Quasiparticle pseudo-Hamiltonian of an infinitesimally polarized Fermi liquid

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We present the microscopic derivation of a quasiparticle pseudo-Hamiltonian for an infinitesimally polarized electron liquid. The Hamiltonian is expressed in terms of suitably defined quasiparticle operators. Our approach is based on a canonical transformation which allows one to replace the bare Coulombic coupling between the interacting electrons with an effective interaction between quasiparticles in which collective charge and spin fluctuations are explicitly accounted for. The relevant matrix elements of the charge and spin-density operators enter our theory via linear-response functions: the charge response, the longitudinal and transverse spin responses, and the mixed charge-spin response. These susceptibilities are in turn expressed in terms of the appropriate many-body local fields. As a consequence our method can be seen as an attempt to satisfactorily include in a self-consistent manner the effects of the vertex corrections associated with charge and spin-fluctuations of the electron liquid. As a result useful expressions for the quasiparticle energy and the effective interaction between two quasiparticles are determined. These can, in turn, be employed in a microscopic determination of the parameters of the Landau theory of the Fermi liquid. The generalization of our results to a multicomponent system is also discussed.

I. INTRODUCTION

Understanding the many-body aspects of an electron gas (EG) has been a subject of steady interest for the past few decades.[1–3] The EG, unlike a system of classical particles, behaves like a gas at high densities and like a solid at low densities.[4] In both these extreme limits, the ground-state energy of the EG has been evaluated exactly. In the high density limit, the ground-state energy was obtained as a series in terms of r_s, the average electronic separation in units of the Bohr radius, in three dimensions (3D) by Gell-Mann and Brueckner[5] and in two dimensions (2D) by Rajagopal and Kimball[6] and by Isihara and Toyoda.[7] Wigner showed that at sufficiently small densities the electrons localize to form a crystal lattice and hypothesized that in 3D a bcc structure is the most stable one.[4] Later on it was verified that, among the simple lattices, the bcc structure has the lowest energy and the ground-state energy for the bcc lattice was obtained as a power series in terms of $r_s^{-1/2}$.[8] In 2D, Bonsall and Maradudin[9] calculated the ground state energy for arbitrary electron lattices, and showed that the triangular one, as expected, has the lowest energy.

In the intermediate density regime, which is relevant in three dimensions for simple metals, and in two dimensions for systems like the electrons in an inversion layer in most density regimes, the usual perturbative techniques are not effective owing to the lack of an expansion parameter. Hence, one has to take recourse to approximate methods which are not completely rigorous but are physically justifiable. A review of a number of these techniques can be found in Ref. 1. Quite useful in this respect are the numerical techniques based on the quantum Monte Carlo methods.[10–12]

Among the various approximate methods designed to deal with the intermediate density regime of particular interest for its physical appeal and elegance is Landau's[13] original phenomenological theory of the Fermi liquids which treats accurately low-lying excitations. Landau called these excitations quasiparticles and postulated a one-to-one correspondence between them and the excited states of a noninteracting Fermi liquid. He wrote down the excitation energy of the system in terms of the energy of the quasiparticles and their effective interaction. The quasiparticle interaction function can be used in turn to obtain various physical properties of the system and can also be parametrized in terms of experimentally obtainable data. Within the framework of perturbative Green's function techniques it was shown by Luttinger and Nozières[14] that the Landau theory is valid in the limit of zero temperature, long wavelength, and zero frequency.

For an EG long-range screening of the Coulombic interaction is an important factor. The simplest approximation that takes this into account is the random phase approximation (RPA). In this approximation the screened charge response of the EG is assumed to be that of the noninteracting system.[15] Using a many-body local field, commonly named after him, Hubbard[16] improved upon this approximation of the screened charge susceptibility by including, in an approximate fashion, some exchange corrections.

More recently, Hubbard approach was generalized in such a way as to include the effects of vertex corrections due to both charge and spin fluctuations in an unpolarized EG.[17–21] In these papers, using formally different approaches, expressions for the quasiparticle self-energy and effective interaction were obtained. As it was pointed out in Ref. 20, these results are basically equivalent.

This body of work showed that the quasiparticle self-energy and effective interaction can be expressed in terms of suitable generalized Hubbard many-body local fields and the charge and spin susceptibilities of the system, which, in turn, can be also expressed in terms of the same quantities. As for the case of an infinitesimally polarized EG, work on the subject was carried out by Ng and Singwi by means of a diagrammatic approach.[21] This approach to the many-body theory of the EG has already found it applications.[22,23]

In this paper, we extend the results obtained previously for the unpolarized system by the present authors[20] and derive a quasiparticle pseudo-Hamiltonian for an infinitesimally polarized liquid in terms of the response functions and many-body local fields of the system. One of the motivating factors for the present work is to arrive at useful expressions that can then be employed within a Landau theory of the Fermi liquid to evaluate various physical quantities of interest.[23]

The basic idea leading to the definition of a pseudo-quasiparticle-Hamiltonian was previously developed by Hamann and Overhauser for the case of an unpolarized system.[24] These authors limited their analysis to the simple case of the RPA. Our treatment, on the other hand, is much more general, and is designed to incorporate the effects of vertex corrections associated with charge and spin-density fluctuations to account for exchange and correlation effects in the infinitesimally polarized electron liquid.

We begin by viewing the electron liquid as a system comprised of a few interacting "test" electrons and a screening dielectric medium characterized only by its collective charge and spin density excitations. The test electrons and the medium interact via effective potentials which we express in terms of (a priori unknown) appropriate local field factors $\tilde{G}$ so as to account for their deviations (due to exchange and correlation effects beyond RPA) from the bare Coulomb potential. By using a canonical transformation this interaction terms are then eliminated to first order, thereby generating an effective coupling between the test electrons. Upon averaging over the coordinates of the screening medium we then obtain the sought renormalization of the test electrons states. An important step in this procedure is represented by the identification of the various a priori unknown matrix elements in terms of appropriate response functions of the medium: the charge, the longitudinal and the transverse spin, and the mixed charge-spin response susceptibilities. These response functions are in turn expressed via the corresponding generalized Hubbard many-body local fields. Finally we show that in order to achieve a physically self-consistent description of the situation the factors $\tilde{G}$ do in fact coincide with the Hubbard many-body local fields appearing in the response functions of the medium.

With this purpose in mind it is important to be able to generalize our treatment of the many-body effects in the electron liquid to the case of a multicomponent system. This is in fact necessary for the case of the electronic system occurring in a silicon inversion layer. There the multicomponent nature of the electronic band structure leads to further interesting and important modifications.

Our paper is structured as follows. In Sec. II, we introduce the total Hamiltonian which we use to model the EG. In Sec. III the bulk of the renormalization procedure is presented and a quasiparticle pseudo-Hamiltonian is arrived at. Next, in Sec. IV we express our results in terms of the quasiparticle energy and the quasiparticle effective interaction. Section V contains a discussion of our results and their implications. The connection of our theory to previous work is also provided there. The paper contains two Appendices. In Appendix A we derive useful expressions for the various response functions that enter our quasiparticle pseudo-Hamiltonian for the case of a multi-component system. Finally, in Appendix B an exact expression for the mixed charge-spin response function is obtained.

II. TOTAL HAMILTONIAN

To describe the excitations of an electron liquid we employ the picture based on the concept of quasiparticle and similar to Landau's phenomenological theory of the normal Fermi liquid. We start by selecting a few electrons from the EG and call them test electrons. The remaining EG is treated as a screening dielectric medium. As the test electrons move through the dielectric medium they produce fluctuations in the density of spin up and spin down electrons. These fluctuations provide virtual clothing to the test electrons and also screen the interaction between them. Thus, the dielectric mimics the true processes in an average way. It is important to realize that in reality the test electrons and the electrons comprising the dielectric are physically the same. This must be taken into account when exchange effects are considered.

The goal is to derive a Hamiltonian containing only the degrees of freedom of the clothed test electrons or quasiparticles. To this end we proceed as follows. We write the total Hamiltonian of the system as

$$H = H_0^{(p)} + H_0^{(m)} + H_1 \, , \tag{1}$$

where $H_0^{(p)}$ is the Hamiltonian of the test electrons and is given by

$$H_0^{(p)} = \sum_{\mathbf{p},\sigma} \epsilon_{\mathbf{p}}^{\sigma} a_{\mathbf{p},\sigma}^{+} a_{\mathbf{p},\sigma} + \frac{1}{2} \sum_{\substack{\mathbf{p},\mathbf{p},\\ \mathbf{q},\sigma,\sigma'}} v(q) a_{\mathbf{p}-\mathbf{q},\sigma}^{+} a_{\mathbf{p}'+\mathbf{q},\sigma'}^{+} a_{\mathbf{p}',\sigma'} a_{\mathbf{p},\sigma} \, , \tag{2}$$

where $q \neq 0$, $a_{\mathbf{p},\sigma}^{+}$ ($a_{\mathbf{p},\sigma}$) creates (destroys) a quasiparticle with momentum $\mathbf{p}$ and spin index σ, ($\sigma = \pm 1$), $v(q)$ is the Fourier transform of the bare Coulomb potential, with $\epsilon_{\mathbf{p}}^{\sigma}$ being the bare (band) energy of the test electron. In Eq. (1) $H_0^{(m)}$ is the Hamiltonian of the dielectric medium and is described by specifying its eigenstates $|n\rangle$ and its eigenvalues ω_n. Furthermore, H_1 is the part of the total Hamiltonian that takes into account the test electron-medium coupling and is given as follows:

$$H_1 = \sum_{\mathbf{p},\mathbf{q},\alpha} v(q)[1 - \tilde{G}_+(\mathbf{q}, \epsilon_{\mathbf{p}+\mathbf{q}}^{\alpha} - \epsilon_{\mathbf{p}}^{\alpha})] \rho_{-\mathbf{q}} a_{\mathbf{p}+\mathbf{q},\alpha}^{+} a_{\mathbf{p},\alpha} - \sum_{\substack{\mathbf{p},\mathbf{q},\\ \alpha,\beta,\mu}} v(q) \tilde{G}_-^{\mu}(\mathbf{q}, \epsilon_{\mathbf{p}+\mathbf{q}}^{\alpha} - \epsilon_{\mathbf{p}}^{\beta}) \sigma_{\alpha\beta}^{\mu} S_{-\mathbf{q}}^{\mu} a_{\mathbf{p}+\mathbf{q},\alpha}^{+} a_{\mathbf{p},\beta} \, , \tag{3}$$

where $q\neq 0$, $\alpha,\beta=\pm 1$; $\mu=x,y,z$; and $\sigma^{\mu}_{\alpha\beta}$ is a Pauli matrix. Furthermore, $\rho_{\mathbf{q}}$ and $S^{\mu}_{\mathbf{q}}$ are the operators associated with charge- and spin-density fluctuations, respectively. Since at this stage the nature and strength of the potentials appearing in H_1 is unknown we have introduced the quantities $\widetilde{G}_+$ and $\widetilde{G}^{\mu}_-$ in order to account for exchange and correlation effects. The $\widetilde{G}$'s are taken to be functions of the change in momentum and the change in energy of the test electron. In the above equation it has been assumed that $\widetilde{G}_+$ and $\widetilde{G}^{\mu}_-$ have reflection symmetry with respect to the plane perpendicular to the axis of polarization, namely the z axis. In general, for a polarized EG in its ground state one expects that $\widetilde{G}^{x}_-=\widetilde{G}^{y}_-=\widetilde{G}^{T}_-\neq\widetilde{G}^{z}_-=\widetilde{G}_-$. Later on we will identify these $\widetilde{G}_\pm$ and $\widetilde{G}^{T}_-$ factors in terms of the true many-body local fields of the EG as a whole (see Sec. III). It is of interest to note that in real space the effective potential felt by a test electron of momentum $\mathbf{p}$ and spin σ as obtained from Eq. (3), can be expressed as (see Appendix A and Ref. 18 for a similar result)

$$\begin{aligned}\phi^{\mathbf{p}}_{\sigma}(q,\omega)=v(q)\{&[\Delta n_\uparrow(q,\omega)+\Delta n_\downarrow(q,\omega)]\\&\times[1-\widetilde{G}_+(\mathbf{q},\epsilon^{\sigma}_{\mathbf{p}+\mathbf{q}}-\epsilon^{\sigma}_{\mathbf{p}})]\\&-\sigma[\Delta n_\uparrow(q,\omega)-\Delta n_\downarrow(q,\omega)]\\&\times\widetilde{G}^{z}_-(\mathbf{q},\epsilon^{\sigma}_{\mathbf{p}+\mathbf{q}}-\epsilon^{\sigma}_{\mathbf{p}})\}\ ,\end{aligned}\tag{4}$$

where z is the quantization axis for the spin σ and Δn_σ represents the density fluctuations of electrons with spin projection σ.

III. RENORMALIZATION PROCEDURE

To obtain the quasiparticle Hamiltonian of an infinitesimally polarized system we adopt the following renormalization procedure. We first perform a canonical transformation on the total Hamiltonian so as to eliminate the term H_1 up to first order. This, in turn, produces an effective coupling between the test electrons through their interaction with the charge and spin-density fluctuations. This is similar to the procedure employed in deriving the Frohlich phonon mediated electron-electron effective interaction.[25] The transformed Hamiltonian is given by $H'=e^{-T}He^{T}$, where the operator T is determined from the requirement

$$H_1+\left[\sum_{\mathbf{p}\sigma}\epsilon^{\sigma}_{\mathbf{p}}a^{+}_{\mathbf{p},\sigma}a_{\mathbf{p},\sigma}+H^{(m)}_0,T\right]=0\ .\tag{5}$$

The above form involving only the Hamiltonians of the noninteracting test electrons and the dielectric medium in the commutator is chosen for the definition of T for the following reasons. First, this form enables us to determine the matrix elements of T with respect to the eigenstates of the dielectric medium. Second, as will be shown below, this definition yields the renormalization term as a combination of identifiable dynamic response functions of the dielectric medium. Last, this form correctly yields the RPA result for the pseudo-Hamiltonian when the exchange and correlation vertex corrections are neglected.

A. Averaging out the medium

The second step involves explicitly removing the degrees of freedom of the dielectric medium. This is done by averaging the transformed Hamiltonian H' over the *uniformly infinitesimally polarized state* $|0\rangle$ of the medium. We thus obtain the following quasiparticle pseudo-Hamiltonian H'_{QP}:

$$H'_{QP}=\langle 0|H'|0\rangle=H^{(p)}_0+\tfrac{1}{2}\langle 0|[H_1,T]|0\rangle\ ,\tag{6}$$

which now contains only the test electron operators. In the pseudo-Hamiltonian given above, constants and higher order terms in $v(q)$ have been omitted. Later on, in Sec. IV we present arguments to show that the neglect of higher-order terms is consistent with the requirement that the correct pseudo-Hamiltonian must contain all the correlation effects. The expectation value with respect to the polarized state $|0\rangle$ on the right-hand side of Eq. (6) is precisely the term that leads to a renormalization of the bare interaction potential and also to the clothing of a bare electron. This term can be evaluated from the matrix elements $\langle 0|H_1|n\rangle$ and $\langle 0|T|n\rangle$. Then, from Eqs. (3) and (5) we obtain

$$\begin{aligned}\langle n|T|0\rangle=\sum_{\mathbf{p},\mathbf{q},\sigma}\Big\{&\{1-\widetilde{G}^{*}_+[\mathbf{q},\Delta^{\sigma}_{\mathbf{p}}(\mathbf{q})]\}\langle n|\rho_{\mathbf{q}}|0\rangle\\&-\sigma\widetilde{G}^{*}_-[\mathbf{q},\Delta^{\sigma}_{\mathbf{p}}(\mathbf{q})]\langle n|S^{z}_{\mathbf{q}}|0\rangle\Big\}\frac{v(q)a^{+}_{\mathbf{p}-\mathbf{q},\sigma}a_{\mathbf{p},\sigma}}{\Delta^{\sigma}_{\mathbf{p}}(\mathbf{q})-\omega_{n0}}\\&-\widetilde{G}^{T*}_-[\mathbf{q},\Delta^{T\sigma}_{\mathbf{p}}(\mathbf{q})]\langle n|S^{\sigma}_{\mathbf{q}}|0\rangle\frac{v(q)a^{+}_{\mathbf{p}-\mathbf{q},-\sigma}a_{\mathbf{p},\sigma}}{\Delta^{T\sigma}_{\mathbf{p}}(\mathbf{q})-\omega_{n0}}\ ,\end{aligned}\tag{7}$$

where for the sake of brevity we have defined

$$\omega_{n0}=\omega_n-\omega_0\ ,\tag{8}$$

$$\Delta^{\sigma}_{\mathbf{p}}(\mathbf{q})\equiv\epsilon^{\sigma}_{\mathbf{p}}-\epsilon^{\sigma}_{\mathbf{p}-\mathbf{q}}\ ,\tag{9}$$

and

$$\Delta^{T\sigma}_{\mathbf{p}}(\mathbf{q})\equiv\epsilon^{\sigma}_{\mathbf{p}}-\epsilon^{-\sigma}_{\mathbf{p}-\mathbf{q}}\ .\tag{10}$$

In obtaining Eq. (7), we used the relationship $\widetilde{G}^{(T)}_\pm(\mathbf{q},\omega)=\widetilde{G}^{(T)*}_\pm(-\mathbf{q},-\omega)$ for which the justification will become clear in Sec. III when the $\widetilde{G}$'s are shown to coincide with the many-body local fields. We now define $S^{\pm}=S^{x}\pm iS^{y}$ and use the fact that the quantities $\langle 0|S^{+}|n\rangle\langle n|S^{+}|0\rangle$, $\langle 0|S^{-}|n\rangle\langle n|S^{-}|0\rangle$, $\langle 0|S^{z}|n\rangle\langle n|S^{\pm}|0\rangle$, and $\langle 0|\rho|n\rangle\langle n|S^{\pm}|0\rangle$ vanish. Furthermore, for $q\neq 0$, we also utilize the fact that $\langle 0|\rho_{\mathbf{q}}|0\rangle$ and $\langle 0|S^{\mu}_{\mathbf{q}}|0\rangle$ vanish whereas $\langle 0|\rho_{\mathbf{q}}|n\rangle\langle n|S^{z}_{-\mathbf{q}}|0\rangle$ has in general a nonzero value for a polarized system. Then from Eqs. (3) and (7) we obtain

$$\langle 0|H_1T|0\rangle = \sum_{\substack{\mathbf{p},\mathbf{p}',\mathbf{q},\\ \sigma,\sigma',n}} [\{1-\tilde{G}_+[\mathbf{q},-\Delta_{\mathbf{p}'}^{\sigma'}(-\mathbf{q})]\}\{1-\tilde{G}_+^*[\mathbf{q},\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]\}|\langle n|\rho_{\mathbf{q}}|0\rangle|^2$$
$$+\sigma\sigma'\tilde{G}_-[\mathbf{q},-\Delta_{\mathbf{p}'}^{\sigma'}(-\mathbf{q})]\tilde{G}_-^*[\mathbf{q},\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]|\langle n|S_{\mathbf{q}}^z|0\rangle|^2$$
$$-(\sigma\{1-\tilde{G}_+[\mathbf{q},-\Delta_{\mathbf{p}'}^{\sigma'}(-\mathbf{q})]\}\tilde{G}_-^*[\mathbf{q},\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]\langle 0|\rho_{-\mathbf{q}}|n\rangle\langle n|S_{\mathbf{p}}^z|0\rangle$$
$$+\sigma'\{1-\tilde{G}_+^*[\mathbf{q},\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]\}\tilde{G}_-[\mathbf{q},-\Delta_{\mathbf{p}'}^{\sigma'}(-\mathbf{q})]\langle 0|S_{-\mathbf{q}}^z|n\rangle\langle n|\rho_{\mathbf{q}}|0\rangle)]$$
$$\times\frac{v(q)^2 a_{\mathbf{p}'+\mathbf{q},\sigma'}^{+}a_{\mathbf{p}',\sigma'}a_{\mathbf{p}-\mathbf{q},\sigma}^{+}a_{\mathbf{p},\sigma}}{\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})-\omega_{n0}}+\tfrac{1}{2}(1-\sigma\sigma')\tilde{G}_-^T[\mathbf{q},-\Delta_{\mathbf{p}'}^{T\sigma'}(-\mathbf{q})]$$
$$\times\tilde{G}_-^{T*}[\mathbf{q},\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})]|\langle n|S_{\mathbf{q}}^{\sigma}|0\rangle|^2$$
$$\times\frac{v(q)^2 a_{\mathbf{p}'+\mathbf{q},-\sigma'}^{+}a_{\mathbf{p}',\sigma'}a_{\mathbf{p}-\mathbf{q},-\sigma}^{+}a_{\mathbf{p},\sigma}}{\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})-\omega_{n0}}\ . \qquad (11)$$

Now, on recognizing that the operator T is anti-Hermitian we obtain from Eq. (11) the renormalization term $\frac{1}{2}\langle 0|[H_1,T]|0\rangle$ of the quasiparticle pseudo-Hamiltonian H'_{QP}. Then, in the renormalization term, upon identifying the various matrix elements of the charge and spin fluctuations in terms of the various dynamic response functions, to be defined below, we obtain the following compact expression:

$$\langle 0|[H_1,T]|0\rangle=2\sum_{\mathbf{p},\sigma}\tilde{E}_{\mathrm{CH}}^{\sigma}(\mathbf{p})a_{\mathbf{p},\sigma}^{+}a_{\mathbf{p},\sigma}+\sum_{\substack{\mathbf{p},\mathbf{p}',\\ \mathbf{q},\sigma,\sigma'}}\{\tilde{V}_{\sigma,\sigma'}[\mathbf{q},-\Delta_{\mathbf{p}'}^{\sigma'}(-\mathbf{q}),\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]a_{\mathbf{p}-\mathbf{q},\sigma}^{+}a_{\mathbf{p}'+\mathbf{q},\sigma'}^{+}a_{\mathbf{p}',\sigma'}a_{\mathbf{p},\sigma}$$
$$+\tilde{V}_{\sigma,\sigma'}^{T}[\mathbf{q},-\Delta_{\mathbf{p}'}^{T\sigma'}(-\mathbf{q}),\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})]a_{\mathbf{p}-\mathbf{q},-\sigma}^{+}a_{\mathbf{p}+\mathbf{q},-\sigma'}^{+}a_{\mathbf{p}',\sigma'}a_{\mathbf{p},\sigma}\}\ , \qquad (12)$$

where the terms $\tilde{V}_{\sigma,\sigma'}$ and $\tilde{V}_{\sigma,\sigma'}^{T}$ are the longitudinal and transverse components of the renormalization part of the effective interaction between two quasiparticles and are given by

$$\tilde{V}_{\sigma,\sigma'}(\mathbf{q},\epsilon,\omega,\delta)\equiv v(q)^2([1-\tilde{G}_+(\mathbf{q},\epsilon)][1-\tilde{G}_+^*(\mathbf{q},\omega)]\mathrm{Re}\chi_C(\mathbf{q},\delta)$$
$$+\sigma\sigma'\tilde{G}_-(\mathbf{q},\epsilon)\tilde{G}_-^*(\mathbf{q},\omega)\mathrm{Re}\chi_S(\mathbf{q},\delta)/(-\mu_B^2)-\{\sigma[1-\tilde{G}_+(\mathbf{q},\epsilon)]\tilde{G}_-^*(\mathbf{q},\omega)$$
$$+\sigma'[1-\tilde{G}_+^*(\mathbf{q},\omega)]\tilde{G}_-(\mathbf{q},\epsilon)\}\mathrm{Re}\chi_{CS}(\mathbf{q},\delta))\ , \qquad (13)$$

and

$$\tilde{V}_{\sigma,\sigma'}^{T}(\mathbf{q},\epsilon,\omega,\delta)\equiv 2(1-\sigma\sigma')v(1)^2\tilde{G}_-^T(\mathbf{q},\epsilon)\tilde{G}_-^{T*}(\mathbf{q},\omega)\mathrm{Re}\chi^{T\sigma}(\mathbf{q},\delta)/(-\mu_B^2)\ , \qquad (14)$$

with μ_B being the Bohr magneton. In Eq. (12) E_{CH}^{σ} is the Coulomb hole part[26] of the renormalization term and is obtained upon rearranging the creation and destruction operators in the usual order of a Hamiltonian expressed in the second quantized form

$$\tilde{E}_{\mathrm{CH}}^{\sigma}(\mathbf{p})=-\sum_{\mathbf{q}}v(q)^2P\int_0^{\infty}\frac{d\omega}{\pi}\mathrm{Im}\left[\frac{|1-\tilde{G}_+|^2\chi_C+|\tilde{G}_-|^2\chi_S/(-\mu_B^2)}{\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})-\omega}-\frac{2\sigma\,\mathrm{Re}[(1-\tilde{G}_+^*)\tilde{G}_-]\chi_{\mathrm{CS}}}{\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})-\omega}+\frac{4|\tilde{G}_-^T|^2\chi^{T\sigma}/(-\mu_B^2)}{\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})-\omega}\right]. \qquad (15)$$

In the above equation it is understood that the factors $\tilde{G}_{\pm}$ are functions of $\mathbf{q}$ and $\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})$, while $\tilde{G}_-^T$ is a function of $\mathbf{q}$ and $\Delta_p^{T\sigma}(\mathbf{q})$. Moreover, the response functions depend on $\mathbf{q}$ and ω.

The various response functions appearing in the effective interaction terms and the Coulomb hole term [see Eqs. (13)–(15)] are χ_C the charge response, χ_S the spin response, $\chi^{T\sigma}$ the transverse spin response, and χ_{CS} the mixed charge-spin response.[27] Exact expressions for these response functions are as follows:

$$\chi_C(\mathbf{q},\nu)=\sum_n\left\{\frac{|\langle n|\rho_{\mathbf{q}}|0\rangle|^2}{\nu-\omega_{n0}+i\eta}-\frac{|\langle n|\rho_{-\mathbf{q}}|0\rangle|^2}{\nu+\omega_{n0}+i\eta}\right\}, \qquad (16)$$

$$\chi_S(\mathbf{q},\nu)=-\mu_B^2\sum_n\left\{\frac{|\langle n|S_{\mathbf{q}}^z|0\rangle|^2}{\nu-\omega_{n0}+i\eta}-\frac{|\langle n|S_{-\mathbf{q}}^z|0\rangle|^2}{\nu+\omega_{n0}+i\eta}\right\}, \qquad (17)$$

$$\chi^{T\sigma}(\mathbf{q},\nu)=\frac{-\mu_B^2}{4}\sum_n\left\{\frac{|\langle n|S_{\mathbf{q}}^{\sigma}|0\rangle|^2}{\nu-\omega_{n0}+i\eta}-\frac{|\langle n|S_{-\mathbf{q}}^{-\sigma}|0\rangle|^2}{n+\omega_{n0}+i\eta}\right\}, \tag{18}$$

and

$$\chi_{\rm CS}(\mathbf{q},\nu)=\sum_n\left\{\frac{\langle 0|\rho_{-\mathbf{q}}|n\rangle\langle n|S_{\mathbf{q}}^z|0\rangle}{\nu-\omega_{n0}+i\eta}-\frac{\langle 0|S_{\mathbf{q}}^z|n\rangle\langle n|\rho_{-\mathbf{q}}|0\rangle}{\nu+\omega_{n0}+i\eta}\right\}$$
$$=\sum_n\left\{\frac{\langle 0|S_{-\mathbf{q}}^z|n\rangle\langle n|\rho_{\mathbf{q}}|0\rangle}{\nu-\omega_{n0}+i\eta}-\frac{\langle n|S_{-\mathbf{q}}^z|0\rangle\langle 0|\rho_{\mathbf{q}}|n\rangle}{\nu+\omega_{n0}+i\eta}\right\}. \tag{19}$$

Details of the derivation of $\chi_{\rm CS}$ are presented in Appendix B.

B. Self-consistent identification of the $\tilde{G}$'s

The last step in the renormalization procedure involves identification of the vertex correction factors $\tilde{G}$. With this goal in mind, we will first express the response functions appearing in the renormalization term [see Eqs. (12)–(15)] as functionals of the many-body local fields $G_\pm$ and G_-^T. Then, based on the formal similarity of the potentials given by Eqs. (4) and (A6) we make the physically reasonable ansatz that the vertex correction factors $\tilde{G}$ coincide identically with the corresponding many-body local fields G that enter the expression for the response functions.

For a single-component system the many-body local fields $G_\pm$ (Ref. 16) are commonly defined through the various response functions of the unpolarized medium as follows:

$$\chi_C\equiv\frac{\chi_0}{1-v(q)(1-G_+)\chi_0}, \tag{20}$$

and

$$\chi_S\equiv\mu_B^2\frac{\chi_0}{1+v(q)G_-\chi_0}. \tag{21}$$

The χ_0 appearing in the above equations differs from the Lindhard[28] polarizability for a noninteracting EG in that here it is defined in terms of exact occupation numbers if the local fields $G_\pm$ are taken to be consistent with Niklasson's definition (see below).[29]

In Appendix A, for an infinitesimally polarized multicomponent system, we have derived expressions for the various response functions in terms of the many-body local fields of a single component. Here, we merely present the results for a single-component system

$$\chi_C=\frac{\chi_0^\uparrow+\chi_0^\downarrow+4v(q)\chi_0^\uparrow\chi_0^\downarrow G_-}{\mathcal{D}}, \tag{22}$$

$$\chi_S=-\mu_B^2\frac{\chi_0^\uparrow+\chi_0^\downarrow-4v(q)\chi_0^\uparrow\chi_0^\downarrow(1-G_+)}{\mathcal{D}}, \tag{23}$$

$$\chi_{\rm CS}=\frac{\chi_0^\uparrow-\chi_0^\downarrow}{\mathcal{D}}, \tag{24}$$

where

$$\mathcal{D}\equiv 1-v(q)(\chi_0^\uparrow+\chi_0^\downarrow)(1-G_+-G_-)-4v(q)^2\chi_0^\uparrow\chi_0^\downarrow G_-(1-G_+). \tag{25}$$

The χ_0^σ appearing in the above equations is the response of a free EG defined in terms of the exact occupation numbers $n_{\mathbf{p}}^\sigma$ as follows:

$$\chi_0^\sigma(\mathbf{q},\omega)\equiv\frac{1}{\Omega}\sum_{\mathbf{p}}\frac{n_{\mathbf{p}-\mathbf{q}}^\sigma-n_{\mathbf{p}}^\sigma}{\omega-\Delta_{\mathbf{p}}^\sigma(\mathbf{q})+i\eta}, \tag{26}$$

with Ω being the volume of the system. For the transverse spin response $\chi^{T\sigma}$, we only present its defining equation in terms of the local field G_-^T as follows:

$$\chi^{T\sigma}\equiv-\mu_B^2\frac{\chi_0^{T\sigma}}{1+2v(q)G_-^T\chi_0^{T\sigma}}, \tag{27}$$

where, similarly to χ_0^σ, the noninteracting transverse response $\chi_0^{T\sigma}$ too is defined in terms of the exact occupation numbers, i.e.,

$$\chi_0^{T\sigma}(\mathbf{q},\omega)\equiv\frac{1}{\Omega}\sum_{\mathbf{p}}\frac{n_{\mathbf{p}-\mathbf{q}}^{-\sigma}-n_{\mathbf{p}}^\sigma}{\omega-\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})+i\eta}. \tag{28}$$

For an unpolarized system, the expressions for the charge and spin responses, given by Eqs. (22) and (23), reduce to the defining equations of $G_\pm$ as given by Eqs. (20) and (21). The mixed charge-spin response $\chi_{\rm CS}$, as expected, reduces to zero for the unpolarized case. As for the transverse spin response, the isotropy of the unpolarized system reduces it to $\frac{1}{2}\chi_S$ with the transverse $\chi_0^{T\sigma}$ and G_-^T coinciding with their unpolarized counterparts $\frac{1}{2}\chi_0$ and G_-.

Now, the $\tilde{G}$'s that enter the expressions for effective interactions given in Eqs. (13) and (14) correspond to the exchange and correlation corrections to the Hartree interaction between a test electron and the charge and spin density fluctuations in the dielectric medium. Similarly, the local fields $G_\pm$ and G_-^T too represent the corrections to the Hartree interaction between an electron and the density fluctuations in the EG. These facts are made explicit in the expressions for the potentials given by Eqs. (4) and (A6). Now, since the test electrons and the screening dielectric medium are one and the same, we expect the $\tilde{G}$'s and the G's to coincide. Hence, we postulate that the $\tilde{G}$'s coincide identically with the corresponding many-body local fields G's for all values of q and ω for an infinitesimally polarized system. Then the quasiparticle pseudo-Hamiltonian is given as follows:

$$H'_{\mathrm{QP}}=\sum_{\mathbf{p},\sigma}[\epsilon_{\mathbf{p}}^{\sigma}+E_{\mathrm{CH}}^{\sigma}(\mathbf{p})]a_{\mathbf{p},\sigma}^{+}a_{\mathbf{p},\sigma}+\frac{1}{2}\sum_{\substack{\mathbf{p},\mathbf{p}'\\ \mathbf{q},\sigma,\sigma'}}(\{v(q)+V_{\sigma,\sigma'}[\mathbf{q},-\Delta_{\mathbf{p}'}^{\sigma'}(-\mathbf{q}),\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]\}a_{\mathbf{p}-\mathbf{q},\sigma}^{+}a_{\mathbf{p}'+\mathbf{q},\sigma'}^{+}a_{\mathbf{p}',\sigma'}a_{\mathbf{p},\sigma}$$
$$+V_{\sigma,\sigma'}^{T}[\mathbf{q},-\Delta_{\mathbf{p}'}^{T\sigma'}(\mathbf{q}),\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})]a_{\mathbf{p}-\mathbf{q},-\sigma}^{+}a_{\mathbf{p}'+\mathbf{q},-\sigma'}^{+}a_{\mathbf{p}',\sigma'}a_{\mathbf{p},\sigma})\ , \qquad (29)$$

where E_{CH}^{σ}, $V_{\sigma,\sigma'}$, and $V_{\sigma,\sigma'}^{T}$ are formally given by Eqs. (13)–(15) with the $\tilde{G}$'s replaced by the corresponding local fields G.

IV. QUASIPARTICLE PSEUDO-HAMILTONIAN

For the sake of clarity, we now express the quasiparticle Hamiltonian in terms of normal products so that the exchange contribution from the effective interaction appears explicitly in the quasiparticle self-energy. We have

$$H_{\mathrm{QP}}=\sum_{\mathbf{p},\sigma}E_{\mathbf{p}}^{\sigma}:a_{\mathbf{p},\sigma}^{+}a_{\mathbf{p},\sigma}:+\frac{1}{2}\sum_{\substack{\mathbf{p},\mathbf{p}',\\ \mathbf{q},\sigma,\sigma'}}(\{v(q)+V_{\sigma,\sigma'}[\mathbf{q},-\Delta_{\mathbf{p}'}^{\sigma'}(-\mathbf{q}),\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]\}:a_{\mathbf{p}-\mathbf{q},\sigma}^{+}a_{\mathbf{p}'+\mathbf{q},\sigma'}^{+}a_{\mathbf{p}',\sigma'}a_{\mathbf{p},\sigma}:$$
$$+V_{\sigma,\sigma'}^{T}[\mathbf{q},-\Delta_{\mathbf{p}'}^{T\sigma'}(-\mathbf{q}),\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})]:a_{\mathbf{p}-\mathbf{q},-\sigma}^{+}a_{\mathbf{p}'+\mathbf{q},-\sigma'}^{+}a_{\mathbf{p}',\sigma'}a_{\mathbf{p},\sigma}:)\ . \qquad (30)$$

Here constant terms have been omitted. In the above equation, the longitudinal component $V_{\sigma,\sigma'}$ corresponds to the case in which the spins of the quasiparticles are unchanged after interaction while the transverse component $V_{\sigma,\sigma'}^{T}$ corresponds to the case where opposite spin quasiparticles interact and flip their spins. For the sake of completeness we provide the expressions for these two as follows:

$$V_{\sigma,\sigma'}(\mathbf{q},\epsilon,\omega,\delta)\equiv v(q)^2([1-G_{+}(\mathbf{q},\epsilon)][1-G_{+}^{*}(\mathbf{q},\omega)]\mathrm{Re}\chi_{C}(\mathbf{q},\delta)+\sigma\sigma'G_{-}(\mathbf{q},\epsilon)G_{-}^{*}(\mathbf{q},\omega)\mathrm{Re}\chi_{S}(\mathbf{q},\delta)/(-\mu_B^2)$$
$$-\{\sigma[1-G_{+}(\mathbf{q},\epsilon)]G_{-}^{*}(\mathbf{q},\omega)+\sigma'[1-G_{+}^{*}(\mathbf{q},\omega)]G_{-}(\mathbf{q},\epsilon)\}\mathrm{Re}\chi_{CS}(\mathbf{q},\delta))\ , \qquad (31)$$

and

$$V_{\sigma,\sigma'}^{T}(\mathbf{q},\epsilon,\omega,\delta)\equiv 2(1-\sigma\sigma')v(q)^2G_{-}^{T}(\mathbf{q},\epsilon)G_{-}^{T*}(\mathbf{q},\omega)\mathrm{Re}\chi^{T\sigma}(\mathbf{q},\delta)/(-\mu_B^2)\ . \qquad (32)$$

The (fully renormalized) quasiparticle energy $E_{\mathbf{p}}^{\sigma}$ occurring in Eq. (30) contains both a dynamically screened exchange part E_{SX}^{σ} and a Coulomb hole part E_{CH}^{σ}

$$E_{\mathbf{p}}^{\sigma}=\epsilon_{\mathbf{p}}^{\sigma}+E_{SX}^{\sigma}(\mathbf{p})+E_{\mathrm{CH}}^{\sigma}(\mathbf{p})\ , \qquad (33)$$

where

$$E_{SX}^{\sigma}(\mathbf{p})=-\sum_{\mathbf{q}}(n_{\mathbf{p}-\mathbf{q}}^{\sigma}\{v(q)+V_{\sigma,\sigma}[\mathbf{q},\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]\}+n_{\mathbf{p}-\mathbf{q}}^{-\sigma}V_{\sigma,-\sigma}^{T}[\mathbf{q},\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})])\ . \qquad (34)$$

In the above expression the first term corresponds to the familiar Hartree-Fock exchange energy while the remaining terms represent exchange contribution from the dynamical screening produced by charge and spin-density fluctuations. In Eq. (33) the Coulomb hole term E_{CH}^{σ} is given by Eq. (15) with the factors $\tilde{G}$ replaced by their corresponding many-body local fields G.

The above derivation of the quasiparticle Hamiltonian is readily extended to the case of an EG with ν_v degenerate components. The problem is considerably simplified under the assumption that the density fluctuations are the same for all the components. Also for the case of a multivalley system where the relevant valleys are separated by a large momentum, an electron retains its valley after being scattered by other electrons. Then the electrons in different valleys can be regarded as different components with the component index v representing an additional quantum number. An additional complication is however represented by the fact that in the derivation of the quasiparticle Hamiltonian, the many-body local fields $G_{\pm}$, must be replaced by $G_{\pm}^{v}$, i.e., those appropriate for a multicomponent system (see Appendix A for further details). The final expression for the quasiparticle Hamiltonian is given as follows:

$$H_{\mathrm{QP}}=\sum_{\mathbf{p},\sigma,v} E_{\mathbf{p}}^{\sigma}:a^{+}_{\mathbf{p},\sigma,v}a_{\mathbf{p},\sigma,v}:+\frac{1}{2}\sum_{\substack{\mathbf{p},\mathbf{p}',\mathbf{q},\\ \sigma,\sigma',v,v'}}\{[v(q)+V_{\sigma,\sigma'}(\mathbf{q},-\Delta_{\mathbf{p}'}^{\sigma'}(-\mathbf{q}),\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q}))]:a^{+}_{\mathbf{p}-\mathbf{q},\sigma,v}a^{+}_{\mathbf{p}'+\mathbf{q},\sigma',v'}a_{\mathbf{p}',\sigma',v'}a_{\mathbf{p},\sigma,v}:$$
$$+V^{T}_{\sigma,\sigma'}(\mathbf{q},-\Delta_{\mathbf{p}'}^{T\sigma'}(-\mathbf{q}),\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q}),\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})):a^{+}_{\mathbf{p}-\mathbf{q},-\sigma,v}a^{+}_{\mathbf{p}'+\mathbf{q},-\sigma',v'}a_{\mathbf{p}',\sigma',v'}a_{\mathbf{p},\sigma,v}:\} \ . \tag{35}$$

The quasiparticle energy and the effective interaction terms are still formally the same as those appearing in Eq. (15) and Eqs. (31)–(34). As for the response functions that enter these terms, expressions are derived in Appendix A.

V. DISCUSSION AND CONCLUSIONS

We have derived a quasiparticle pseudo-Hamiltonian for a multicomponent infinitesimally polarized Fermi liquid. This quasiparticle Hamiltonian is constructed in such a way as to properly account in an averaged way for the usually unwieldy effects of correlations beyond the popular RPA. This was achieved through the approximate use of Hubbard generalized many-body local fields associated with both charge and spin fluctuations. Our results can at this point be used to perform explicit calculations of many-body effects in the EG. To this purpose suitable approximations to the Hubbard local fields must be used. Such approximate expressions involve in turn the knowledge of the exact limits acquired by such quantities.[29–32] The alternative is to use for these functions the output of numerical work.[12] Lately a elegant self-consistent method for the evaluation of useful expressions of the Hubbard local fields has been devised.[23] The results of such an analysis will be presented elsewhere.[33]

Our results of Eqs. (15), (33), and (34) for the self-energy can be recast in the following transparent and useful form (see Ref. 33):

$$\Sigma^{\sigma}(\mathbf{p},\omega)=-\sum_{\mathbf{q}}\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi i}\{[v(q)+v(q)^2(|1-G_{+}|^2_{\chi C}+|G_{-}|^2_{\chi C}/(-\mu_B^2)$$
$$-2\sigma\,\mathrm{Re}[(1-G_{+}^{*})G_{-}]\chi_{\mathrm{CS}})]g^{\sigma}(\mathbf{p}-\mathbf{q},\omega-\epsilon)+4v(q)^2|G_{-}^{T}|^2\chi^{T\sigma}/(-\mu_B^2)g^{-\sigma}(\mathbf{p}-\mathbf{q},\omega-\epsilon)\} \ , \tag{36}$$

where in order to recover our results one must set $\omega=\epsilon_{\mathbf{p}}^{\sigma}$. Here $g^{\sigma}(\mathbf{k},\omega)$ is the bare one electron Green's function and is defined as follows:

$$g^{\sigma}(\mathbf{p},\omega)\equiv\frac{n_{\mathbf{p}}^{\sigma}}{\omega-\epsilon_{\mathbf{p}}-i\eta}+\frac{1-n_{p}^{\sigma}}{\omega-\epsilon_{\mathbf{p}}+i\eta} \ . \tag{37}$$

Furthermore, in Eq. (36) it is understood that the $G_{\pm}$ are functions of $\mathbf{q}$ and $\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})$, that the G_{-}^{T} is a function of $\mathbf{q}$ and $\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})$, and that the response functions depend on $\mathbf{q}$ and ϵ. The expressions for the effective interaction terms in Eqs. (31) and (32) and that for the self-energy in Eq. (36) can be seen to be equivalent to the corresponding results of Ng and Singwi,[21] who however did not attempt to express their expressions in a transparent form (see also Ref. 33). There are also some differences mostly associated with the frequency dependence of the various quantities. The results of Ref. 21 can in fact be recovered if in our Eqs. (31), (32), and (36) in the many-body local fields that are prefactors to the response functions the complex conjugate forms are replaced by the corresponding complex forms, and the frequencies of all the local fields are replaced by the frequency appearing in the response functions.

It is of interest to note that for the special case of an unpolarized electron liquid, the charge and spin fluctuations are not coupled and also, owing to the isotropy, the effective potential due to the transverse spin fluctuations is just twice that due to the longitudinal spin fluctuations. Accordingly the quasiparticle self-energy of Eq. (36) simplifies to the following form for the unpolarized case:

$$\Sigma^{\sigma}(\mathbf{p},\omega)=-\sum_{\mathbf{q}}\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi i}\{v(q)+v(q)^2[|1-G_{+}|^2_{\chi_C}+3|\,G_{-}|^2_{\chi_S}/(-\mu_B^2)]\}g(\mathbf{p}-\mathbf{q},\omega-\epsilon) \ . \tag{38}$$

As for the quasiparticle Hamiltonian, it is still given by Eq. (30) with $\Delta_{\mathbf{p}}^{T\sigma}(\mathbf{q})=\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})$ and the following simplifications for the screened exchange, Coulomb hole and the effective interaction terms:

$$E_{SX}^{\sigma}(\mathbf{p})=-\sum_{\mathbf{q}}[n_{\mathbf{p}-\mathbf{q}}(v(q)+v(q)^2\{|1-G_+[\mathbf{q},\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]|^2\mathrm{Re}\chi_C[\mathbf{q},\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]+3|G_-[\mathbf{q},\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]|^2\mathrm{Re}\chi_S[\mathbf{q},\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})]/(-\mu_B^2)\})] , \tag{39}$$

$$E_{\mathrm{CH}}^{\sigma}(\mathbf{p})=-\sum_{\mathbf{q}} v(q)^2 P\int_0^{\infty}\frac{d\omega}{\pi}\mathrm{Im}\left[\frac{|1-G_+|^2\chi_C+3|G_-|^2\chi_S/(-\mu_B^2)}{\Delta_{\mathbf{p}}^{\sigma}(\mathbf{q})-\omega}\right] , \tag{40}$$

$$V_{\sigma,\sigma'}(\mathbf{q},\epsilon,\omega,\delta)=v(q)^2\{[1-G_+(\mathbf{q},\epsilon)][1-G_+^*(\mathbf{q},\omega)]\mathrm{Re}\chi_C(\mathbf{q},\delta)+\sigma\sigma' G_-(\mathbf{q},\epsilon)G_-^*(\mathbf{q},\omega)\mathrm{Re}\chi_S(\mathbf{q},\delta)/(-\mu_B^2)\} , \tag{41}$$

and

$$V_{\sigma,\sigma'}^T(\mathbf{q},\epsilon,\omega,\delta)=(1-\sigma\sigma')v(q)^2G_-(\mathbf{q},\epsilon)G_-^*(\mathbf{q},\omega)\mathrm{Re}\chi_S(\mathbf{q},\delta)/(-\mu_B^2) . \tag{42}$$

For this case the quasiparticle self-energy and effective interaction terms of Ref. 21 are obtained form Eqs. (38), (41), and (42) by making the same modifications as in the polarized case. The above results for the effective interaction terms of an unpolarized system agree with those derived in Refs. 17, 19, and 20. In these papers however, the question of the proper frequency dependence of the various response functions and the corresponding local fields was not tackled.

It must be mentioned that our quasiparticle energy yields what in a diagrammatic analysis would amount to the on-shell value of the self-energy. Furthermore, and in connection with the above, it should be made clear that the quasiparticle Hamiltonian derived here should only be used for calculations carried out to first order. This prescription is the consequence of the fact that in our derivation the renormalization term is explicitly constructed so as to ignore higher-order terms in H'. On the other hand, we believe that such a procedure is appropriate in view of the fact that, as readily verified, our pseudo-Hamiltonian leads by construction to the expected RPA results when the many-body local-field corrections are neglected.[35]

For the case of a multi-component system our approach does not take into account the effects due to the difference in the density fluctuations between components. Future work that includes these effects is in progress.

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APPENDIX A

In this appendix, using arguments based on linear response theory, we will derive the response functions for an infinitesimally polarized EG with ν_v degenerate components. We will refer here to the various components as valleys (as in band valleys). We will assume that the valleys are separated by large vectors in momentum space and it is therefore reasonable to assume that electrons retain their valley index after a scattering process.

When an external potential $\phi_{\mathrm{ext}}(\mathbf{q},\omega)$ is applied to the electronic system it sets up density fluctuations $\Delta n_{\uparrow}$ of spin-up and $\Delta n_{\downarrow}$ of spin-down electrons. Assuming that these density fluctuations are equal for all the valleys, the total effective potential felt by a spin up electron can be written by generalizing the procedure of Refs. 17 and 34 as follows:

$$\phi^{\uparrow}=\phi_{\mathrm{ext}}+v(q)\left\{[\Delta n_{\uparrow}+\Delta n_{\downarrow}]-[G_{x,\mathrm{intra}}^{\uparrow\uparrow}+G_{c,\mathrm{intra}}^{\uparrow,\uparrow}+G_{c,\mathrm{inter}}^{\uparrow,\uparrow}(\nu_v-1)]\frac{2\Delta n_{\uparrow}}{\nu_v}-[G_{c,\mathrm{intra}}^{\uparrow,\downarrow}+G_{c,\mathrm{inter}}^{\uparrow,\downarrow}(\nu_v-1)]\frac{2\Delta n_{\downarrow}}{\nu_v}\right\} , \tag{A1}$$

where for the sake of brevity the $\mathbf{q}$ and ω dependence of the potentials, the density fluctuations, and the many-body local fields has not been displayed. In the above equation the G's are assumed to be the same for each valley, the subscripts x and c denote exchange and correlation effects, and the labels *intra* and *inter* refer to intravalley and intervalley processes. Furthermore, among the terms containing the Coulombic potentials $v(q)$, the sum of the first two terms involving the density fluctuations represents the Hartree term, the next one is the exchange term, and the remaining ones are correlation terms. The total effective potential felt by a spin down electron can be similarly written as

$$\phi^{\downarrow}=\phi_{\mathrm{ext}}+v(q)\left\{[\Delta n_{\uparrow}+\Delta n_{\downarrow}]-[G_{x,\mathrm{intra}}^{\downarrow,\downarrow}+G_{c,\mathrm{intra}}^{\downarrow,\downarrow}+G_{c,\mathrm{inter}}^{\downarrow,\downarrow}(\nu_v-1)]\frac{2\Delta n_{\downarrow}}{\nu_v}-[G_{c,\mathrm{intra}}^{\downarrow,\uparrow}+G_{c,\mathrm{inter}}^{\downarrow,\uparrow}(\nu_v-1)]\frac{2\Delta n_{\uparrow}}{\nu_v}\right\} . \tag{A2}$$

Furthermore, for an unpolarized system, we note that for symmetry reasons

$$G^{\uparrow\uparrow}_{x,\mathrm{intra}}=G^{\downarrow\downarrow}_{x,\mathrm{intra}} , \tag{A3}$$

and

$$G^{\uparrow\uparrow(\downarrow)}_{c,\mathrm{intra(inter)}}=G^{\downarrow\downarrow(\uparrow)}_{c,\mathrm{intra(inter)}} . \tag{A4}$$

We assume that the above relations remain approximately valid for an infinitesimally polarized system. Also, when the electrons after interacting with each other scatter back to their original valleys, we can write

$$G^{\uparrow\uparrow}_{c,\mathrm{inter}}=G^{\uparrow\downarrow}_{c,\mathrm{inter}}=G^{\uparrow\downarrow}_{c,\mathrm{intra}} . \tag{A5}$$

With these approximations the potential felt by an electron with spin $\sigma(=\pm1)$ can be cast in the following compact form:

$$\phi_\sigma=\phi_{\mathrm{ext}}+v(q)[\Delta n_\uparrow+\Delta n_\downarrow](1-G^v_+) -\sigma v(q)[\Delta n_\uparrow-\Delta n_\downarrow]G^n_- , \tag{A6}$$

where the quantities $G^v_\pm$ are defined as follows:

$$G^v_\pm\equiv\frac{1}{\nu_v}[G^{\uparrow\uparrow}_{x,\mathrm{intra}}+G^{\uparrow\uparrow}_{c,\mathrm{intra}}+(\nu_v\pm\nu v_v-1)G^{\uparrow\downarrow}_{c,\mathrm{intra}}] . \tag{A7}$$

Then on defining the single valley local fields $G_\pm$ as

$$G_\pm\equiv G^{\uparrow\uparrow}_{x,\mathrm{intra}}+G^{\uparrow\uparrow}_{c,\mathrm{intra}}\pm G^{\uparrow\downarrow}_{c,\mathrm{intra}} , \tag{A8}$$

we obtain the following useful form for the multivalley local fields $G^v_\pm$ defined in Eq. (A7):

$$G^v_{+(-)}=G_+-G_{-(+)}+\frac{1}{\nu_v}G_- . \tag{A9}$$

We will now derive the expression for the charge response of an infinitesimally polarized system. In the presence of a spin independent infinitesimal external potential ϕ_{ext}, the total effective potential ϕ_σ felt by an electron of spin σ is still given by Eq. (A6). The density fluctuation Δn_σ is related to ϕ_σ via the relation

$$\Delta n_\sigma=\nu_v\chi^\sigma_0\phi_\sigma , \tag{A10}$$

where χ^σ_0 is the response for a free EG as defined in Eq. (26) in the text. Then using Eqs. (A6) and (A10) we obtain the expression for charge response from its definition

$$\chi_C\equiv\frac{\Delta n_\uparrow+\Delta n_\downarrow}{\phi_{\mathrm{ext}}} =\frac{\nu_v\chi^\uparrow_0+\nu_v\chi^\downarrow_0+4v(q)\nu_v^2\chi^\uparrow_0\chi^\downarrow_0G^v_-}{\mathcal{D}_v} , \tag{A11}$$

where

$$\mathcal{D}_v\equiv1-v(q)(\nu_v\chi^\downarrow_0+\nu_v\chi^\uparrow_0)(1-G^v_+-G^v_-) -4v(q)^2\nu_v^2\chi^\uparrow_0\chi^\downarrow_0G^v_-(1-G^v_+) . \tag{A12}$$

For an unpolarized system, for which $\chi^\uparrow_0=\chi^\downarrow_0$, the expression for χ_C simplifies to the following

$$\chi_C=\frac{\nu_v\chi_0}{1-v(q)(1-G^v_+)\nu_v\chi_0} , \tag{A13}$$

where $\chi_0=\chi^\uparrow_0+\chi^\downarrow_0$.

To derive the spin response function, we consider the case of an infinitesimal external magnetic field H^z_{ext} setting up density fluctuations $\Delta n_\uparrow$ and $\Delta n_\downarrow$. Using Eq. (A6) we get the relevant effective potential to be

$$\phi_\sigma=\mu_B\sigma H^z_{\mathrm{ext}}+v(q)[\Delta n_\uparrow+\Delta n_\downarrow](1-G^v_+) -\sigma v(q)[\Delta n_\uparrow-\Delta n_\downarrow]G^v_- . \tag{A14}$$

Then from the definition of spin response and from Eqs. (A10) and (A14) it follows that

$$\chi_S\equiv\mu_B\frac{\Delta n_\downarrow-\Delta n_\uparrow}{H^z_{\mathrm{ext}}} =-\mu_B^2\frac{\nu_v\chi^\uparrow_0+\nu_v\chi^\downarrow_0-4v(q)\nu_v^2\chi^\uparrow_0\chi^\downarrow_0(1-G^v_+)}{\mathcal{D}_v} . \tag{A15}$$

For an unpolarized system this reduces to the familiar form

$$\chi_S=-\mu_B^2\frac{\nu_v\chi_0}{1-v(q)G^v_-\nu_v\chi_0} . \tag{A16}$$

The mixed charge-spin response function is obtained by using Eqs. (A6), (A10), and (A14)

$$\chi_{CS}\equiv\frac{\Delta n_\uparrow-\Delta n_\downarrow}{\phi_{\mathrm{ext}}}\equiv\frac{\Delta n_\uparrow+\Delta n_\downarrow}{\mu_BH^z_{\mathrm{ext}}}=\frac{\nu_v\chi^\uparrow_0-\nu_v\chi^\downarrow_0}{\mathcal{D}_v} . \tag{A17}$$

It should be noted that for an unpolarized system, as can be expected from symmetry considerations, $\chi_{CS}=0$.

The transverse spin response $\chi^{T\sigma}$ can be defined for a multivalley system as follows:

$$\chi^{T\sigma}\equiv-\mu_B^2\frac{\nu_v\chi^{T\sigma}_0}{1+2v(q)G^{Tv}_-\nu_v\chi^{T\sigma}_0} , \tag{A18}$$

where $\chi^{T\sigma}_0(\mathbf{q},\omega)$ is defined in Eq. (28) in the text. The only unknown quantity in Eq. (A18) is the transverse many-body local field G^{Tv}_- for which we propose the following ansatz:

$$G^{Tv}_-\equiv\frac{1}{2\nu_v}[G^{\uparrow\uparrow}_{x,\mathrm{intra}}+G^{\uparrow\uparrow}_{c,\mathrm{intra}}+G^{\downarrow\downarrow}_{x,\mathrm{intra}} +G^{\downarrow\downarrow}_{c,\mathrm{intra}}-2G^{\uparrow\downarrow}_{c,\mathrm{intra}}] . \tag{A19}$$

Now, for an unpolarized system $\chi^{T\sigma}$ simplifies to $\frac{1}{2}\chi_S$ with $\chi^{T\sigma}_0$ and G^{Tv}_- reducing to their unpolarized forms χ^σ_0 and G^v_-, respectively. However, it should be noted that within the present approximation of $G^{\uparrow\uparrow}_{x(c),\mathrm{intra}}=G^{\downarrow\downarrow}_{x(c),\mathrm{intra}}$, and the transverse field G^{Tv}_- coincides with the longitudinal field G^v_-.

APPENDIX B

We will consider here a system with arbitrary polarization and derive the expression for the mixed charge-spin response function χ_{CS}. Let the initial state of the system be $|0\rangle$ and let the Hamiltonian H be characterized by eigenstates $|n\rangle$ having excitation energies ω_{n0}. Let the ground state of the system be $|G\rangle$. In order to obtain an expression of χ_{CS}, we begin by considering the spin response of the system due to an external spin symmetric potential $\phi(r,t)$. The Hamiltonian corresponding to the perturbation is (with standard notation) given by

$$H_e=[\rho_{\mathbf{q}}\phi(\mathbf{q},\omega)e^{-i\omega t}+\text{c.c.}]e^{\eta t} . \tag{B1}$$

Then the Schrödinger equation can be written as

$$i\frac{\partial}{\partial t}|\psi(t)\rangle=(H+H_e)|\psi(t)\rangle , \tag{B2}$$

where $|\psi(t)\rangle$ is an eigenstate of the total Hamiltonian and can be projected onto the states $|n\rangle$ as follows:

$$|\psi(t)\rangle=\sum_n a_n(t)e^{-i\omega_n t}|n\rangle . \tag{B3}$$

The boundary conditions are given by

$$a_n(-\infty)=\begin{cases}1 & \text{if } n=0\\ 0 & \text{otherwise.}\end{cases} \tag{B4}$$

Substituting Eq. (B3) in Eq. (B2) and retaining only the terms that are first order in $\phi(\mathbf{q},\omega)$, we obtain after integration

$$a_n(t)=\frac{\langle n|\rho_{\mathbf{q}}|0\rangle\phi(\mathbf{q},\omega)e^{(-i\omega+i\omega_{n0}+\eta)t}}{\omega-\omega_{n0}+i\eta}-\frac{\langle n|\rho_{-\mathbf{q}}|0\rangle\phi(\mathbf{q},\omega)^*e^{(i\omega+i\omega_{n0}+\eta)t}}{\omega+\omega_{n0}-i\eta} , \tag{B5}$$

where $n\neq0$. Then on defining

$$\langle S^z_{-\mathbf{q}}(t)\rangle\equiv\langle\psi(t)|S^z_{-\mathbf{q}}|\psi(t)\rangle , \tag{B6}$$

with $S^z_{-\mathbf{q}}$ being the induced spin density fluctuation operator and using Eqs. (B3), (B5), and (B6), we obtain

$$\langle S^z_{-\mathbf{q}}(t)\rangle=\sum_n\left\{\frac{\langle0|S^z_{-\mathbf{q}}|n\rangle\langle n|\rho_{\mathbf{q}}|0\rangle\phi(\mathbf{q},\omega)e^{(-i\omega+\eta)t}}{\omega-\omega_{n0}+i\eta}-\frac{\langle0|S^z_{-\mathbf{q}}|n\rangle\langle n|\rho_{-\mathbf{q}}|0\rangle\phi(\mathbf{q},\omega)^*e^{(i\omega+\eta)t}}{\omega+\omega_{n0}-i\eta}\right.$$
$$\left.+\frac{\langle n|S^z_{-\mathbf{q}}|0\rangle\langle0|\rho_{-\mathbf{q}}|n\rangle\phi(\mathbf{q},\omega)^*e^{(i\omega+\eta)t}}{\omega-\omega_{n0}-i\eta}-\frac{\langle n|S^z_{-\mathbf{q}}|0\rangle\langle0|\rho_{\mathbf{q}}|n\rangle\phi(\mathbf{q},\omega)e^{(-i\omega+\eta)t}}{\omega+\omega_{n0}+i\eta}\right\} , \tag{B7}$$

where use has been made of the fact that

$$\langle0|S^z_{\mathbf{q}}|0\rangle=0 . \tag{B8}$$

In Eq. (B7) the second and third terms on the right hand side vanish since $|n\rangle$ cannot be coupled to $|0\rangle$ by both $S^z_{\mathbf{q}}$ and $\rho_{-\mathbf{q}}$ since the former has momentum $\mathbf{q}$ whereas the latter has momentum $-\mathbf{q}$. Then we obtain the following expression for the spin response:

$$\frac{\langle S^z_{-\mathbf{q}}(\omega)\rangle}{\phi(\mathbf{q},\omega)}=e^{i\omega t-\eta t}\frac{\langle S^z_{-\mathbf{q}}(t)\rangle}{\phi(\mathbf{q},\omega)}=\sum_n\left\{\frac{\langle0|S^z_{-\mathbf{q}}|n\rangle\langle n|\rho_{\mathbf{p}}|0\rangle}{\omega-\omega_{n0}+i\eta}-\frac{\langle n|S^z_{-\mathbf{q}}|0\rangle\langle0|\rho_{\mathbf{q}}|n\rangle}{\omega+\omega_{n0}+i\eta}\right\} . \tag{B9}$$

Similarly, upon applying an external magnetic field $h^z_{\text{ext}}(r,t)$, the charge response is given by

$$\frac{\langle\rho_{-\mathbf{q}}(\omega)\rangle}{\mu_B h^z_{\text{ext}}(\mathbf{q},\omega)}=\sum_n\left\{\frac{\langle0|\rho_{-\mathbf{q}}|n\rangle\langle n|S^z_{\mathbf{q}}|0\rangle}{\omega-\omega_{n0}+i\eta}-\frac{\langle0|S^z_{\mathbf{q}}|n\rangle\langle n|\rho_{-\mathbf{q}}|0\rangle}{\omega+\omega_{n0}+i\eta}\right\} . \tag{B10}$$

Then on using the definition of χ_{CS} given in Eq. (A17) we obtain the following relationship for the mixed charge-spin response function:

$$\chi_{CS}(\mathbf{q},\nu)=\sum_n\left\{\frac{\langle0|\rho_{-\mathbf{q}}|n\rangle\langle n|S^z_{\mathbf{q}}|0\rangle}{\nu-\omega_{no}+i\eta}-\frac{\langle0|S^z_{\mathbf{q}}|n\rangle\langle n|\rho_{-\mathbf{q}}|0\rangle}{\nu+\omega_{no}+i\eta}\right\}=\sum_n\left\{\frac{\langle0|S^z_{-\mathbf{q}}|n\rangle\langle n|\rho_{\mathbf{q}}|0\rangle}{\nu-\omega_{n0}+i\eta}-\frac{\langle n|S^z_{-\mathbf{q}}|0\rangle\langle0|\rho_{\mathbf{q}}|n\rangle}{\nu+\omega_{n0}+i\eta}\right\} . \tag{B11}$$

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