Semiclassical magnetothermopower of a quasi-two-dimensional electron gas

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Recent studies of the magnetothermopower of a two-dimensional electron gas have concentrated on measurements in either the low-field weak localization regime or the high-field Landau quantization regime. In this paper we emphasize that for magnetic fields between these two limits there is an interesting intermediate regime in which the total thermopower is dominated by semiclassical effects. Detailed expressions are derived for both the diffusion and phonon-drag contributions to the thermopower and a number of important observations are made that should be directly amenable to experimental verification.

I. INTRODUCTION

The physical effect of a perpendicular magnetic field on the thermoelectric transport properties of a two-dimensional electron gas (2DEG) at low temperatures depends upon the strength of the applied field. Weak localization effects are important below a cut-off field $B_{\rm WL} \sim (4\mu k_F l)^{-1}$, where μ is the mobility, k_f is the Fermi wave vector and l is the mean free path.[1] Recent experiments on highly disordered Si-on-sapphire metal-oxide-semiconductor field-effect transitors (MOSFET's) ($\mu \sim 500$ cm^2 V^{-1} s^{-1}, $k_f l \sim 3$) have clearly demonstrated the importance of weak localization effects up to fields ~ 1 T (in these samples $B_{\rm WL} \sim 1.5$ T).[2] Of course, in higher-mobility samples localization effects are much less important and $B_{\rm WL}$ is correspondingly a lot smaller. Here, much attention has been given to the behavior in the Landau quantization regime,[3–5] which may be defined as occurring for fields $B > \mu^{-1}\tau_t/\tau_s$, where τ_t is the transport relaxation time and τ_s is the single-particle (state) relaxation time.[6] What has been largely ignored, however, both experimentally and theoretically, is the intermediate regime which lies between the weak localization regime and the Landau quantization regime, in which the transport behavior is dominated by scattering processes which can be treated semiclassically. For a typical Si MOSFET one might have $\mu \sim 1$ m^2 V^{-1} s^{-1}, $k_f l \sim 10$, and $\tau_t/\tau_s \sim 1$ (since short-range potential fluctuations dominate the scattering), in which case this intermediate regime extends from ~ 25 mT to ~ 1 T. In modulation-doped GaAs/Al$_x$Ga$_{1-x}$As heterojunctions one might have $\mu \sim 30$ m^2 V^{-1} s^{-1}, $k_f l \sim 100$, and $\tau_t/\tau_s \sim 10$ (since long-range potential fluctuations dominate the scattering), whereupon the intermediate regime extends from ~ 0.083 mT to ~ 300 mT. In this paper, a Boltzmann equation approach is used to calculate the relevant magnetothermopower tensor of an isotropic 2DEG in this semiclassical intermediate regime. Detailed consideration is given not only to the diffusion thermopower, but also to the less well understood contribution arising from phonon-drag. A number of important observations are made which should be directly amenable to experimental verification.

The paper is set out as follows. The Boltzmann equation and its formal solutions are given in the following section. In Sec. III these solutions are used to derive the thermopower tensor. Detailed results are presented in Sec. IV for typical "higher mobility" samples such as those discussed above. Finally, in Sec. V, conclusions are given.

II. THE BOLTZMANN EQUATION

The motion of the electrons can be described statistically by the electron distribution function $f(\mathbf{k},\mathbf{r},t)$, where $\mathbf{k}$ and $\mathbf{r}$ are the 2D electron wave and position vectors respectively, and we suppose that only the ground subband is occupied. The steady-state Boltzmann equation for the electrons is[7]

$$\left(\frac{\partial f}{\partial t}\right)_c = \mathbf{v}\cdot\nabla_{\mathbf{r}} f + \dot{\mathbf{k}}\cdot\nabla_{\mathbf{k}} f, \qquad (1)$$

where $\mathbf{v}$ is the velocity of the electrons in the x-y plane.

The left-hand side of Eq. (1) is the rate of change of f due to collisions. The contributions due to static imperfections, ionized impurities, and phonon emission and absorption are taken into account in what follows.

The applied electric fields and temperature gradients are assumed small and therefore we assume that they cause a small linear perturbation to both the electron distribution f and the phonon distribution function $N_{\mathbf{Q}}$, denoted as f^1 and $N^1_{\mathbf{Q}}$, respectively. A relaxation time $\tau(\varepsilon)$ is associated with the electron scattering by ionized impurities and imperfections and another, $\tau_p(Q)$, with the phonon-phonon and phonon-boundary interaction. For weak electron-phonon interaction it can be shown by examining the coupled electron and phonon Boltzmann equations that[8,9]

$$\left(\frac{\partial f}{\partial t}\right)_c = -\frac{f^1}{\tau(\varepsilon)} + U(\mathbf{k}), \qquad (2)$$

where

$$U(\mathbf{k}) = \frac{1}{k_BT}\sum_{\mathbf{k}'\mathbf{Q}}\frac{1}{F}\left(\frac{\partial N_{\mathbf{Q}}}{\partial t}\right)_c(\Gamma_{\mathbf{k}'\mathbf{k}} - \Gamma_{\mathbf{k}\mathbf{k}'}), \qquad (3)$$

$$F = -\frac{dN^o_{\mathbf{Q}}}{d\hbar\omega_{\mathbf{Q}}}\frac{1}{\tau_p(\mathbf{Q})}, \tag{4}$$

and

$$\Gamma_{\mathbf{k}'\mathbf{k}} = f^o_{\mathbf{k}}(1 - f^o_{\mathbf{k}'})P^{ao}_{\mathbf{k}\mathbf{k}'}(\mathbf{Q}) \tag{5}$$

is the average rate of absorption of phonons with three-dimensional (3D) wave vector $\mathbf{Q} = (\mathbf{q}, q_z)$ resulting in electron transitions from $\mathbf{k}$ to $\mathbf{k}'$ when the whole system is in thermal equilibrium. In accordance with the standard methods for solving the linearized Boltzmann equation, use has been made of the detailed balance relationship to simplify the formulas by expressing the phonon emission term in terms of the phonon absorption term.[7,8] $N_{\mathbf{Q}}$ is the nonequilibrium phonon distribution obtained from the phonon Boltzmann equation[8–11] and f^o and $N^o_{\mathbf{Q}}$ are respectively the distributions of electrons and phonons in thermal equilibrium.

An explicit expression for the rate at which the electron will transfer from $\mathbf{k}$ to $\mathbf{k}'$ by absorbing one phonon with wave vector $\mathbf{Q}$ is obtained from the golden rule:[10]

$$P^{ao}_{\mathbf{k}\mathbf{k}'} = \frac{\pi E_1^2 Q^2 N^o_Q}{\rho V \omega_{\mathbf{Q}} \varepsilon^2(q,T)}|Z_{11}|^2 \delta(\varepsilon_{k'} - \varepsilon_k - \hbar\omega_{\mathbf{Q}})\delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}, \tag{6a}$$

$$|Z_{11}|^2 = \frac{b^6}{(b^2 + q_z^2)^3}. \tag{6b}$$

The form factor $|Z_{11}|^2$ accounts for the finite extent of the 2DEG in the confinement direction; b is the variational parameter in the Fang and Howard wave function.[10] ρ is the density of the material and it is 2.39 g/cm^3 for Si and 5.3 g/cm^3 for GaAs. V is the volume of the material and E_1 is a spherically symmetric acoustic-phonon deformation potential. Finally, $\varepsilon(q,T)$ is the temperature-dependent dielectric function, which is calculated in the single-subband, random-phase approximation.[10] Since we are restricting ourselves to the regime where Landau quantization is unimportant, we assume that the electron states themselves are not affected by the field. Consequently, both $\varepsilon(q,T)$ and $P^{ao}_{\mathbf{k}\mathbf{k}'}$ are independent of magnetic field.

To maintain simplicity of notation, the above expressions (and those that follow) have been written down only for the simplest possible single-branch phonon process. It is implicitly understood throughout the paper that different phonon processes must be summed over according to the following rules. At low temperatures we need consider only long-wavelength acoustic phonons whose dispersion is characterized by an appropriate velocity v. In GaAs only longitudinal phonons ($v = 5140$ m s^{-1}) contribute to the deformation potential interaction and $E_1 = -11$ eV.[12] In Si, in order to account for both longitudinal (LA) and transverse (TA) acoustic phonon modes (with velocities $v = 8831$ m s^{-1} and $v = 5281$ m s^{-1} respectively), E_1 is replaced in the calculations by $\Xi_u(q_z^2/Q^2 + D)$ and $\Xi_u q q_z/Q^2$ for LA and TA phonon modes respectively.[10,13,14] Ξ_u is the deformation potential for pure shear strain and $D = \Xi_d/\Xi_u$, with Ξ_d denoting the deformation potential for pure dilation. The values used in the calculations for Si are $\Xi_u = 9.0$ eV and $\Xi_d = -6.0$ eV. In GaAs the piezoelectric scattering interaction should also be added to the deformation potential interaction and so the function $(eh_{14})^2 A/Q^2$ has to be added to E_1^2 in Eq. (6a) for LA phonons and replaces E_1^2 for TA phonons ($v = 3040$ m s^{-1}). The parameter h_{14} is equal to 1.2×10^7 V/cm, and A is $9q^4q_z^2/2Q^6$ for LA phonons and $(8q^2q_z^4 + q^6)/4Q^6$ for TA phonons.[5]

In response to an in-plane electromotive force $\mathbf{E}$, an in-plane temperature gradient ∇T, and a perpendicular magnetic field $\mathbf{B}$, the linearized Boltzmann equation for the electrons can be written

$$-\frac{f^1}{\tau(\varepsilon)} - \mathbf{v}\cdot\nabla_{\mathbf{r}} f^o - \frac{e}{\hbar}\mathbf{E}\cdot\nabla_{\mathbf{k}} f^o - \frac{e}{\hbar}(\mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{k}} f^1_{\mathbf{k}} + U(\mathbf{k}) = 0 \tag{7}$$

where the charge on the electron is denoted by e, which is negative. Equation (7) is solved in the Appendix. For weak electron-phonon interactions the external fields and the electron-phonon interaction contribute independently to the perturbation of the electron distribution and the solution of (7) can be formally written as:

$$f^1 = f^1_d + f^1_g, \tag{8}$$

where the diffusion and phonon-drag terms respectively are given by the following equations:

$$f^1_d(\mathbf{k}) = -\frac{df^o}{d\varepsilon}\frac{\mathbf{v}\cdot\mathbf{A} - \omega\tau_f(\mathbf{v}\times\mathbf{A})_z}{1 + \omega^2\tau_f^2} \tag{9}$$

and

$$f^1_g(\mathbf{k}) = \int_0^{2\pi}\frac{d\theta^*}{\omega}G(\theta,\theta^*)U(k,\theta^*). \tag{10}$$

Here, ω is the cyclotron frequency, τ_f is the zero-magnetic-field (scattering) relaxation time at the Fermi level, $\mathbf{A}$ is defined in the Appendix, and finally $G(\theta,\theta^*)$ is a Green's function also defined in the Appendix.

III. CALCULATION OF THE THERMOPOWER AND THE NERNST-ETTINGSHAUSEN COEFFICIENT

The total 2D current density is

$$\mathbf{J}_{2D} = \frac{2e}{A}\sum_{\mathbf{k}}\mathbf{v}f^1(\mathbf{r},\mathbf{k}) = \sigma\mathbf{E} + L\nabla_{\mathbf{r}}T, \tag{11}$$

where A is the area of the 2DEG. The transport tensors, σ and L, are deduced by comparing the second part with the third (phenomenological) part of Eq. (11). Equation (8) shows that the current (and consequently the transport coefficients) arise from two distinct processes: electron diffusion and electron drag by phonons. The phonon-drag contribution to σ is usually negligible but phonon-drag plays an important role in thermoelectric transport. The thermoelectric tensor obtained from the two processes is considered below.

A. Diffusion

A straightforward modification of the standard 3D theory (see, e.g., Ref. 15) gives for the 2D transport coefficients

$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_o}{1+\omega^2\tau_f^2} \equiv \sigma_B, \tag{12}$$

$$\sigma_{xy} = -\sigma_{yx} = -(\omega\tau_f)\sigma_{xx}, \tag{13}$$

and

$$L_{xx} = L_{yy} = \frac{\pi^2 k_B}{3|e|} k_B T \left.\frac{d\sigma_B}{d\varepsilon}\right|_{\varepsilon_f}, \tag{14}$$

$$L_{yx} = -L_{xy} = \frac{\pi^2 B}{3m^*} k_B^2 T \left.\frac{d(\tau\sigma_B)}{d\varepsilon}\right|_{\varepsilon_f}, \tag{15}$$

where $\sigma_o = e^2\tau_f N_s/m^*$ is the zero-magnetic-field conductivity, m^* is the effective mass, and N_s is the area density of electrons. The thermopower may be expressed in terms of L and σ:[7]

$$S_{xx} = -\frac{\sigma_{xx}L_{xx} - \sigma_{xy}L_{yx}}{\sigma_{xx}^2 + \sigma_{xy}^2}, \tag{16}$$

$$S_{xy} = \frac{\sigma_{xx}L_{yx} + \sigma_{xy}L_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}. \tag{17}$$

We easily find from Eqs. (12)–(17) that

$$S_{xx}^d = -C_d\left(1 + \frac{p}{1+\omega^2\tau_f^2}\right), \tag{18}$$

and

$$S_{xy}^d = pC_d\frac{\omega\tau_f}{1+\omega^2\tau_f^2} \tag{19}$$

with

$$C_d = \frac{\pi^2 k_B}{3|e|}\frac{k_B T}{\varepsilon_f} \tag{20}$$

where the energy dependence of τ is expressed through the quantity p: $p = \varepsilon_f(d\ln\tau/d\varepsilon)|_{\varepsilon_f}$.

The equations for S^d show that all the magnetic field dependence in the thermoelectric power tensor is proportional to p. We see that, for a strongly degenerate electron gas, the changes in L in a magnetic field result in zero magnetothermopower $\Delta S_{xx} \equiv [S_{xx}(B) - S_{xx}^o]$ and Nernst-Ettingshausen coefficient S_{xy} when $p = 0$.

B. Phonon-drag

The contribution to the current due to electron-phonon drag can be obtained by combining Eqs. (3), (10), and (11):

$$J_{2D}^g = -\frac{2|e|}{k_B T^2 A}\sum_{\mathbf{k}} \mathbf{v}\int_0^{2\pi}\frac{d\theta^*}{\omega}G(\theta,\theta^*) \times \sum_{\mathbf{k}',\mathbf{Q}} \tau_p(Q)\hbar\omega_{\mathbf{Q}}(\Gamma_{\mathbf{k}'\mathbf{k}^*} - \Gamma_{\mathbf{k}^*\mathbf{k}'})\mathbf{v}_p\cdot\nabla_{\mathbf{r}}T, \tag{21}$$

where $\mathbf{k}^* = (k,\theta^*)$ and $\mathbf{v}_p$ denotes the phonon velocity.

The components of L are therefore given by

$$L_{ij} = -\frac{2|e|}{k_B T^2 A}\sum_{\mathbf{k}} v_i\int_0^{2\pi}\frac{d\theta^*}{\omega}G(\theta,\theta^*) \times \sum_{\mathbf{k}',\mathbf{Q}} \tau_p(Q)\hbar\omega_{\mathbf{Q}}(\Gamma_{\mathbf{k}'\mathbf{k}^*} - \Gamma_{\mathbf{k}^*\mathbf{k}'})v_{pj}. \tag{22}$$

The application of a magnetic field perpendicular to the electron gas does not affect the uniformity of the electron-phonon interaction in the x-y plane. For given electron and phonon energies this interaction depends only on the angle between the phonon and the electron wave vectors. This fact simplifies the analytical expressions obtained for L. The two independent coefficients are given by

$$L_{xx} = L_{yy} = -\int\int dq\,dq_z C_g F(\mathbf{Q}) \times\left[\left\{\left(\frac{2m^*}{\hbar}\omega_{\mathbf{Q}} - q^2\right)I_{00} - \left(\frac{2m^*}{\hbar}\omega_{\mathbf{Q}} + q^2\right)I_{01}\right\}\right], \tag{23}$$

and

$$L_{yx} = -L_{xy} = -\int\int dq\,dq_z C_g F(\mathbf{Q}) \times\left[\left\{\left(\frac{2m^*}{\hbar}\omega_{\mathbf{Q}} - q^2\right)I_{10} - \left(\frac{2m^*}{\hbar}\omega_{\mathbf{Q}} + q^2\right)I_{11}\right\}\right], \tag{24}$$

where

$$C_g = \frac{|e| l_p E_1^2 g_v (2m^*)^{\frac{1}{2}}}{32\pi^3\hbar k_B T^2\rho} \tag{25}$$

and

$$F(\mathbf{Q}) = \frac{Q|Z_{11}|^2}{\varepsilon^2(q,T)}\frac{1-e^{-\hbar\omega_{\mathbf{Q}}/k_B T}}{\sinh^2(\hbar\omega_{\mathbf{Q}}/2k_B T)}, \tag{26}$$

with

$$I_{nm} = \int_0^\infty du\,O_n(u^2+\gamma+m\hbar\omega_{\mathbf{Q}})f(u^2+\gamma) \times[1 - f(u^2+\gamma+\hbar\omega_{\mathbf{Q}})]. \tag{27}$$

Here

$$O_n = \frac{\omega^n\tau^{n+1}}{1+\omega^2\tau^2}, \tag{28}$$

$$\varepsilon(k) = u^2 + \gamma, \tag{29}$$

where

$$\gamma = \frac{(\hbar\omega_{\mathbf{Q}} - \hbar^2 q^2/2m^*)^2}{4(\hbar^2 q^2/2m^*)}. \tag{30}$$

In Eq. (25), g_v is the valley degeneracy and it is 2 for Si and 1 for the $GaAs/Al_xGa_{1-x}As$ heterojunction, and the in-plane effective masses m^* are $0.19m_e$ and $0.067m_e$ respectively, with m_e being the free electron mass. We retain C_g within the integral as a reminder that for different phonon processes E_1^2 is, in general, replaced by a wave-vector-dependent function according to the rules prescribed earlier. These processes are then summed over. It is assumed that, at low temperatures, boundary scattering dominates the phonon-phonon interaction. If v_s is a phonon velocity and $\tau_p(Q)$ is the corresponding phonon relaxation time, then the phonon mean free path $l_p = v_s\tau_p(Q)$ is determined by the dimensions of the sample and is assumed to be independent of phonon mode and wave vector. The values of l_p used in the calculations were determined experimentally. They are 1.08 mm at 1.5 K and 0.92 mm at 7 K for a Si MOSFET (Ref. 16) and 0.41 mm at 3 K for the $GaAs/Al_xG_{1-x}As$ heterojunction.[3] The occupancy factor $f(u^2+\gamma)[1-f(u^2+\gamma+\hbar\omega_{\mathbf{Q}})]$ in Eq. (27) is, for $k_BT << \varepsilon_f$, nonzero only in a restricted region around the Fermi level. Moreover the relaxation time does not change rapidly near ε_f. Consequently we can approximate using first-order expansions at ε_f or $\varepsilon_f+\hbar\omega_{\mathbf{Q}}$ depending on which integral I_{nm} is involved. By proceeding in this way and using Eqs. (16) and (17) we find that:

$$S^g_{xx} = S^o_g + p\frac{\omega^2\tau_f^2}{1+\omega^2\tau_f^2}S^1_g, \tag{31}$$

and

$$S^g_{xy} = p\frac{\omega\tau_f}{1+\omega^2\tau_f^2}S^1_g \tag{32}$$

where

$$S^o_g = -\frac{2\tau_f}{\sigma_o}\int\int dqdq_z q^2 C_g F(\mathbf{Q})\lambda^o(\mathbf{Q}) \times \int_0^\infty du f(u^2+\gamma)[1-f(u^2+\gamma+\hbar\omega_{\mathbf{Q}})], \tag{33}$$

and

$$S^1_g = \frac{2\tau_f}{\sigma_o}\int\int dqdq_z q^2 C_g F(\mathbf{Q})\lambda^1(\mathbf{Q}) \times \int_0^\infty du f(u^2+\gamma)[1-f(u^2+\gamma+\hbar\omega_{\mathbf{Q}})], \tag{34}$$

with

$$\lambda^o(\mathbf{Q}) = 1 + p\lambda^1(\mathbf{Q}) \tag{35}$$

and

$$\lambda^1(\mathbf{Q}) = \frac{1}{2}\frac{\hbar\omega_{\mathbf{Q}}}{\varepsilon_f}\left[1+\frac{\hbar\omega_{\mathbf{Q}}}{\hbar^2q^2/2m^*}\right]. \tag{36}$$

By summing the contributions to the thermopower tensor elements given in these equations and in Eqs. (18) and (19) we see that the total magnetothermopower and the Nernst-Ettingshausen coefficient are

$$\Delta S_{xx} = p\frac{\omega^2\tau_f^2}{1+\omega^2\tau_f^2}C, \tag{37}$$

$$S_{xy} = p\frac{\omega\tau_f}{1+\omega^2\tau_f^2}C, \tag{38}$$

with

$$C = S^1_g + C_d. \tag{39}$$

It can be seen from Eqs. (20), (25), and (34) that C is independent of the magnetic field B.

IV. DISCUSSION

The magnetothermopower tensor dependence upon B is predicted by Eqs. (37) and (38). Interestingly, the structural simplicity of the diffusion terms is maintained when drag processes are taken into account. Since ΔS_{xx} is proportional to $\omega^2\tau_f^2/(1+\omega^2\tau_f^2)$, the rate of increase of $|\Delta S_{xx}|$ increases below $\omega\tau_f = 1/\sqrt{3}$ and decreases above it with the sign of ΔS_{xx} being determined by the sign of p. S_{xy} is proportional to $\omega\tau_f/(1+\omega^2\tau_f^2)$. Hence, $\omega\tau_f = 1$ is predicted as a stationary point with the extreme value being $Cp/2$. S_{xy} is maximum at $\omega\tau_f = 1$ when $p > 0$ ($GaAs/Al_xGa_{1-x}As$ heterojunction)[3,13] and minimum when $p < 0$ (Si MOSFET).[13]

Both the magnitude and the sign of the derivative of τ with respect to the energy in the neighborhood of ε_f are important in determining the behavior of the elements of the magnetothermopower tensor. Their sign is determined by the sign of p, as can be seen in Eqs. (37) and (38). In Fig. 1, S^d_{xx} and S^d_{xy} are plotted against $\omega\tau_f$ for general values of p. The diffusive case has been chosen for qualitative discussion. The magnitude of p is particularly important for the sign of the diffusion thermopower, as is predicted by Eq. (18). If $p < -(1+\omega_c^2\tau_f^2)$, S^d_{xx} changes

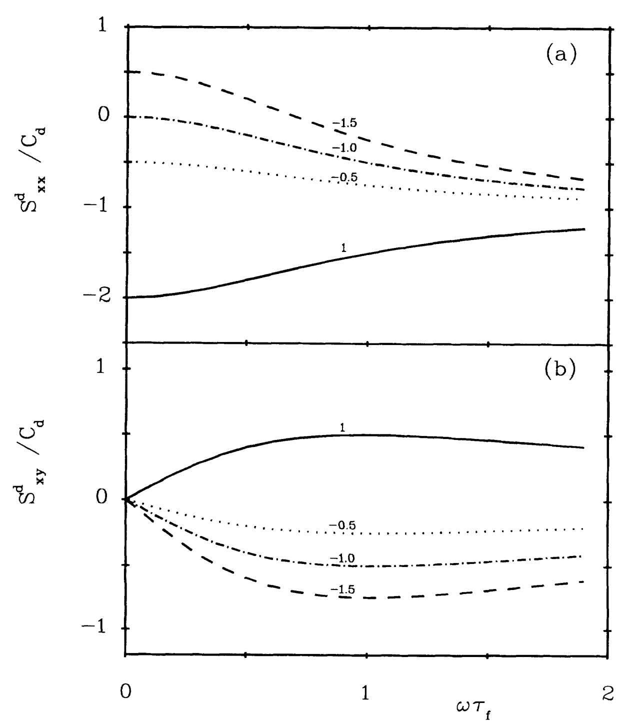

FIG. 1. S^d_{xx}/C_d (a) and S^d_{xy}/C_d (b) are plotted versus $\omega\tau_f$ for different values of p.

sign (Fig. 1). This can lead to the observation of a positive S_{xx} at low temperatures, where diffusion dominates. Such an observation has already been reported for the electron gas embedded in a Si MOSFET in zero magnetic field.[16,17] Here, it is predicted that the electron density at which the thermopower vanishes changes with the applied magnetic field.

Detailed numerical calculations for a Si MOSFET used in recent experiments are presented in Figs. 2, 3, and 5. The electron density is $N_s = 10.7 \times 10^{15}$ m^{-2} and the measured mobility at T=1 K is $\mu \sim 1$ m^2/V s,[16,17] which is consistent with theoretical calculations based on dominant interface-ionized impurities and interface roughness scattering.[13] The calculated value for p is -0.6. Since $\mu B \sim 1$ at $B \sim 1$ T the predicted semiclassical behavior is expected to be observed for this Si MOSFET, at low temperatures and for magnetic fields above the localization regime until the onset of Landau localization. The high-field limit of the semiclassical regime coincides with the onset of Landau quantization and measurements there would indicate the nature of the crossover.

As is illustrated in Fig. 2, the main contribution to both the magnetothermopower ΔS_{xx} and the Nernst-Ettingshausen S_{xy} coefficients is due to electron diffusion when $T < 2$ K. We see from Eqs. (31) and (32) that the phonon-drag contribution is determined by the magnitude of S_g^1, which depends on the ratio $\hbar\omega_{\mathbf{Q}}/\varepsilon_f$. At low T the allowed phonon energies are small, resulting in a small value of S_g^1. Increasing T increases the allowed phonon energies with the result that S_g^1 goes up. We see from Fig. 3 that the phonon-drag contribution dominates when $T = 7$ K.

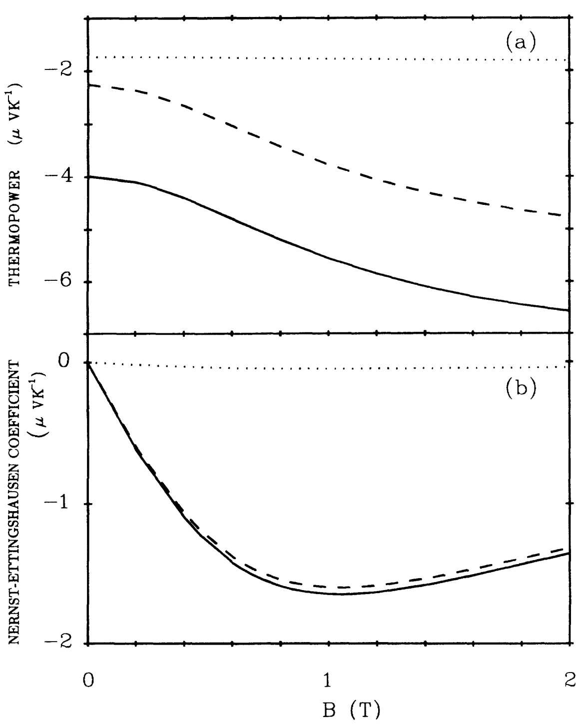

FIG. 2. S_{xx} (a) and S_{xy} (b) are plotted versus B for drag (dotted line), diffusion (dashed line), and total (solid line). Si MOSFET with $N_s = 10.7 \times 10^{15}$ m^{-2} at $T = 1.5$ K.

The same magnetic-field dependence is predicted for GaAs/Al$_x$Ga$_{1-x}$As heterojunctions from Eqs. (37) and (38) and is shown in Figs. 4(a) and 4(b) respectively. In this case, both the magnetothermopower ΔS_{xx} and the Nernst-Ettingshausen coefficient S_{xy} are positive because p is positive (the calculated value for p is 0.9). Moreover, as a result of modulation doping and low interface roughness the mobility of electrons in a heterojunction is much higher than that of electrons in a Si MOSFET. Typically for $N_s = 6.82 \times 10^{15}$ m^{-2} and at $T = 4.2$ K, $\mu \sim 25$ m^2 V^{-1} s^{-1}.[3] Semiclassical effects are expected to be dominant up to $B \sim \mu^{-1}\tau_t/\tau_s \sim 400$ mT. The characteristic behavior is analogous to that described for the Si MOSFET but the peak in S_{xy} and the turning point in S_{xx} are observed at $B \sim 150$ mT (Fig. 4). Because of the higher electron mobility in the heterojunction, most of the variation with magnetic field is now observed in a narrow range of values around $B \sim 150$ mT, which is well below the onset of Landau quantization. On the other hand, weak localization is only important for $B < 0.1$ mT. Therefore the region where pronounced semiclassical effects occur is completely distinct and direct measurements should confirm the above predictions. Drag dominates at almost all temperatures in GaAs/Al$_x$Ga$_{1-x}$As heterojunctions because m^* is considerably smaller than in Si MOSFET's.

Equations (37) and (38) can be expressed in the following way:

$$\frac{\Delta S_{xx}}{C} = p\frac{\mu^2 B^2}{1 + \mu^2 B^2}, \tag{40}$$

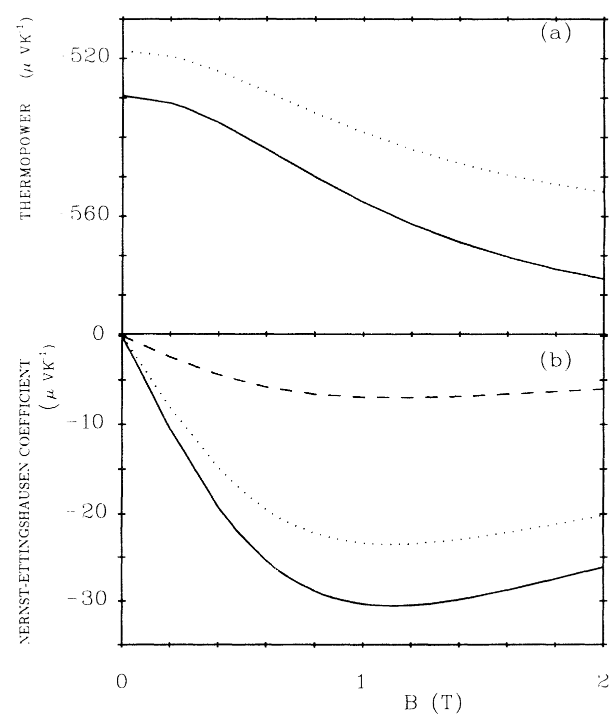

FIG. 3. (a) S_{xx} is plotted versus B for drag (dotted line) and for total thermopower (solid line). (b) S_{xy} is plotted versus B for diffusion (dashed line), drag (dotted line), and total thermopower (solid line). Si MOSFET with $N_s = 10.7 \times 10^{15}$ m^{-2} at $T = 7$ K.

$$\frac{S_{xy}}{C} = p\frac{\mu B}{1+\mu^2 B^2}. \tag{41}$$

The quantity p reflects the dominant scattering process and will vary from system to system. The direct relation between magnetothermopower effects and p is expected to prove useful in determining p experimentally. At a particular temperature, both μ and N_s (and consequently ε_f, since it can be expressed in terms of N_s) are usually known from Hall effect and conductivity measurements. The parameters l_p and E_1 in Eq. (25) can be estimated experimentally. Therefore C in the above equations can be calculated for a given T and N_s. Hence, we can determine the value of p by fitting either Eq. (40) or Eq. (41) to experimental data points for either ΔS_{xx} or S_{xy} as functions of B. At low temperatures ($T < 2$ K for a Si MOSFET and $T < 0.4$ K for a GaAs/Al$_x$Ga$_{1-x}$As heterojunction) phonon-drag effects are negligible. Therefore, the value of p can be deduced from diffusion magnetothermopower effects. It should be noticed that even zero-magnetic-field thermopower data at low temperatures (where the diffusion thermopower is practically the only contribution to the total thermopower) can give the value of p, although here localization effects need to be considered carefully. The new prediction of the present theory is that the magnetothermopower effects due to phonon-drag depend on p in the same way as those due to diffusion. Thus, even at comparatively high temperatures the experimental data can be processed to yield a value of p in a similar way to that for low temperatures. At higher temperatures the values of both ΔS_{xx} and S_{xy} are much larger than they are at low temperatures, which makes their experimental determination as functions of B much easier.

Another consequence of Eqs. (37) and (38) is:

$$\frac{\Delta S_{xx}(B)}{S_{xy}(B)} = \mu B. \tag{42}$$

Equation (42) implies that a plot of $\Delta S_{xx}/S_{xy}$ versus B should be a straight line with slope μ. Since the mobility is usually known from other experiments, this plot provides a simple check of the theory.

The accuracy of the analytical results expressed in Eqs. (31) and (32) remains to be discussed. The discussion is concentrated on the situation when phonon drag dominates ($T > 2$ K for a Si MOSFET and 0.4 K for a GaAs/Al$_x$Ga$_{1-x}$As heterojunction). The deviation of the results calculated from Eqs. (31) and (32) from exact results depends on k_BT/ε_f. We compare approximate and exact results in Fig. 5 for a Si MOSFET for $k_BT/\varepsilon_f = 0.09$ and 0.24. From those and similar plots we conclude that the approximate phonon-drag formulas are valid when $k_BT/\varepsilon_f < 0.1$.

V. CONCLUSIONS

In this paper thermoelectric effects have been examined in the presence of a magnetic field perpendicular

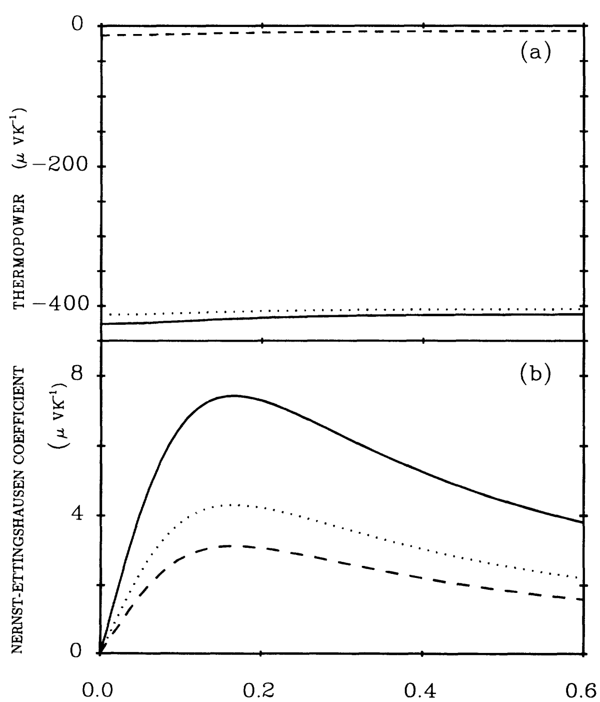

FIG. 4. S_{xx} (a) and S_{xy} (b) are plotted versus B for drag (dotted line), diffusion (dashed line), and total (solid line). GaAs/Al$_x$Ga$_{1-x}$As heterojunction with $N_s = 6.82\times10^{15}$ m^{-2} at $T = 7$ K.

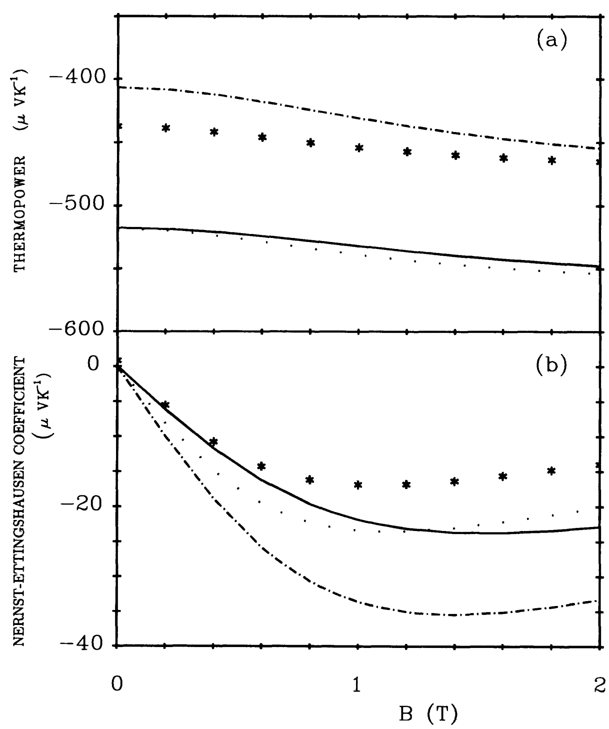

FIG. 5. Comparison of exact drag results with approximations for a Si MOSFET in which functions involving the relaxation time are expanded about the Fermi level. (a) S_{xx} for $T = 7$ K. The full line is exact and the dots are approximations for $N_s = 10.7\times10^{15}$ m^{-2} and $k_BT/\varepsilon_f = 0.09$. The dash-dot line is exact and the stars are approximations for $N_s = 4\times10^{15}$ m^{-2} and $k_BT/\varepsilon_f = 0.24$. (b) S_{xy} for $T = 7$ K for the same two cases using the same notation.

to a 2DEG when the magnitude of B is above the weak localization regime and below the Landau quantization regime. The Boltzmann equation has been used to obtain the transport coefficients when electron scattering by ionized impurities and imperfections as well as electron-phonon interactions are taken into account. When the electron-phonon coupling is weak and the elastic scattering of the electrons is described by a relaxation time, the magnetothermoelectric effects are similar in the diffusion- and phonon-drag-dominated regimes. The predicted behavior is easily verified experimentally, and such experiments provide a convenient route to the direct determination of the logarithmic derivative of the relaxation time at the Fermi level.

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APPENDIX: SOLUTION OF THE BOLTZMANN EQUATION

We assume that the electron gas is located in the x-y plane. The Boltzmann equation when only the ground subband is occupied can be written as

$$-\frac{f^1}{\tau} - \frac{e}{\hbar}B(\nabla_{\mathbf{k}} f^1 \times \mathbf{v})_z = -\frac{\partial f^o}{\partial \varepsilon}\frac{\varepsilon-\varepsilon_f}{T}\mathbf{v}\cdot\nabla_{\mathbf{r}}T + e\frac{\partial f^o}{\partial \varepsilon}\mathbf{v}\cdot\mathbf{E} - U. \quad (A1)$$

Defining $\lambda = (\omega\tau)^{-1}$, where $\omega = |e|B/m^*$ is the cyclotron frequency, and expressing the electron wave vector as $\mathbf{k} = (k\cos\theta, k\sin\theta)$, Eq. (A1) becomes

$$\lambda f^1 + \frac{\partial f^1}{\partial \theta} = g_1(k)\cos\theta + g_2(k)\sin\theta + g_3(k,\theta), \quad (A2)$$

where

$$g_1(k) = \frac{\partial f^o}{\partial \varepsilon}\left[\frac{\varepsilon-\varepsilon_f}{\omega T}v\frac{\partial T}{\partial x} - \frac{e}{\omega}vE_x\right], \quad (A3)$$

$$g_2(k) = \frac{\partial f^o}{\partial \varepsilon}\left[\frac{\varepsilon-\varepsilon_f}{\omega T}v\frac{\partial T}{\partial y} - \frac{e}{\omega}vE_y\right], \quad (A4)$$

and

$$g_3(k,\theta) = \frac{U(k,\theta)}{\omega}. \quad (A5)$$

The differential equation (A2) is solved by using a Green's function, which is defined by the differential equation

$$\frac{\partial G}{\partial \theta} + \lambda G = \delta(\theta - \theta^*) \quad (A6)$$

and the periodic boundary condition: $G(2\pi) = G(0)$. We find that

$$G(\theta,\theta^*) = \begin{cases} \dfrac{e^{-\lambda(\theta-\theta^*)}}{1-e^{-2\pi\lambda}}, & \theta > \theta^*, \\ \dfrac{e^{-\lambda[2\pi+(\theta-\theta^*)]}}{1-e^{-2\pi\lambda}}, & \theta < \theta^*, \end{cases} \quad (A7)$$

and the solution of (A2) is seen to be:

$$f^1(k,\theta) = \int_0^{2\pi} d\theta^* G(\theta,\theta^*)[g_1(k)\cos\theta^* + g_2(k)\sin\theta^* + g_3(k,\theta^*)] = f_d^1 + f_g^1, \quad (A8)$$

where

$$f_d^1 = \frac{\omega\tau}{1+\omega^2\tau^2}\{g_1(k)[\cos\theta + (\omega\tau)\sin\theta] + g_2(k)[\sin\theta - (\omega\tau)\cos\theta]\} \quad (A9)$$

is due to electron diffusion processes and

$$f_g^1 = \int_0^{2\pi} d\theta^* G(\theta,\theta^*) g_3(k,\theta^*) \quad (A10)$$

is due to phonon-drag.

We note that f_d^1 can be written in the following form:

$$f_d^1(\mathbf{k}) = -\frac{\partial f^o}{\partial \varepsilon}\frac{\mathbf{v}\cdot\mathbf{A} - \omega\tau_f(\mathbf{v}\times\mathbf{A})_z}{1+\omega^2\tau_f^2}, \quad (A11)$$

with

$$\mathbf{A} = \tau(k)\left[e\mathbf{E} - (\varepsilon-\varepsilon_f)\frac{\nabla_{\mathbf{r}}T}{T}\right]. \quad (A12)$$

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