the surface. Integra-

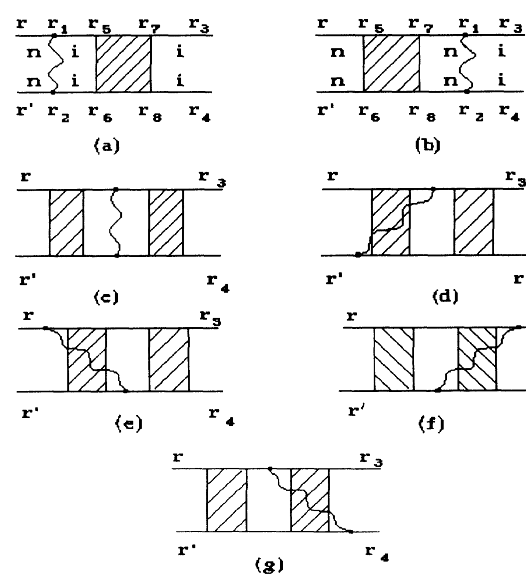

FIG. 2. Diagrammatic representation of the multiple scattering. These diagrams are not connected to the weak localization.

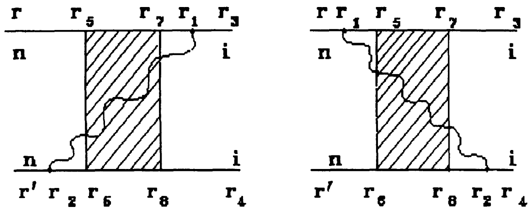

FIG. 3. Diagrammatic representation of the processes of quantum transport.

tion with respect to the azimuth angle was carried out in Eq. (20).

Equation (20) has been used in Ref. 15 and one can easily obtain this formula as follows. The Fourier transform $\rho_{nn}(\mathbf{q},z;\mathbf{q},z)$ of the density matrix $\rho_{nn}(\rho,z;\rho',z')$ corresponds to transition from the position vectors $\boldsymbol{\rho}$ parallel to the surface, to the components $\mathbf{q}$ of the wave vector that is also parallel to the surface. $\rho_{nn}(\mathbf{q},z;\mathbf{q},z)$ is the probability of finding the particle within the plane of the surface $z=z_0$ with $\mathbf{q}$, which is the component parallel to the surface, of the wave vector k_n. Therefore, $\rho_{nn}(q,z_0;q,z_0)A^{-1}(2\pi)^2 d^2q$ is the probability of finding the component of the wave vector $\mathbf{k}_n$ parallel to the surface within the interval $d^2q=q\,dq\,d\phi=-k_n^2\mu\,d\mu\,d\phi$, ϕ being the azimuth angle. The probability is the one per unit area of the surface. Therefore, in the equation

$$\frac{\rho_{nn}(\mathbf{q},z_0;\mathbf{q},z_0)}{A}\frac{d^2q}{(2\pi)^2}=S(\mu,\phi)d\mu\,d\phi$$

the function $S(\mu,\phi)$ is the probability of the particle escaping from the surface within a unit solid angle. On integrating with respect to ϕ we obtain the function

$$S(\mu)=\int_0^{2\pi}S(\mu,\phi)d\phi\ ,$$

which is identical to Eq. (20).

III. DENSITY MATRIX OF ZEROTH ORDER

In the density matrix of the zeroth order,

$$\rho_{in}^0(\mathbf{r},\mathbf{r}')=\langle\psi_i(\mathbf{r})\rangle\langle\psi_n^*(\mathbf{r}')\rangle\ , \tag{21}$$

the coherent component $\langle\psi_i(\mathbf{r})\rangle$ of the electron wave field $\psi_i(\mathbf{r})$, which corresponds to the motion of the electron in the matter without energy loss and change in the direction of motion, obeys the equation

$$\Delta\langle\psi_i(\mathbf{r})\rangle+\frac{2m}{\hbar^2}(E-u_0n)\langle\psi_i(\mathbf{r})\rangle=0\ . \tag{22}$$

The potential of all the force centers can be written in the form

$$U(\mathbf{r})=u_0\sum_{j=1}^{N}\delta(\mathbf{r}-\mathbf{r}_j)\ ,$$

as it is often written in transport theories.[15] So far as the main point of interest in this paper is to show that multiple collisions do not destroy the different type of weak localization, it is sufficient to take the potential of the force centers in the simplest form.

This form is a reasonable approximation when wavelength is long compared to the atomic scale, as in the case of light, for example. As regards fast electrons, the approximation appears to leave out of account the possibility of the anisotropy of elastic scattering. However, in Sec. VII we shall show the present theory in the case of fast electrons to be sound, too.

Further the value u_0 will be understood, in a sense, as an optical potential. Equation (22) must be solved under the boundary condition

$$\langle\psi(\mathbf{r})\rangle|_{z=z_0}=\exp(i\mathbf{k}_0\mathbf{r})\ . \tag{23}$$

If the medium occupies the whole space the plane $z=z_0$ will coincide with a remote plane boundary of the medium.

The solution of Eq. (22) under boundary condition, Eq. (23), has the form

$$\langle\psi_i(\mathbf{r})\rangle=\exp\{i(\mathbf{k}_{0\parallel}\boldsymbol{\rho}+k_z z)\}\ , \tag{24}$$

where $k=\sqrt{k_0^2-2m\hbar^{-2}nu_0}$. $\boldsymbol{\rho}$ is the component parallel to the boundary surface of the position vector $\mathbf{r}$. We assume that the direct backscattering of electron waves from the surface is negligible.

The diagonal element of the density matrix with respect to the entrance channel is

$$\rho_{ii}^0(\mathbf{r},\mathbf{r}')=\exp\{i[\mathbf{k}_{0\parallel}(\boldsymbol{\rho}-\boldsymbol{\rho}')+k_z(z-z')]\}\ . \tag{25}$$

Here

$$k_z=\sqrt{k_0^2\mu_i^2-2m\hbar^{-2}nu_0},\quad \mu_i=\cos\vartheta_i\ ,$$

ϑ_i being the angle of the particle incidence onto the surface. $\mathbf{k}_{0\parallel}$ is the component of the wave vector of the incident particle, parallel to the boundary plane.

The elements of the density matrix of the zeroth order with the subscripts $i\neq n$, which refer to channels of scattering are defined as

$$\rho_{in}^0(\mathbf{r},\mathbf{r}')=\frac{2m}{\hbar^2}\exp\{i\mathbf{k}_0\mathbf{r}\}\times\left[\int G_n(\mathbf{r},\mathbf{r}_1)\psi_i(\mathbf{r}_1)T(\mathbf{r}_1,i\to n)d\mathbf{r}_1\right]^*\ . \tag{26}$$

IV. THE *M* BLOCK

As far as the different type of weak localization and the usual weak localization are certain to display[14] themselves in the range of the different angles of scattering, there is no need to take into account the interference of the electron waves, when they undergo elastic scattering only. That is to say, one may consider this type of weak localization and the usual weak localization independently. Therefore the multiple elastic scattering in the M block may be treated in the ladder approximation.

The diagrammatic equation, Fig. 4, for the M block corresponds to the analytical expression which, on averaging, reduces to

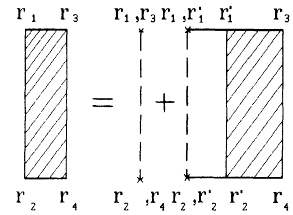

FIG. 4. Diagrammatic representation of the equation for the M block.

$$M(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)= n\int d\mathbf{r}_j\langle \mathbf{r}_1|t_j|\mathbf{r}_3\rangle\langle \mathbf{r}_2|t_j^+|\mathbf{r}_4\rangle + n\int d\mathbf{r}_j d\mathbf{r}_1' d\mathbf{r}_2' d\mathbf{r}_1'' d\mathbf{r}_2''\langle \mathbf{r}_1|t_j|\mathbf{r}_1'\rangle\langle \mathbf{r}_2|t_j^+|\mathbf{r}_2'\rangle G(\mathbf{r}_1',\mathbf{r}_1'')G^*(\mathbf{r}_2',\mathbf{r}_2'')M(\mathbf{r}_1'',\mathbf{r}_2'',\mathbf{r}_3,\mathbf{r}_4)\ . \tag{27}$$

Here t_j is the operator of elastic scattering from the jth center, n is the force center concentration. For potentials of small radius[4]

$$\langle \mathbf{r}_1|t_j|\mathbf{r}_2\rangle = -4\pi f\delta(\mathbf{r}_1-\mathbf{r}_j)\delta(\mathbf{r}_2-\mathbf{r}_j)\ , \tag{28}$$

and Eq. (27) is given as

$$M(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)=\frac{4\pi}{l_{\rm el}}\left[\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_2-\mathbf{r}_4)+\delta(\mathbf{r}_1-\mathbf{r}_2)\int G(\mathbf{r}_1-\mathbf{r}_1'')G^*(\mathbf{r}_2-\mathbf{r}_2'')M(\mathbf{r}_1'',\mathbf{r}_2'';\mathbf{r}_3,\mathbf{r}_4)d\mathbf{r}_1''d\mathbf{r}_2''\right]\ . \tag{29}$$

Here $l_{\rm el}$ is the mean free path for elastic collision. When carrying out the estimations one may consider that

$$n|u_0|^2m^2l_{\rm el}=\pi\hbar^4\ , \tag{30}$$

at $U=u_0\sum_j\delta(\mathbf{r}-\mathbf{r}_j)$, the value u_0 being connected with the scattering amplitude f by the relation

$$(4\pi)^2|f|^2\hbar^4=4m^2|u_0|^2\ . \tag{31}$$

The M block may be represented by the expression

$$M(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)= 4\pi l_{\rm el}^{-1}\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_3-\mathbf{r}_4)\times[\delta(\mathbf{r}_1-\mathbf{r}_3)+F(\mathbf{r}_1,\mathbf{r}_3)]\ . \tag{32}$$

Substituting Eq. (32) in Eq. (29) we infer that the function F is governed by the equation

$$F(\mathbf{r}_1,\mathbf{r}_2)=4\pi l_{\rm el}^{-1}\left\{|G(\mathbf{r}_1,\mathbf{r}_2)|^2+\int d\mathbf{r}'|G(\mathbf{r}_1,\mathbf{r}')|^2F(\mathbf{r}',\mathbf{r}_2)\right\}\ . \tag{33}$$

The equation has the structure of transport equation, provided there are only elastic collisions, and it has already been used in Ref. 15. It should be noted that the function F is not equal to zero, only when multiple elastic collisions exist.

V. A DIFFERENT TYPE OF WEAK LOCALIZATION UNDER MULTIPLE COLLISIONS IN AN INFINITE MEDIUM

In the case of an infinite medium the Green's function, contained in Eq. (33), is given by

$$G(\mathbf{r}_1,\mathbf{r}_2)=-\frac{1}{4\pi|\mathbf{r}_1-\mathbf{r}_2|}\exp\{-\tfrac{1}{2}n\sigma_t|\mathbf{r}_1-\mathbf{r}_2|\}\ , \tag{34}$$

where σ_t is the total scattering cross section, $\sigma_t=\sigma_{\rm el}+\sigma_{\rm inel}$. In such a case Eq. (33) can be written as

$$F(\mathbf{r})=\frac{1}{4\pi l_{\rm el}}\left[\frac{1}{r^2}\exp(-n\sigma_t r)+\int d\mathbf{r}'\frac{\exp(-n\sigma_t|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|^2}F(\mathbf{r}')\right]\ . \tag{35}$$

The Fourier transform of the function $F(\mathbf{r})$, in accordance with the Eq. (35), is

$$F(K)=\frac{\tan^{-1}(Kl_t)}{Kl_{\rm el}-\tan^{-1}(Kl_t)}\ . \tag{36}$$

On letting $Kl_t\to 0$ the function $F(K)$ reduces to

$$(\sigma_t/\sigma_{\rm el}-1)^{-1}\ .$$

Taking $Kl_t,Kl_{\rm el}\gg 1$, we have

$$F(K)\approx\pi/2Kl_{\rm el}\ .$$

In the absence of multiple collisions $F(K)$ equals zero.

Now we shall analyze how the contributions of different diagrams to the density matrix depend on the function F. As the phenomenon of the new type of quantum transport is due to the diagrams shown in Fig. 3, we shall consider those first. Having in mind Eq. (32) we can write the contribution of the diagram 3(a) to the density matrix as follows

$$\frac{4\pi}{l_{el}}\left[\int G_n(\mathbf{r},\mathbf{r}_3)G_n^*(\mathbf{r}',\mathbf{r}_2)T(\mathbf{r}_1)T^*(\mathbf{r}_2)G_i^*(\mathbf{r}_2,\mathbf{r}_3)G_n(\mathbf{r}_3,\mathbf{r}_1)\rho_{ii}^0(\mathbf{r}_1,\mathbf{r}_3)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3 + \int G_n(\mathbf{r},\mathbf{r}_3)G_n^*(\mathbf{r}',\mathbf{r}_2)T(\mathbf{r}_1)T^*(\mathbf{r}_2)G_i^*(\mathbf{r}_2,\mathbf{r}_3)G_n(\mathbf{r}_5,\mathbf{r}_1)F(\mathbf{r}_3-\mathbf{r}_5)\rho_{ii}^0(\mathbf{r}_1,\mathbf{r}_5)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3d\mathbf{r}_5\right] . \quad (37)$$

Here we have changed variables in the integrals. The first addent in the formula corresponds to the case in which there is the only event of elastic collision. The second addent is due to multiple elastic collisions, and can be obtained from the first one by a formal substitution

$$G_n(\mathbf{r}_3,\mathbf{r}_1)\rho_{ii}^0(\mathbf{r}_1,\mathbf{r}_3)\rightarrow \int G_n(\mathbf{r}_5,\mathbf{r}_1)F(\mathbf{r}_5-\mathbf{r}_3)\rho_{ii}^0(\mathbf{r}_1,\mathbf{r}_5)d\mathbf{r}_5 . \quad (38)$$

The last integral can be written as

$$\int G_n(\mathbf{r}_3+\mathbf{s},\mathbf{r}_1)F(\mathbf{s})\rho_{ii}^0(\mathbf{r}_1,\mathbf{r}_3+\mathbf{s})d\mathbf{s} , \quad (39)$$

where $\mathbf{s}=\mathbf{r}_5-\mathbf{r}_3$, the new variable of integration.

By virtue of the well-known integral representation of the Green's function $G_n(\mathbf{r}_3+\mathbf{s},\mathbf{r}_1)$ and making use of Eq. (25) we are led to a form of Eq. (39), which contains an integration in momentum space. One may write it as

$$\rho_{ii}^0(\mathbf{r}_1,\mathbf{r}_3)\int \frac{d^3k_1}{(2\pi)^3}\frac{\exp[i\mathbf{k}_1(\mathbf{r}_3-\mathbf{r}_1)]}{E_n-E_{k_1}+i\Gamma}F(\mathbf{k}-\mathbf{k}_1) . \quad (40)$$

Here $\mathbf{k}$ is the initial momentum of the particle. It can be shown from Eq. (40) and the left-hand side of the Eq. (38) that the function

$$\tilde{G}_n(\mathbf{r}_3-\mathbf{r}_1)=\int \frac{d^3k_1}{(2\pi)^3}\frac{\exp[i\mathbf{k}_1(\mathbf{r}_3-\mathbf{r}_1)]}{E_n-E_{\mathbf{k}_1}+i\Gamma}F(\mathbf{k}-\mathbf{k}_1) \quad (41)$$

might be interpreted as a function taking into account multiple elastic scattering.

It suffices for our purposes to show that multiple elastic collisions do not destroy the new type of weak localization. This problem can be solved by studying the form of the function that is given by Eq. (41).

The function $F(K)$ will just describe multiple collisions if $F(K)>1$. It is the case that corresponds to the inequality $Kl_t \ll 1$. When $K=|\mathbf{k}_1-\mathbf{k}|$ is small, the function

$$F(K)=\left[\frac{l_{el}}{l_t}-1+\frac{1}{3}(Kl_t)^2\right]^{-1} . \quad (42)$$

If the difference $l_{el}-l_t$ is not equal to zero, the function $F(K)$ will be constant under the condition

$$(Kl_t)^2<\frac{l_{el}}{l_t}-1 .$$

Therefore in the multiple-collision approximation we have to reduce to a finite value the upper limit of the integration in Eq. (41) in momentum space. This limit is $\tilde{k}_1$, a certain value that may be determined as follows:

$$\tilde{k}_1\approx k+\frac{1}{l_t}\sqrt{l_{el}/l_t-1} .$$

It yields

$$\tilde{G}_n(R)=F(K=0)G_n(R) . \quad (43)$$

We have been neglecting the term that is due to the part of the integration path from $\tilde{k}_1$ to ∞. The integral that is due to $k_1>\tilde{k}_1$ can be estimated as follows. We write this integral in the form

$$\int_{|k_1|>\tilde{k}_1}\frac{d^3k_1}{(2\pi)^3}\frac{e^{i\mathbf{k}_1\mathbf{R}}}{E_n-E_{k_1}-i\Gamma}F(\mathbf{k}_1-\mathbf{k})=-\frac{3}{l_t^2}\int_{|k_1|>\tilde{k}_1}\frac{d^3k_1}{(2\pi)^3}\frac{e^{ik_1R\cos\vartheta_1}}{k_1^2E_{k_1}} = -\frac{3m}{\pi^2l_t^2}R\left[\frac{\sin x_1}{2x_1^2}+\frac{\cos x_1}{2x_1}-\frac{1}{2}[Si(\infty)-Si(x_1)]\right] , \quad (44)$$

where $x_1=\tilde{k}_1R$. Let us estimate the terms of Eq. (44) with respect to the leading term that is given by Eq. (43). There are four terms in Eq. (44). The ratio of the first term to the leading term is

$$\frac{3}{\pi l_t^2\tilde{k}_1^2F(K=0)}e^{-ik_nR}\sin\tilde{k}_1R ,$$

where $\tilde{k}_1=k+l_t^{-1}(l_{el}/l_t-1)^{1/2}\approx k$, since $kl_t \ll 1$. Therefore the ratio is of the order

$$(kl_t)^{-2}\left[\frac{l_{el}}{l_t}-1\right]\left[e^{i(k-k_n)R}-e^{-i(k+k_n)R}\right] .$$

Since $(kl_t)^2\gg 1$, this ratio is small.

The ratio of the second term in Eq. (44) to the leading one is

$$\frac{3m}{\pi(\tilde{k}_1l_t)F(K=0)}\left[\frac{R}{l_t}\right]e^{-ik_nR}\cos\tilde{k}_1R .$$

At lengths of the order $R \sim l_t$ this ratio is small because of the presence of $\tilde{k}_1 l \approx k l_t \gg 1$ in the denominator.

The ratio of the terms containing the integral sines to the leading term is of the order of

$$\frac{3}{\pi F(K=0)} \left[\frac{R}{l_t} \right]^2 e^{-ik_n R} [Si(\infty) - Si(kR)] \; . \tag{45}$$

Since, at $kR \gg 1$,

$$Si(\infty) - Si(kR) \approx \frac{1}{kR}(\cos kR + \sin kR) \; ,$$

the ratio Eq. (45) is also small at distances of the order $R \sim l_t$ because of the presence of $k l_t \gg 1$ in the denominator.

Thus,

$$\tilde{G}_n(R) = \frac{m}{2\pi R} \left[\frac{l_{\rm el}}{l_t} - 1 \right]^{-1} e^{ik_n R} \; . \tag{46}$$

The contribution of the diagram 3(b) may be expressed in terms of the contribution of the diagram 3(a), so far as the diagram 3(b) is obtained from the diagram 3(a) by complex conjugation with mutual replacement of the variables $\mathbf{r}$ and $\mathbf{r}'$. Therefore the diagram 3(b) has the same properties as the diagram 3(a).

From Eq. (46) we conclude that the density matrix in the multiple-collision approach equals the density matrix taking into account only a single elastic collision multiplied by the factor

$$F(K=0) = \left[\frac{l_{\rm el}}{l_t} - 1 \right]^{-1} \; . \tag{47}$$

The value of this factor is above unit, and does not depend on coordinates, and, therefore, does not alter the angular dependence of scattered particles.

It is easy to demonstrate that diagrams shown in Fig. 2 have the same property, which is a precise result. For example, one may represent the contribution of the diagram 2(a) to the density matrix as follows:

$$\frac{4\pi}{l_{\rm el}} \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_5 d\mathbf{r}_6 d\mathbf{r}_7 d\mathbf{r}_8 G_n(\mathbf{r},\mathbf{r}_1) G_n^*(\mathbf{r}',\mathbf{r}_2) T(\mathbf{r}_1) T^*(\mathbf{r}_2) G_1(\mathbf{r}_1,\mathbf{r}_5) G_i^*(\mathbf{r}_2,\mathbf{r}_6)$$
$$\times \delta(\mathbf{r}_5-\mathbf{r}_6)\delta(\mathbf{r}_7-\mathbf{r}_8)[\delta(\mathbf{r}_5-\mathbf{r}_7)+F(\mathbf{r}_5-\mathbf{r}_7)]\rho_{ii}^0(\mathbf{r}_7,\mathbf{r}_8) \; .$$

Let us distinguish a combination C of functions, which is given by

$$C(\mathbf{r}_1,\mathbf{r}_5,\mathbf{r}_6,\mathbf{r}_7) = \int d\mathbf{r}_8 G_i(\mathbf{r}_1,\mathbf{r}_5)\delta(\mathbf{r}_5-\mathbf{r}_6)\delta(\mathbf{r}_7-\mathbf{r}_8)[\delta(\mathbf{r}_5-\mathbf{r}_7)+F(\mathbf{r}_5-\mathbf{r}_7)]\rho_{ii}^0(\mathbf{r}_7,\mathbf{r}_8) \; .$$

Making the variable change

$$\mathbf{S} = \mathbf{r}_5 + \mathbf{r}_7 \text{ and } \mathbf{s} = \mathbf{r}_5 - \mathbf{r}_7 \; ,$$

we shall write the integral of C

$$\int d\mathbf{s}\, d\mathbf{S} C = \int d\mathbf{s}\, d\mathbf{S}\, d\mathbf{r}_8 G_i[\mathbf{r}_1, \tfrac{1}{2}(\mathbf{s}+\mathbf{S})]\delta(\tfrac{1}{2}(\mathbf{s}+\mathbf{S})-\mathbf{r}_6)\delta(\tfrac{1}{2}(\mathbf{S}-\mathbf{s})-\mathbf{r}_8)[\delta(\mathbf{s})+F(\mathbf{s})]\rho_{ii}^0(\tfrac{1}{2}(\mathbf{S}-\mathbf{s}),\mathbf{r}_8)$$
$$= G_i(\mathbf{r}_1,\mathbf{r}_6) \int d\mathbf{s}\, d\mathbf{r}_8 \rho_{ii}^0(\mathbf{r}_6-\mathbf{s},\mathbf{r}_8)\delta(\mathbf{s}-\mathbf{r}_6-\mathbf{r}_8)[\delta(\mathbf{s})+F(\mathbf{s})] \; .$$

Having in mind that $\rho_{ii}^0(0)=1$, after integration with respect to $\mathbf{r}_8$ we obtain

$$G_i(\mathbf{r}_1,\mathbf{r}_6) \int d\mathbf{s}[\delta(\mathbf{s})+F(\mathbf{s})]\rho_{ii}^0(0) = G_i(\mathbf{r}_1,\mathbf{r}_6) \left[1 + \int F(\mathbf{s}) d\mathbf{s} \right] = G_i(\mathbf{r}_1,\mathbf{r}_6)[1+F(K=0)] \; .$$

Thus, the influence of multiple elastic collisions on the contribution of both the diagram depicted in Fig. 2 and the diagrams depicted in Fig. 3 to the density matrix reduces to the multiplication of the old density matrix (which takes into account a single elastic scattering through large angle only and an event of inelastic collision with a fixed energy loss) with the factor $1+F(K=0)$, $F(K=0)$ being independent of scattering angle. Thus we may conclude that the different type of weak localization is not suppressed by elastic multiple scattering at arbitrary angles.

VI. EFFECTS OF A SURFACE ON THE ANGULAR DISTRIBUTION OF PARTICLES IN THE THEORY OF A DIFFERENT TYPE OF QUANTUM TRANSPORT

The surface of the disordered medium affects the angular distribution of scattered particles. First of all, changes in the angles of the particle incidence or escape cause changes in the path the particle runs in the matter. Since the total attenuation of the intensity of electron waves depends on this path, one may expect the existence of an additional, with respect to weak localization in endless media, angular dependence of scattering probability. Secondly, the probability of inelastic scattering with a fixed energy loss also depends on the time the particle spends in the medium. Lastly, this dependence in the ladder and crossed diagrams is different. All these circumstances can be taken into account by a simple model in which the change in the direction of the particle's motion is due to the single elastic scattering through a large angle. Such approach has been applied to the theory of the different[14] and the usual[16] weak localization in endless media.

In this treatment the contribution of the diagrams, Fig. 3, to the density matrix in accordance with Eq. (19) is given by

$$(4\pi)^2\left[\frac{2m}{\hbar^2}\right]n|f|^2\int G_n(\mathbf{r},\mathbf{r}_1)G_n^*(\mathbf{r}',\mathbf{r}_2)G_i^*(\mathbf{r}_2,\mathbf{r}_1)\times T^*(\mathbf{r}_2,i\to n)\rho_{ni}^0(\mathbf{r}_1,\mathbf{r}_1)d\mathbf{r}_1 d\mathbf{r}_2 + \text{complex conjugation}\ . \quad (48)$$

In studying Eq. (48), for simplicity let us assume that the operators of the inelastic collisions possess the same property as the operators of elastic collisions, i.e.,

$$\left[\frac{2m}{\hbar^2}\right]^2 T^*(\mathbf{r}_2,i\to n)T(\mathbf{r}_1,i\to n)=w\delta(\mathbf{r}_1-\mathbf{r}_2)\ .$$

Then w is the probability of inelastic scattering per length in an endless medium. Now Eq. (48) may be rewritten in the form

$$(4\pi)^2 n|f|^2 w\int d\mathbf{r}_1 d\mathbf{r}_2 G_n(\mathbf{r},\mathbf{r}_1)G_n^*(\mathbf{r}',\mathbf{r}_2)G_n(\mathbf{r}_1,\mathbf{r}_2)\times G_i^*(\mathbf{r}_2,\mathbf{r}_1)\rho_{ii}^0(\mathbf{r}_2,\mathbf{r}_1)+\text{c.c.} \quad (49)$$

Here the product of the Green's functions

$$G_n(\mathbf{r}_1,\mathbf{r}_2)G_i^*(\mathbf{r}_2,\mathbf{r}_1)=\frac{1}{(4\pi)^2|\mathbf{r}_1-\mathbf{r}_2|^2}\exp\left[i\left(|\mathbf{k}-\mathbf{q}|+\frac{2\pi nf}{|\mathbf{k}-\mathbf{q}|}-|\mathbf{k}_f+\mathbf{q}|-\frac{2\pi nf}{|\mathbf{k}_f+\mathbf{q}|}\right)|\mathbf{r}_2-\mathbf{r}_1|\right]$$
$$=\frac{1}{(4\pi)^2|\mathbf{r}_1-\mathbf{r}_2|^2}\exp\{i(|\mathbf{k}-\mathbf{q}|-|\mathbf{k}_f+\mathbf{q}|)(1-2\pi nk^{-2}\text{Re}f)|\mathbf{r}_2-\mathbf{r}_1|\}\exp(-n\sigma_t|\mathbf{r}_2-\mathbf{r}_1|)\ . \quad (50)$$

Here f is the elastic scattering amplitude, $\hbar\mathbf{k}$ and $\hbar\mathbf{k}_f$ are the initial and final momenta of the particle, respectively. $\sigma_t=(4\pi/k)\text{Im}f$ is the total scattering cross section. For convenience we write the absolute values of the momenta $\hbar\mathbf{k}$ and $\hbar k_n$ of the particle in the states before and after the event of the inelastic collision in the form $k=|\mathbf{k}_f+\mathbf{q}|$ and $k_n=|\mathbf{k}-\mathbf{q}|$, where $\mathbf{q}$ is the momentum transfer during the event of inelastic scattering with a fixed energy loss because it allows us to perform the summing up over the final states of the medium in the simplest way.

On expending in powers of $\mathbf{q}$ in Eq. (50), we shall have

$$n|f|^2 w\int G_n(\mathbf{r},\mathbf{r}_1)G_n^*(\mathbf{r}',\mathbf{r}_2)|\mathbf{r}_1-\mathbf{r}_2|^{-2}\exp(-n\sigma_t|\mathbf{r}_1-\mathbf{r}_2|)\exp\left[i\frac{\omega}{v}|\mathbf{r}_2-\mathbf{r}_1|\right]\times\exp[-q(\nu_i+\nu_f)|\mathbf{r}_1-\mathbf{r}_2|]\rho_{ii}^0(\mathbf{r}_1,\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2+\text{c.c.} \quad (51)$$

Here $\mathbf{v}$ is the particle velocity, $\nu_i=\cos(\widehat{\mathbf{q},\mathbf{k}})$ and $\nu_f=\cos(\widehat{\mathbf{q},\mathbf{k}_f})$. For convenience, we usually suppress the terms with Ref.

Since the angular distribution of backscattering particles is determined by the integral [see Eq. (20)]

$$\int d\boldsymbol{\rho}\, d\boldsymbol{\rho}'\rho(\mathbf{r},\mathbf{r}')\exp[-i\mathbf{k}_\parallel(\boldsymbol{\rho}-\boldsymbol{\rho}')]\ ,$$

it is necessary to perform a Fourier transformation of Eq. (51) taking into account the relation

$$\int d\boldsymbol{\rho}\exp(-i\mathbf{k}_\parallel\boldsymbol{\rho})G_n(\boldsymbol{\rho},z;\mathbf{r}')|_{z=z_0}=\frac{\exp(-ik_z z)}{2ik_z}\langle\psi_0(\mathbf{r}',\tilde{\mathbf{k}})\rangle\ . \quad (52)$$

Here $\tilde{\mathbf{k}}=(-\mathbf{k}_\parallel,k_z)$. Equation (52) can be easily verified by direct calculation, bearing in mind, that in the medium the Green's function obeys the equation

$$[\Delta+k^2+4\pi nf]G(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}')$$

and has the form

$$G(\mathbf{r},\mathbf{r}')=-\frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}\exp\left[i\left(k_0+\frac{2\pi nf}{k_0}\right)|\mathbf{r}-\mathbf{r}'|\right]\ ,$$

whereas the function $\psi_0(\mathbf{r},\mathbf{k})$ is the solution of the equation

$$[\Delta+k^2+4\pi nf]\psi_0(\mathbf{r},\mathbf{k})=0\ ,$$

the function $\psi_0(\mathbf{r},\mathbf{k})$ on a boundary surface having the form

$$\exp[i(k_z z-\mathbf{k}_\parallel\boldsymbol{\rho})]\ .$$

In performing the Fourier transformation of Eq. (51) mentioned above we find that

$$\int d\boldsymbol{\rho}\, d\boldsymbol{\rho}'\rho(\mathbf{r},\mathbf{r}')\exp[-i\mathbf{k}_\parallel(\boldsymbol{\rho}-\boldsymbol{\rho}')]|_{z,z'\to z_0}=\frac{n|f|^2 w}{4k_z^2}\int\langle\psi_0(\mathbf{r}_2,\tilde{\mathbf{k}})\rangle\langle\psi_0^*(\mathbf{r}_1,\tilde{\mathbf{k}})\rangle|\mathbf{r}_2-\mathbf{r}_1|^{-2}$$
$$\times\exp(-n\sigma_t|\mathbf{r}_2-\mathbf{r}_1|)\exp\left[i\frac{\omega}{v}|\mathbf{r}_2-\mathbf{r}_1|\right]\exp[-q(\nu_i+\nu_f)|\mathbf{r}_2-\mathbf{r}_1|]\langle\psi_0(\mathbf{r}_1)\rangle\langle\psi_0^*(\mathbf{r}_2)\rangle d\mathbf{r}_1 d\mathbf{r}_2+\text{c.c.} \quad (53)$$

The integrand of Eq. (53) contains the product of functions

$$\langle\psi_0(\mathbf{r}_2,\tilde{\mathbf{k}})\rangle\langle\psi_0^*(\mathbf{r}_1,\tilde{\mathbf{k}})\rangle\langle\psi_0(\mathbf{r}_1,\mathbf{k})\rangle\langle\psi_0^*(\mathbf{r}_2,\mathbf{k})\rangle\ . \quad (54)$$

From Eq. (54) one can factor out a multiplier

$$\exp\left[-\frac{n\sigma_t}{2}\left[\frac{1}{\mu_i}+\frac{1}{|\mu_f|}\right](z_1+z_2)\right], \tag{55}$$

where μ_i and μ_f are the cosines of the angles of incidence and escape, respectively. Now Eq. (53) can be written in the form

$$\frac{A}{4k_z^2}n|f|^2w\int dz_1dz_2d\rho\frac{\exp(-\{n\sigma_t-i[\omega/v-q(v_i+v_f)]\}\sqrt{\rho^2+(z_1-z_2)^2})}{\rho^2+(z_1-z_2)^2}$$
$$\times\exp\{i(\mathbf{k}_f+\mathbf{k})_{\|}\rho+i(k_{fz}-k_z)(z_2-z_1)-(n\sigma_t/2\tilde{\mu})(z_1+z_2)\}+\text{c.c.} \tag{56}$$

Here A is the area of the surface.

$$\tilde{\mu}=\frac{\mu_i|\mu_f|}{\mu_i+|\mu_f|}. \tag{57}$$

Having in mind Eqs. (56) and (20) one can write an expression for the number of particles with energy E_n moving from the surface at an angle whose value is $\cos^{-1}|\mu_f|$, to the surface normal

$$S(\mu_i,\mu_f)=\frac{n|f|^2wk_n^2|\mu_f|}{(4\pi)^2k_z^2}\int d\mathbf{q}\,d\rho\,dz_1dz_2\frac{\exp(-\{n\sigma_t-i[\omega/v-q(v_i+v_f)]\}\sqrt{\rho^2+(z_1-z_2)^2})}{\rho^2+(z_1-z_2)^2}$$
$$\times\exp\{i(\mathbf{k}_{f\|}-\mathbf{k}_{\|})\rho+i(k_{fz}-k_z)(z_2-z_1)-(n\sigma_t/2)(z_1+z_2)\}+\text{c.c.} \tag{58}$$

Here the integral with respect to $\mathbf{q}$ corresponds to summing up over the final states of the medium.

On setting $q\rightarrow 0$ Eq. (58) is simplified. In this case the integration in Eq. (58) can be performed in an analytical form, and the angular part of the Eq. (58) is given by

$$g_c(\mu_i,\mu_f)=\tilde{\mu}\{1-\mu_i|\mu_f|+[(1-\mu_i^2)(1-\mu_f^2)]^{1/2}\}^{-1/2}. \tag{59}$$

If all angles are situated in the same plane then we shall have

$$g_c=\frac{\tilde{\mu}}{\sqrt{1-\cos\chi}},$$

where χ is the angle of scattering.

In contrast to the crossed diagrams at $q=0$ the ladder diagrams turned out to correspond to the angular part

$$g_L(\mu_i,\mu_f)=\tilde{\mu}\ln\left[\left[1+\frac{1}{\mu_i}\right]^{\mu_i}\left[1+\frac{1}{|\mu_f|}\right]^{|\mu_f|}\right]. \tag{60}$$

The quotient of the contribution of the crossed diagrams to the contribution of the ladder diagrams is

$$\frac{J_c}{J_L}=\pi\sqrt{2}\frac{\lambda}{l_t}\frac{g_c(\mu_i,\mu_f)}{g_L(\mu_i,\mu_f)}. \tag{61}$$

At $q\neq 0$, when the contribution of the crossed diagrams is described by Eq. (58), the quotient

$$\frac{J_c}{J_L}=\frac{2}{ql_t\cos(\chi/2)\ln[(1+1/\mu_i)^{\mu_i}(1+1/|\mu_f|)^{|\mu_f|}]}$$
$$\times\int_0^\infty\frac{d\zeta}{\zeta}\exp\left[-\left[\sqrt{1-\mu_i^2}+\sqrt{1-\mu_f^2}-\frac{\lambda}{l_t}\frac{1+\tilde{\mu}}{\tilde{\mu}}\right]\zeta\right]\cos\frac{\omega\zeta}{2E}\cos[(\mu_i-|\mu_f|)\zeta]\sin\left[2q\lambda\zeta\cos\frac{\chi}{2}\right]. \tag{62}$$

This last expression takes into consideration the different type of weak localization. The integral with respect to ζ has been found by numerical methods. The plots obtained by this method will be represented and discussed in the following section.

When the absolute value of the vector $\mathbf{q}$ is not fixed, the integration over q will be performed. On making natural assumption that $w\propto q^{-2}$ and performing the integration we find that instead of Eq. (62) we have

$$\frac{J_c}{J_L}=\frac{\lambda}{l_t\cos(\chi/2)\ln\left[(1+1/\mu_i)^{\mu_i}(1+1/|\mu_f|)^{|\mu_f|}\right]}$$
$$\times\int_0^\infty\frac{d\zeta}{\zeta}\cos\left[\frac{\omega\zeta}{2E}\right]\cos[(\mu_i-|\mu_f|)\zeta]\exp[-(\sqrt{1-\mu_i^2}+\sqrt{1-\mu_f^2})\zeta]Si[2q_c\lambda\zeta\cos(\chi/2)], \tag{63}$$

where *Si* is the integral sine and q_c is the limit value of the momentum q.

VII. CONCLUSION

Now it can be stated with assurance that multiple collisions do not destroy this type of weak localization, which is our major accomplishment in this study. The results obtained for the simplest case,[14] in which there is only one event of elastic scattering, hold in the circumstances of multiple elastic scattering in an infinite disordered medium, and are universal for any mechanism of electron-energy loss.

As the steady-state Schrödinger equation and steady-state Maxwell's equations may be written in the same mathematical form, it implies that the particles undergoing collisions are either electrons or photons, i.e., the formalism built in the assumption that the particles are electrons can be adopted in the theory of light scattering by disordered centers without changes.

If the particle undergoing collisions is an electron, the theory will describe the case in which the motion of the particle is due to a sufficient initial kinetic energy. Since the energy of such an electron is usually well above the Fermi energy, there will be no trouble with the Pauli principle. A fixed amount of energy is lost owing to the excitation of a plasmon, or photon, or optical phonon, or electrons in an atom.

In Sec. IV the assumption was made that the atomic potentials are of zero range. This is true when the wavelength of the particle is long with respect to the atomic scale, as in the case of light, though not so in the case of fast electrons.

However, there is strong reason to think that treatment based on such an assumption could provide basic understanding of the quantum transport of sufficient fast electrons. In fact, the consideration of quantum transport in the formalism, which takes into account multiple collisions, leads us to the same features of the new localization as in the case of a single elastic scattering through a large angle, from a potential of zero range. On the other hand, the theory[14] that describes the phenomenon under conditions of a single scattering event does not involve the mentioned assumption, and the characteristics of the new localization do not depend on an explicit form of the potential $U(\mathbf{r})$, provided that the value of the transferred momentum associated with the energy loss is small as compared with the total momentum transferred during the event of elastic collision. Thus, the theory taking into account multiple collisions is credible at $q \ll Q$, where **q** and **Q** are the momenta transferred as a result of some excitation and during scattering by the force center, respectively. Often q is of the order ω/v, $\hbar\omega$ being the energy loss and v the velocity of the particle. Then for an semi-infinite medium the condition $q \ll Q$ yields the inequality

$$n \ll E/\hbar\omega \;, \tag{64}$$

where n is the number of elastic collision events and E is the energy of fast electron. At $E/\hbar\omega \simeq 100$, for example, condition (64) will be fulfilled owing to the attenuation of

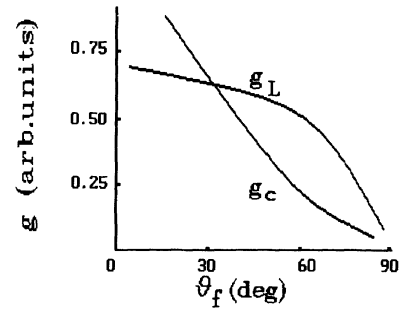

FIG. 5. The angular dependence of the functions g_c and g_L.

the wave field of the incident particles.

However, although such treatment facilitates insight into the new phenomena, the description of its peculiar features demands a more realistic model. The important point to be noted that it is not the issue of the theory of the new localization only. As regards the assumption that each individual elastic scattering is isotropic, an analysis of the real significance of this assumption has not yet been performed in terms of the usual weak localization of sufficient fast electrons too.

In the second part of this paper another limiting case is considered in which quantum transport could be suppressed on account of the proximity of the medium surface. That is the case when the attenuation of the wave field is so strong that the particle suffers only one event of elastic scattering through a large angle in a semi-infinite disordered medium. At such a case the angular dependence is for the first approximation determined by angular factors $g(\mu_i,\mu_f)$, which decrease monotonically with increase of the angle of incidence or escape (Fig. 5).

Weak localization is not the only manifestation of quantum transport. Sometimes[14] interference is great, but localization does not show up. The dependence of the function given by Eq. (61) and shown in Fig. 6, on the incidence and escape angles, results from the phenomenon of quantum transport, while weak localization is not taken into account.

At $q \neq 0$ (the new type of weak localization taken into consideration), the behavior of the curve J_c/J_L as a function of the escape angle θ_f, depends on the incidence angle θ_i. At $\theta_i < 15°$ the curve is a monotonic. At a fixed

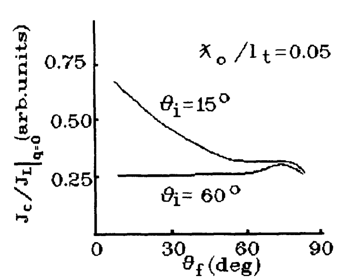

FIG. 6. The angular dependence of the quotient J_c/J_L in the case $q=0$.

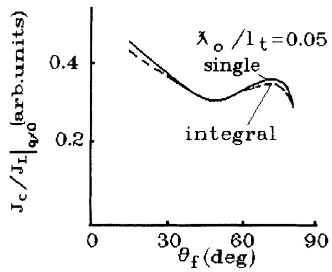

FIG. 7. The angular dependence of the quotient J_c/J_L in the case $q \neq 0$. $\theta_i = 60°$. Solid line corresponds to a fixed q; dashed line—the integration with respect to q.

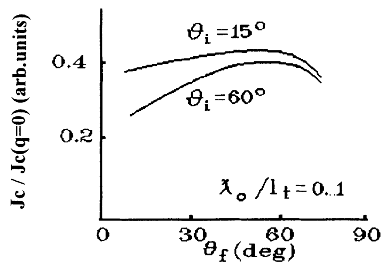

FIG. 8. Degree of localization at different angles of the particle escape.

value of q and when we integrate with respect to q, the results are similar (Fig. 7).

The localization shows up at $q \neq 0$, so the ratio $[J_c(q \neq 0) - J_c(q=0)]/J_c$ plotted in Fig. 8 characterizes its manifestation. There is a pronounced maximum of the curve.

In conclusion, we would like to stress the point that in the case of photons existence of this type of weak localization of light under Raman scattering may be expected. This type of localization may also appear in problems associated with the theory of conductivity under conditions of hot-electron transport in disordered or doped semiconductors.

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