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Feedback effects in superconductors

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We calculate corrections to the BCS gap equation caused by the interaction of electrons with the collective phase and amplitude modes in the superconducting state. This feedback reduces the BCS gap parameter Δ and leaves the critical temperature T_c unchanged. The feedback effect is proportional to $(\Delta/\varepsilon_F)^2$, where ε_F is the Fermi energy. This is a negligible correction for type-I superconductors. However, in type-II superconductors the feedback effect is greatly enhanced due to smaller Fermi velocities v_F, and may be responsible for effects seen in recent experimental data on organic superconductors.

In the BCS theory of superconductivity,[1] there exist two distinct collective modes corresponding to the fluctuations of the phase and amplitude of the superconducting gap. The phase or Anderson-Bogoliubov mode[2] has been known for a very long time to be important in maintaining gauge invariance in the BCS theory.[3] In the presence of a Coulomb field, the phase mode (π) interacts strongly with the Coulomb field to become the plasmon mode. On the other hand, the amplitude mode (σ) is largely unaffected by Coulomb interactions, so that this mode remains intact, except for mixing effects to be discussed later. This decoupling feature of the amplitude mode means that it is not easily observable, and it was only recently that such a mode was discovered in the charge-density-wave compound $NbSe_2$ through the coupling to long-wavelength optical phonons.[4,5]

In this paper we wish to consider the effects of these collective modes back on the superconducting state. In the effective four-Fermi interaction BCS theory, an effective coupling between the collective modes and the quasiparticles induces self-energy corrections to the quasiparticle propagator. These corrections can either enhance the attraction between Cooper pairs and so contribute positively to the superconducting state, or they can act negatively on the superconducting state and reduce the gap parameter Δ. The magnitude of these corrections is proportional to $(\Delta/\varepsilon_F)^2$. These are negligible corrections for type-I superconductors where typically $\Delta/\varepsilon_F \sim 10^{-3}$. However, the feedback effects may become important if the typical Fermi energies are much smaller. This is the case in type-II superconductors where $v_F \simeq 10^6$ cm s^{-1}. Recent experiments in organic superconductors,[6] where typical Fermi energies are small, hint at the possibility that such a scenario may be at work. We will now present a calculation of these corrections and show how the superconducting state is affected.

Let us first recall some basic features of the field theoretic formulation of BCS superconductivity.[7,8] In the BCS ansatz[1] the Fröhlich effective electron-electron interaction[9] is replaced by a contact potential

$$V(\mathbf{x}-\mathbf{x}') = -V\delta^3(\mathbf{x}-\mathbf{x}') \tag{1}$$

where $V>0$. The effective Lagrangian is given by

$$\mathcal{L} = i\Psi^\dagger\dot{\Psi} - \Psi^\dagger\varepsilon\tau_3\Psi + \tfrac{1}{2}V\Psi^\dagger\tau_3\Psi\Psi^\dagger\tau_3\Psi \tag{2}$$

where ε is the electron kinetic energy measured from the Fermi energy and we have used the two-component notation[3]

$$\Psi = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow^\dagger \end{pmatrix} \tag{3}$$

to represent the Bogoliubov-Valatin fermionic quasiparticle modes. In the superconducting state the Lagrangian (2) is written as a sum of a free term $\mathcal{L}_0$ plus an interaction piece $\mathcal{L}_I$,

$$\begin{aligned} \mathcal{L}_0 &= i\Psi^\dagger\dot{\Psi} - \Psi^\dagger\epsilon\tau_3\Psi - \Delta\Psi^\dagger\tau_1\Psi , \\ \mathcal{L}_I &= \tfrac{1}{2}V\Psi^\dagger\tau_3\Psi\Psi^\dagger\tau_3\Psi + \Delta\Psi^\dagger\tau_1\Psi , \end{aligned} \tag{4}$$

where we have introduced the mass gap Δ. The bare quasiparticle Green's function corresponding to $\mathcal{L}_0$ is

$$G(k) = i\frac{k^0 1 + \epsilon\tau_3 + \Delta\tau_1}{(k^0)^2 - E^2 + i\epsilon} \tag{5}$$

where $E^2 = \varepsilon^2 + \Delta^2$ is the quasiparticle excitation energy. In $\mathcal{L}_I$ we have to ensure that there are no self-energy corrections proportional to τ_1 in order to maintain consistency with the ansatz that $\mathcal{L}_0$ describes the superconducting ground state with mass gap Δ. Using a Fierz identity for the Pauli matrices,[8] this leads directly to the BCS gap equation[1]

$$1 = \tfrac{1}{2}V\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{\varepsilon^2+\Delta^2}} \equiv J(\Delta) , \tag{6}$$

where the integral is cut off at the Debye energy, ω_D.

To exhibit the collective modes of the superconducting state, let us examine the quasiparticle-quasiparticle scattering amplitude generated by the infinite sum of bubble diagrams, as shown in Fig. 1. The scattering amplitude is a simple geometric series and is easily summed to give

$$\mathcal{A}_{\pi,\sigma}(k) = \frac{-(i/2)V}{1 - I_{\pi,\sigma}(k)} \tag{7}$$

where

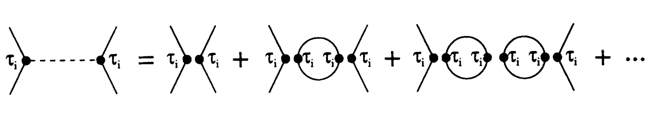

FIG. 1. The infinite sum of bubble diagrams for the quasiparticle-quasiparticle scattering amplitude. The τ_i represents either τ_1 (σ mode) or τ_2 (π mode).

$$I_\pi(k)=-i\tfrac{1}{2}V\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\tau_2 G(p+k/2)\tau_2 G(p-k/2)] \tag{8}$$

and

$$I_\sigma(k)=-i\tfrac{1}{2}V\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\tau_1 G(p+k/2)\tau_1 G(p-k/2)] \tag{9}$$

are the integrals for the two types of single bubble diagrams. The poles of the scattering amplitude (7) occur when $I_{\pi,\sigma}(k)=1$. At zero momentum transfer ($\mathbf{k}=0$), the integrals (8) and (9) can be written in the form

$$I_\pi(k_0^2=\omega^2)=1-\tfrac{1}{2}V\int\frac{d^3p}{(2\pi)^3}\frac{1}{E}\frac{\omega^2}{\omega^2-4E^2}\ , \tag{10}$$

$$I_\sigma(k_0^2=\omega^2)=1-\tfrac{1}{2}V\int\frac{d^3p}{(2\pi)^3}\frac{1}{E}\frac{\omega^2-4\Delta^2}{\omega^2-4E^2}\ , \tag{11}$$

where we have used the BCS gap equation (6). It is then obvious that $I_\pi(\omega^2=0)=I_\sigma(\omega^2=4\Delta^2)=1$ and the quasiparticle-quasiparticle scattering amplitude has poles at $\omega^2=0$ and $\omega^2=4\Delta^2$ which represent the phase (π) and amplitude (σ) modes, respectively.[10] For nonzero momentum transfers $\mathbf{k}$, we can Taylor expand the integrands in (8) and (9) to obtain the dispersion relations for the collective modes[5]

$$E_\pi^2(\mathbf{k})=\tfrac{1}{3}v_F^2\mathbf{k}^2\ , \tag{12}$$

$$E_\sigma^2(\mathbf{k})=4\Delta^2+\tfrac{1}{3}v_F^2\mathbf{k}\ , \tag{13}$$

where v_F is the Fermi velocity.

The effective quasiparticle-collective mode coupling is obtained from the residue at the pole of the scattering amplitude (7). Using (10), the quasiparticle-π mode coupling is

$$f_\pi^2=-\tfrac{1}{2}V\left[\left.\frac{dI_\pi}{d\omega^2}\right|_{\omega^2=0}\right]^{-1}\underset{\omega_D\gg\Delta}{=}\frac{4\Delta^2}{N(\varepsilon_F)} \tag{14}$$

where $N(\varepsilon_F)=mk_F/\pi^2$ is the density of states at the Fermi surface. If we attempt a similar procedure for the σ mode then it turns out that the corresponding integral in (14) is divergent, because the pole coincides with the two-particle threshold. This is the inadequacy of modeling the BCS theory by the σ model. We will simply circumvent this problem by assuming $f_\sigma=f_\pi$ as in the Ginzburg-Landau theory. This is a good approximation in the weak-coupling limit.

What are the effects of the collective modes on the quasiparticle self-energy? First, we assume that a Coulomb field is present, so that the Goldstone π mode turns into the massive plasmon mode.[7] In order to correctly take into account the plasmon mode, we need to start with the original Coulomb and phonon interactions instead of the effective four-Fermi interaction, V. This is beyond the scope of this paper, so we will ignore its effects on the quasiparticle self-energy. However we expect this contribution to be small because the plasmon mass is large compared to Δ and the Debye energy, ω_D.

For the massive σ mode there will be two contributions to the quasiparticle self-energy. The first contribution comes from the tadpole term shown in Fig. 2. It is given by

$$\Sigma_T=-i\frac{\Delta J(\Delta)}{1-I_\sigma(0)}\ . \tag{15}$$

However, this term is already implicitly included in the BCS gap equation and its inclusion would amount to a double counting of diagrams. To see this more clearly, consider the effect of adding a small bare term Δ_0 to the gap equation (6)

$$\Delta=\Delta_0+\Delta J(\Delta)\ . \tag{16}$$

If we now seek a perturbative solution of (16) of the form $\Delta+\delta\Delta$ then we obtain

$$\delta\Delta=\Delta_0+\frac{\partial[\Delta J(\Delta)]}{\partial\Delta}\delta\Delta=\frac{\Delta_0}{1-I_\sigma(0)}\ . \tag{17}$$

Thus comparing (15) and (17) we see that the tadpole term appears only as the response of Δ to a nonzero Δ_0.

The second contribution results from contracting the crossed tree diagram, leading to the "Weisskopf term" shown in Fig. 2. This contribution will act negatively on the superconducting state because contracting the crossed diagram involves a sign change. Hence the Weisskopf term will act to reduce the gap Δ. In order to calculate the Weisskopf term we reinterpret the quasiparticle-quasiparticle scattering amplitude, $\mathcal{A}_\sigma$ as arising from the exchange of the σ mode with propagator

$$G_\sigma(k)=i\frac{1}{(k^0)^2-E_\sigma^2(\mathbf{k})} \tag{18}$$

where we have approximated the continuum cut solely by the σ mode pole. This considerably simplifies the equations and the corrections arising from the continuum contribution in (7) do not affect the qualitative behavior. The Weisskopf self-energy term may now be written as

$$\Sigma_W(k)=if_\pi^2\int\frac{d^4p}{(2\pi)^4}\tau_1 G(p)G_\sigma(p-k)\tau_1 \tag{19}$$

and we will evaluate it at the Fermi surface: $k_0=\Delta$, $|\mathbf{k}|=k_F$. The term proportional to τ_1 which gives a contribution to the gap in the limit $\varepsilon_F\gg\omega_D\gg\Delta$ is

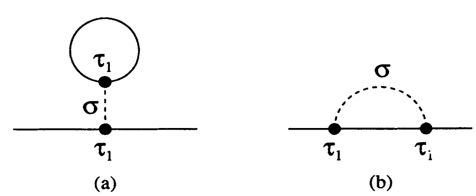

FIG. 2. The quasiparticle self-energy diagrams arising from the σ mode coupling, where (a) depicts the tadpole diagram and (b) shows the Weisskopf term.

$$\delta Z_\Delta=\frac{3}{8}\left[\frac{\Delta}{\varepsilon_F}\right]^2\int_0^{\omega_D}d\varepsilon\frac{1}{\sqrt{\varepsilon^2+\Delta^2}}\ln\left[\frac{1}{4}\left[\frac{\varepsilon}{\varepsilon_F}\right]^2+\frac{3}{8\varepsilon_F^2}\sqrt{(\varepsilon^2+\Delta^2)(\tfrac{1}{3}\varepsilon^2+4\Delta^2)}+\frac{3}{4}\left[\frac{\Delta}{\varepsilon_F}\right]^2\right] \tag{20}$$

where ω_D is the Debye frequency cutoff.

The Weisskopf term will also give corrections to the τ_3 and 1 terms in $\mathcal{L}_0$. The corrections to τ_3 will renormalize the chemical potential and the electron mass and give rise to an effective electron mass, m^*. The term proportional to the identity matrix 1 adds a contribution $k_0(\delta Z_\Psi)$ to the energy k_0. Defining $Z_\Psi=1-\delta Z_\Psi$, this corresponds to a wave-function renormalization $\Psi\to Z_\Psi^{-1/2}\Psi$ and modifies the mass gap term by Z_Ψ^{-1}. Evaluating the wave-function renormalization constant at the Fermi surface in the limit $\varepsilon_F\gg\omega_D\gg\Delta$ gives

$$\delta Z_\Psi=-\frac{3}{8}\left[\frac{\Delta}{\varepsilon_F}\right]^2\left[\left[3-\sqrt{3}\right]\ln\frac{2\omega_D}{\Delta}-0.844+\mathcal{O}\left[\frac{\omega_D}{\varepsilon_F}\right]\right]. \tag{21}$$

Thus the total self-energy contributions to the gap arising from the Weisskopf term will be $\Delta_W=\Delta(\delta Z_\Delta+\delta Z_\Psi)$ where we have kept terms to lowest order in the correction parameter $(\Delta/\varepsilon_F)^2$. The BCS gap equation with the Weisskopf corrections in the limit $\varepsilon_F\gg\omega_D\gg\Delta$ is

$$1=\tfrac{1}{2}VN(\varepsilon_F)\ln\frac{2\omega_D}{\Delta}-\frac{3}{8}\left[\frac{\Delta}{\varepsilon_F}\right]^2\left[\left[\ln\frac{2\omega_D}{\Delta}\right]^2+\ln\frac{2\omega_D}{\Delta}\left[2\ln\frac{\varepsilon_F}{\omega_D}+0.762\right]-2.389\right]$$
$$-\frac{3}{8}\left[\frac{\Delta}{\varepsilon_F}\right]^2\left[\left[3-\sqrt{3}\right]\ln\frac{2\omega_D}{\Delta}-0.844\right] \tag{22}$$

where we have evaluated the integral in (20) to $\mathcal{O}(\ln x/x^2)$ where $x=\omega_D/\Delta$. In normal type-I superconductors $(\Delta/\varepsilon_F)^2\sim10^{-6}$ because $v_F\simeq10^8\ \mathrm{cm\,s^{-1}}$. This is quite a small correction compared to $VN(\varepsilon_F)$ which is typically ~0.25. However, in type-II superconductors the Fermi velocity is smaller: $v_F\simeq10^6\,\mathrm{cm\,s^{-1}}$, and the gap parameter is larger, so that the correction $(\Delta/\varepsilon_F)^2\sim10^{-2}$. Note that this does not contradict the fact that we assumed $\varepsilon_F\gg\omega_D\gg\Delta$, because the corrections are always proportional to $(\Delta/\varepsilon_F)^2$. However, we need to obtain the coefficients of $(\Delta/\varepsilon_F)^2$ in the limit $\varepsilon_F\gtrsim\omega_D$. We present the full exact expressions for δZ_Δ and δZ_Ψ below.

All the above results are for $T=0$. The results at finite temperature are obtained by using the imaginary time formalism. The quasiparticle-π mode coupling constant (14) at finite T becomes

$$\frac{1}{f_\pi^2(T)}=\frac{N(\varepsilon_F)}{8\sqrt{\varepsilon_f}}\int_{-\omega_D}^{\omega_D}\frac{d\varepsilon}{E^3}\sqrt{\varepsilon+\varepsilon_F}\tanh\frac{\beta}{2}E \tag{23}$$

where $\beta=1/k_BT$. It is interesting to note that in the limit $\varepsilon_F\gg\omega_D\gg\Delta$ the coupling (23) is simply $f_\pi^2(T)=4\Delta^2(T)/N(\varepsilon_F)$ as $T\to0$. One immediate consequence of (23) is that at $T=T_c$ the integral is divergent and so the coupling constant vanishes, i.e., $f_\pi(T_c)=0$. Hence the determination of T_c remains unaffected by the amplitude mode correction.

Similarly we can obtain the finite-temperature expressions for δZ_Δ and δZ_Ψ. At the Fermi surface we have

$$\delta Z_\Delta(T)=f_\pi^2(T)\frac{N(\varepsilon_F)}{8\sqrt{\varepsilon_F}}\int_{-\omega_D}^{\omega_D}d\varepsilon\sqrt{\varepsilon+\varepsilon_F}\int_{-1}^{1}d(\cos\theta)\frac{1}{\Delta^2-(E+E_\sigma)^2}\frac{1}{\Delta^2-(E-E_\sigma)^2}$$
$$\times\left[\frac{1}{E}(\Delta^2+E^2-E_\sigma^2)\tanh\frac{\beta E}{2}+\frac{1}{E_\sigma}(\Delta^2-E^2+E_\sigma^2)\coth\frac{\beta E_\sigma}{2}\right], \tag{24}$$

$$\delta Z_\Psi(T)=f_\pi^2(T)\frac{N(\varepsilon_F)}{8\sqrt{\varepsilon_F}}\int_{-\omega_D}^{\omega_D}d\varepsilon\sqrt{\varepsilon+\varepsilon_F}\int_{-1}^{1}d(\cos\theta)\frac{1}{\Delta^2-(E+E_\sigma)^2}\frac{1}{\Delta^2-(E-E_\sigma)^2}$$
$$\times\frac{1}{E_\sigma}\left[2EE_\sigma\tanh\frac{\beta E}{2}+(\Delta^2-E^2-E_\sigma^2)\coth\frac{\beta E_\sigma}{2}\right], \tag{25}$$

where the coupling $f_\pi(T)$ is determined from (23) and

$$E^2=\varepsilon^2+\Delta^2 \; ,$$
$$E_\sigma^2=\tfrac{8}{3}\varepsilon_F^2\left[1+\frac{1}{2}\frac{\varepsilon}{\varepsilon_F}\right]-\tfrac{8}{3}\varepsilon_F^2\sqrt{1+(\varepsilon/\varepsilon_F)}\cos\theta+4\Delta^2 \; . \tag{26}$$

In the limit $\varepsilon_F \gg \omega_D \gg \Delta$ and $T=0$ the expressions (24) and (25) reduce to (20) and (21). Combining these corrections with the finite-temperature BCS gap equation,[7] we obtain the complete finite-temperature equation for $\Delta(T)$:

$$1=\frac{1}{4\sqrt{\varepsilon_F}}VN(\varepsilon_F)\int_{-\omega_D}^{\omega_D}\frac{d\varepsilon}{E}\sqrt{\varepsilon+\varepsilon_F}\tanh\frac{\beta}{2}E + \delta Z_\Delta(T)+\delta Z_\Psi(T) \; . \tag{27}$$

The solution of (27) is shown in Fig. 3 for typical values of the parameters in a type-II superconductor. The biggest deviation is at $T=0$ and decreases until $T=T_c$, where there is no change from the BCS result. Such a scenario may be occurring in the organic superconductor $(BEDT\text{-}TTF)_2I_3$ where a proposal,[6] to place the gap at 6 cm^{-1} may be consistent with the observation that the gap below T_c is reduced from the BCS value $[2\Delta(T=0)=20$ $cm^{-1}]$, while T_c remains unchanged.

For a thorough discussion of the realistic cases, however, one has to take into account the complexities of the electronic band structure and phonon spectra. One of the important effects is the mixing of the amplitude mode with the original Coulomb and phonon interactions.[5] This mixing is proportional to Δ/ε_F and occurs because of the intrinsic particle-hole asymmetry relative to the Fermi surface. It again results in a reduction of the pairing forces and is most significant in type-II or organic superconductors.

It should be noted that if one considers so-called "neutral" superconductors and includes the effect of the pure Goldstone τ_2 mode then one finds that the feedback is positive for the τ_1 term. This will almost cancel against the negative feedback of the amplitude mode (20). However, the contribution to the 1 term is the same sign as for the amplitude mode and will add to the amplitude correction to give $\Delta_W^{\tau_2}=2\Delta\delta Z_\Psi$.

The feedback effects of the collective bosonic modes on the superconducting state may also be relevant for the recent high-T_c superconductors where $\Delta/\varepsilon_F \sim 0.1$.[11] However, without a complete understanding of the mechanism involved in high-T_c materials at present, we can only but speculate on these effects.

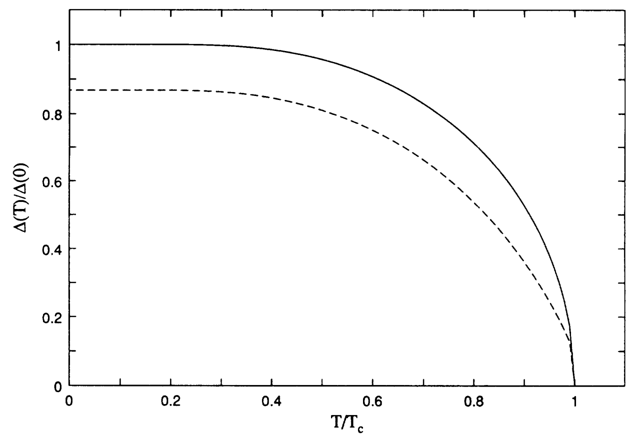

FIG. 3. Comparison of the solution of the gap equation with feedback effect (dashed line) with the normal BCS result (solid line) for typical type-II superconductor parameter values ($m^*/m_e=15$, $v_F/c=10^{-4}$, $\Lambda/\varepsilon_F=0.9$).

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