Correction to the decay rate of nonequilibrium carrier distributions due to scattering-in processes

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(Received 8 July 1993)

We show that, for doped semiconductor structures at nonzero temperatures, processes which scatter electrons *into* a state $|\mathbf{k}\rangle$ can contribute strongly to the decay of a nonequilibrium electron occupation of $|\mathbf{k}\rangle$. For electrons, the decay rate $\gamma(\mathbf{k})$ is given by the sum of the total scattering-out and *scattering-in* rates of state $|\mathbf{k}\rangle$. The scattering-in term is safely neglected in the nondegenerate and highly degenerate limits, but it increases $\gamma(\mathbf{k})$ of low-energy electrons injected into semidegenerate systems (between these limits) by the factor $[1-f_0(\mathbf{k})]^{-1}$. We show that the degree of degeneracy of a system of fermions depends strongly on the system dimension, so that doped systems of reduced dimension can be semidegenerate, with the two scattering rates comparable, even at high temperature.

Inelastic scattering of electrons in doped semiconductor systems is of interest because of its technological importance in device performance and because it provides information on the fundamental carrier interactions. For example, it is critical in determining whether substantial ballistic transport of electrons is possible through devices of small dimension[1,2] and whether coherent resonant tunneling conduction is possible in double-barrier and superlattice devices.[3] It is also used for interpretation of hot-electron[4,5] and ultrafast optical[6] spectroscopy experiments.

In these situations, a *nonequilibrium* distribution of carriers is injected into a system in equilibrium, and it is the *decay* of this nonequilibrium distribution, either temporally or spatially, that determines the measured experimental quantities. It has generally been assumed that the decay rate of a distribution that is peaked around momentum $\mathbf{k}$ will be given completely by the total scattering-out rate of state $|\mathbf{k}\rangle$. In this paper, we point out that, while this is a good approximation for most experimental situations in metals, as well as the low-temperature and completely nondegenerate regimes in doped semiconductors, it is *not* true in general. When electrons are injected into states that have a significant thermal electron occupation, the decay rate is significantly *larger* than the scattering-out rate. The decay of the nonequilibrium distribution function (for fermions) is actually given by the *sum* of the total scattering-out *and total scattering-in* rates.

There are three important energy scales in the problem: the chemical potential μ and temperature T of the equilibrium system, and the electron injection energy E. We show that in situations where $E-\mu$ is much larger than k_BT the scattering-in rate is negligible. However, when $E-\mu$ is less than or on the order of several k_BT, it becomes significant. The scattering-in rate can substantially affect the interpretation of experimental results, especially in lower-dimensional semiconductor nanostructures.

We derive this result using the Boltzmann equation. Collision processes produce the time evolution of the distribution function $f(\mathbf{k},t)$ given by

$$\frac{\partial f(\mathbf{k},t)}{\partial t}=-\int\frac{d^dq}{(2\pi)^d}\{P(\mathbf{q},\omega_{\mathbf{k},\mathbf{k}-\mathbf{q}})f(\mathbf{k},t)\times[1-f(\mathbf{k}-\mathbf{q},t)]-P(\mathbf{q},\omega_{\mathbf{k}-\mathbf{q},\mathbf{k}})f(\mathbf{k}-\mathbf{q},t)\times[1-f(\mathbf{k},t)]\}\ , \tag{1}$$

where d is the system dimensionality, $P(\mathbf{q},\omega)$ gives the probability per unit time for a carrier to scatter with change of momentum $\mathbf{q}$ and energy $\hbar\omega$, and $\omega_{\mathbf{k},\mathbf{k}'}=\{E(\mathbf{k})-E(\mathbf{k}')\}/\hbar$.

In most of the experiments described above, the nonequilibrium part of the distribution function is monoenergetic and hence sharply peaked in $\mathbf{k}$ space. Let us assume a perturbation from the equilibrium distribution $f_0(\mathbf{k})$ that is sharply peaked around $\mathbf{k}=\mathbf{k}_0$, so that it can be approximated by a δ function:

$$f(\mathbf{k},t)=f_0(\mathbf{k})+f_1(\mathbf{k},t)\delta(\mathbf{k}-\mathbf{k}_0)\ . \tag{2}$$

Substituting Eq. (2) into Eq. (1) gives the decay rate γ of the nonequilibrium distribution f_1. Noting that the terms not involving f_1 cancel (because there is no net change in f_0 due to scattering), we obtain

$$\frac{\partial}{\partial t}f_1(\mathbf{k}_0,t)=-\gamma(\mathbf{k}_0)f_1(\mathbf{k}_0,t)\ , \tag{3}$$

$$\gamma(\mathbf{k}_0)\equiv\gamma_{\text{out}}(\mathbf{k}_0)+\gamma_{\text{in}}(\mathbf{k}_0)\ , \tag{4}$$

$$\gamma_{\text{out}}(\mathbf{k})=\int\frac{d^dq}{(2\pi)^d}P(\mathbf{q},\omega_{\mathbf{k},\mathbf{k}-\mathbf{q}})[1-f_0(\mathbf{k}-\mathbf{q})]\ , \tag{5}$$

$$\gamma_{\text{in}}(\mathbf{k})=\int\frac{d^dq}{(2\pi)^d}P(\mathbf{q},\omega_{\mathbf{k}-\mathbf{q},\mathbf{k}})f_0(\mathbf{k}-\mathbf{q})\ . \tag{6}$$

Here, $\gamma_{\text{out}}(\mathbf{k})$ can be identified as the total equilibrium electron scattering-out rate from an occupied state $\mathbf{k}$.

Similarly, $\gamma_{in}(\mathbf{k})$ is the total equilibrium rate for electron scattering into an unoccupied state $|\mathbf{k}\rangle$ (or, equivalently, the equilibrium conduction-band *hole* scattering-out rate from $|\mathbf{k}\rangle$).[7] The origin of the familiar $\gamma_{out}(\mathbf{k})$ term is obvious. The $\gamma_{in}(\mathbf{k})$ term is due to the increased nonequilibrium occupation of $|\mathbf{k}\rangle$ by the injected electron, which blocks by Pauli exclusion those processes that scatter electrons into $|\mathbf{k}\rangle$. The electron scattering-in rate for the state is thereby reduced below the equilibrium rate, causing $f_1(\mathbf{k},t)$ to decay *faster* than $\gamma_{out}(\mathbf{k})$ alone would indicate.

Though the existence of the scattering-in term $\gamma_{in}(\mathbf{k})$ has been recognized for many years,[8] it is often neglected in calculations of $\gamma(\mathbf{k})$. We now show that this is justified only for nondegenerate systems or for injection energies large compared to the Fermi-surface thermal width above a Fermi sea (e.g., electron-energy-loss experiments in metals). The condition that there is no net change in the distribution function at equilibrium implies that the left-hand side of Eq. (1) is zero when $f(\mathbf{k},t)=f_0(\mathbf{k})$, yielding

$$\gamma_{out}(\mathbf{k})f_0(\mathbf{k})=\gamma_{in}(\mathbf{k})[1-f_0(\mathbf{k})] , \tag{7}$$

which implies, from Eq. (4),

$$\gamma(\mathbf{k})=\frac{\gamma_{out}(\mathbf{k})}{1-f_0(\mathbf{k})} . \tag{8}$$

That is, the decay rate $\gamma(\mathbf{k})$ is *enhanced by the factor* $[1-f_0(\mathbf{k})]^{-1}$ *over the scattering-out rate* $\gamma_{out}(\mathbf{k})$. Thus, the $\gamma_{in}(\mathbf{k})$ contribution to the decay of a single-electron excitation of energy $E(\mathbf{k})$ is significant only if $f_0(\mathbf{k})$ is not small compared to unity. This justifies the common practice of neglecting $\gamma_{in}(\mathbf{k})$ in the classical nondegenerate limit[10] or when the excitation energy is much larger than the thermal width above the equilibrium-system chemical potential.[1,4,9] Also, while the enhancement in γ is very large for $E(\mathbf{k})$ below μ, it is very difficult to perform experiments involving injection of electrons below μ. However, for low-excitation energies above μ in the semidegenerate or degenerate regimes, $\gamma_{in}(\mathbf{k})$ can be significant, as we show below.

First, consider the criterion for the validity of nondegenerate statistics, where the enhancement factor is negligible. The degree of degeneracy of an electron gas may be characterized by the dimensionless temperature $\Theta=k_BT/E_{F,0}$ where $E_{F,0}$ is μ at $T=0$. The Fermi-Dirac distribution function may be approximated safely by the Maxwell-Boltzmann distribution if

$$\exp\{(E-\mu)/k_BT\}\gg 1 \quad \text{or} \quad -\mu/k_BT>x , \tag{9}$$

where $x\sim 3$–5. When this condition holds, one can show that, for a parabolic band,

$$\Theta=\left[\frac{\exp\left[-\frac{\mu}{k_BT}\right]}{\frac{d}{2}\Gamma\left[\frac{d}{2}\right]}\right]^{2/d} . \tag{10}$$

Here Γ is the gamma function and μ is measured with respect to the band edge. Equations (9) and (10) with $x=3$ imply that for $\Theta<6,20$, and 512 in three, two, and one dimensions, respectively, the system cannot be considered nondegenerate and γ_{in} may be significant. Since the system-dimension dependence of the criterion is approximately $\Theta>\exp(2x/d)$, the condition is much more stringent in lower dimensions, and, therefore, Γ_{in} can contribute significantly even at relatively high temperatures in low-dimensional systems.

Recently, two of us[11] presented a calculation of the finite-temperature scattering rate for hot electrons injected into n-type GaAs which neglected the hole-scattering term. The transition rates $P(\mathbf{q},\omega)$ were calculated in the Born approximation using the total dielectric function of a coupled electron-phonon system and the random-phase approximation for the electron polarizability. Figure 1 shows how the result is changed when the neglected term is included for $n=8\times10^{17}$ cm^{-3} and temperatures 0–300 K. At 300 K at this density, $\Theta=0.55$ and the system is semidegenerate. A comparison of the results at low energies above $E_{F,0}=47$ meV is of interest since Heiblum, Galbi, and Weckwerth[5] experimentally determined γ for low-energy electrons in n-type GaAs at this doping level and low temperature (4.2 K). Figure 1 shows that as the temperature is increased and f_0 becomes non-negligible above μ, the decay rate γ can be significantly larger than γ_{out}.

Even a relatively small enhancement of γ due to γ_{in} can have important experimental consequences. For example, in ballistic-electron transport and spectroscopy experiments, one measures the fraction α of injected electrons that traverse a thin transistor base without scattering. This quantity α is determined by the part of f_1 that does not decay during the time t_0 it takes for the elec-

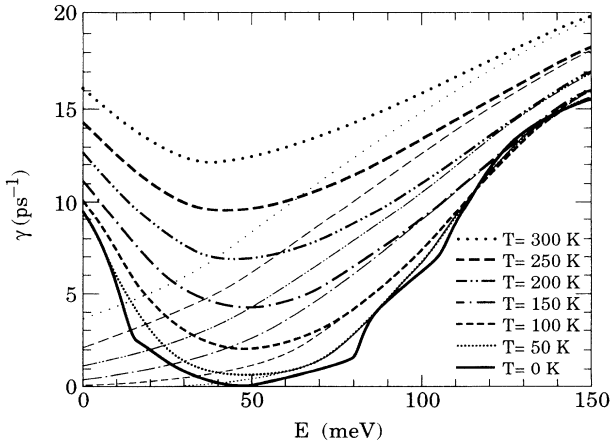

FIG. 1. Comparison of the scattering-out rate $\gamma_{out}(\mathbf{k})$ (thin lines), and the decay rate of the nonequilibrium part of the distribution function, $\gamma(\mathbf{k})=\gamma_{out}(\mathbf{k})+\gamma_{in}(\mathbf{k})$ (bold lines), in n-type GaAs doped at 8×10^{17} cm^{-3}, for various temperatures. The enhancement of $\gamma(\mathbf{k})$ over $\gamma_{out}(\mathbf{k})$ is due to the Pauli principle, which [for $f_1(\mathbf{k})>0$] restricts particles from scattering *into* state $|\mathbf{k}\rangle$, thus reducing the replenishment rate of $|\mathbf{k}\rangle$. The cusps in the $T=0$ curve are due to plasmon and phonon emission thresholds (Ref. 11).

trons to traverse the base. The solution of Eq. (3) shows that $\alpha \propto \exp(-\gamma t_0)$ is *exponentially* sensitive to the scattering rate. Therefore, a small change in γ can imply a substantial change in α or in the differential current gain $\beta=\alpha/(1-\alpha)$ of a hot-electron transistor. The enhancement of γ due to γ_{in} makes devices of this type with useful gain at room temperature less feasible than the earlier calculation[11] indicates.

The correction to γ also can be significant in femtosecond optical spectroscopy, which measures carrier thermalization due to the fastest relaxation mechanisms. Knox[6] used near-band-gap excitation and detection of carriers to study carrier-carrier scattering in modulation-doped quantum wells. The experiments were performed at room temperature and doping densities $n=3.5\times10^{11}$ cm^{-2} such that $\Theta\sim2$. Thus the photoexcited carriers scatter from a sea of equilibrium carriers in the semidegenerate regime and the photexcitation energy ($\sim$20 meV) is within the thermal width of the Fermi surface, making the scattering-in contribution to the thermalization rate significant.

Inelastic scattering determines whether coherent or incoherent tunneling occurs in double-barrier structures.[3] The scattering-in contribution could be important when there is a large buildup of charge in the structure at the resonant bias voltage (as evidenced by increased photoluminescence linewidths[12]), which implies a large $f_0(\mathbf{k})$ in the well, where $\mathbf{k}$ is the momentum parallel to the barrier planes.

Finally, we mention that for bosons, the scattering-in term has the *opposite* sign from the scattering-out term since an extra nonequilibrium occupation of a boson state *enhances* rather than restricts processes which further increase that occupation. Thus the decay of a nonequilibrium exciton or phonon population will be slower (by a factor of $[1+f_0(\mathbf{k})]^{-1}$) than if no scattering-in processes occurred. This is relevant to experiments on excitonic absorption in the presence of an exciton gas or an electron-hole pair gas.[13]

In conclusion, the decay of a nonequilibrium perturbation of an electron-distribution function is given by the sum of *both* the equilibrium scattering-out *and* scattering-in rates. We find that the scattering-in term, which is usually ignored in scattering-rate calculations, enhances the decay rate $\gamma(\mathbf{k})$ by the factor $[1-f_0(\mathbf{k})]^{-1}$ over $\gamma_{\text{out}}(\mathbf{k})$ and can be significant for low-energy electrons injected into semidegenerate systems, especially in lower-dimensional systems. Workers in the semiconductor field should be alerted to the possible significance of scattering-in processes in modern device and experimental structures that are commonly in the semidegenerate regime. While the extremely degenerate ($k_BT/E_F\ll1$) and nondegenerate ($k_BT/E_F\gg1$) cases, applying, respectively, to metals and lightly doped semiconductor devices, do not involve significant scattering-in effects, heavily doped semiconductors at room temperatures or below may be significantly affected.

B.A.S. thanks J. R. Lowney and P. B. Allen for helpful discussions. Work by B.Y-K.H. and S.D.S. was supported by the U.S. ONR and the U.S. ARO.

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$$\tau(\mathbf{k})=1/\gamma(\mathbf{k})=\hbar/[2|\mathrm{Im}\{\Sigma^{\mathrm{ret}}(\mathbf{k},E(\mathbf{k}))\}|] .$$

Σ^{ret} is the retarded self-energy, which can be written as

$$\mathrm{Im}[\Sigma^{\mathrm{ret}}]=(\Sigma^{>}+\Sigma^{<})/2 ,$$

where $\Sigma^{>}/\hbar$ and $\Sigma^{<}/\hbar$ are physically equivalent to the scattering-out and scattering-in rates, respectively [see, e.g., R. Jalabert and S. Das Sarma, Phys. Rev. B **40**, 9723 (1989), or Ref. 8].

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