Magnetic self-field entry into a current-carrying type-II superconductor

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A problem of the irreversible entry of the Abrikosov vortex ring, induced by the self-field of a transport current, into a long type-II superconductor cylinder of arbitrary radius R is solved exactly in the London approximation. The magnetic field and current distribution for the toroidal vortex inside the cylinder are evaluated. The Gibbs free energy of the system is calculated when the transport current is applied. The critical current j_c of a spontaneous vortex penetration into the superconductor through a surface Bean-Livingston barrier is found to be independent of the radius of the cylinder and close to the depairing current. However, the dependence of the width of edge barrier on the transport current in a thin cylinder is found to differ qualitatively from that in a thick cylinder. As a result of this, in the first case there is no characteristic current but j_c, while in the second case a characteristic current $j_{c_1} \cong j_c/\kappa$ arises (κ the Ginzburg-Landau parameter) at which the barrier width drops down to values of the order of the magnetic-field-penetration depth λ, which allows for vortex entry on the surface defects of the size of λ. The latter result is discussed in reference to the high-critical-current observations on the microbridges of high-temperature superconductors.

I. INTRODUCTION

Resistivity onset in superconductors at currents small with respect to the Ginzburg-Landau depairing current j_{GL} is believed to be connected with the motion of magnetic flux in them.[1-3] In the absence of an external magnetic field, the resistivity is provided by processes of two kinds: (i) penetration into the superconductor of the vortices of the self-field of the transport current (in the case of type-II superconductors[1-4]) or of the normal domains carrying the magnetic flux (in the case of type-I superconductors[5]) and (ii) nucleation and the consequent expansion of vortices of vorticity opposite to that of the self-field of the current.[6-8]

Both mechanisms predicts a rather high value of the critical-current density $j_c \sim 10^9$ A/cm^2 for a perfect superconducting sample.

Some experiments on a high-quality microbridges of high-T_c superconductors have demonstrated a very high current-carrying capacity that seemed to increase when the transverse size of bridges decreased.[9,10] A current density as high as 1.3×10^9 A/cm^2 was achieved in Ref. 10. This magnitude as well as the dependence on size obtained in Ref. 10 is consistent with the Onsager-Feynman vortex-ring-creation mechanism[6] as was discussed in Ref. 10, which is in favor of dissipation process (ii).

The possible relation of mechanism (i) to the results of Refs. 9 and 10 is so far questionable. A correct account of the edge-barrier effect on the magnetic-flux penetration into the sample may lead to a critical current of the same order. In fact, the theory of flux entry into type-I superconductors advanced by Clem and co-workers[5] allows one to estimate the critical current for type-II superconductors too. The main results of Ref. 5 was that the critical current of a long strip is considerably enhanced above that calculated using Silsbee's rule.[11] The latter states that the onset of electrical resistance occurs when the self-field produced by the current first attains the critical value of H_c, the thermodynamic critical field, at the surface of the specimen. Another important result of Clem and co-workers[5] was in the increase of the value of the critical current with the growth of the transverse size of the sample that takes place due to the contribution to the energy from the out-of-specimen change in the field. This result is likely to be in direct contradiction with experiment.[10]

It should be taken into account, however, that an out-of-specimen change in the field vanishes provided the flux lines are closed inside the sample, as it may be for circular cross sections (see Ref. 5). Another essential point is that the calculations in Ref. 5 were carried out only for the case of a large specimen with characteristic lengths much greater than λ, the magnetic-field-penetration depth. In type-II superconductors, the edge-barrier width in a wide-field range is of the order of λ. Thus, for a sample cross-section size of the order of λ, the process of flux entry may differ from that in large samples. Therefore, to discern between the two above resistivity mechanisms, it is interesting to consider the self-field magnetic-vortex entry into a type-II superconductor sample of arbitrary transverse size.

In the present work vortex-ring penetration into a long cylindrical sample of isotropic type-II superconductors is considered. The structure of the vortex inside a cylinder is found exactly in the framework of the London approach for an arbitrary cylinder radius $R \gg \xi$. We have calculated the Gibbs free-energy barrier against irreversible vortex entry into the current-carrying superconductor, the barrier width dependence on the cylinder radius R, and the critical-current density on the sample surface at various R.

The paper is organized as follows. In Sec. II we introduce the London equation for a closed magnetic vortex in a superconducting cylinder and outline the solution to it. We give exact expressions for the magnetic field, current distribution, and magnetic flux flowing through the vortex. In this section we calculate also the free energy of the vortex ring in an exact form and approximations for the case of thin and thick cylinders. In Sec. III the Gibbs free energy is found for the case when the external transport current is applied to the cylinder. Making use of the exact expression for the Gibbs energy, we consider the problem of the irreversible entry of the self-field vortex ring into the superconductor against the edge Bean-Livingston barrier. In this section we evaluate the barrier width dependence on the transport current for different values of the cylinder radius and stress the qualitative difference in barrier behavior of thick and thin samples. We determine the critical current for the spontaneous entry of self-field vortices as one at which the edge barrier vanishes. For the case of thick wires, one more characteristic current is shown to arise that determines the vortex entry on the surface defects. Finally, in Sec. IV we discuss the obtained results in the context of high-T_c superconductors.

II. STRUCTURE OF THE MAGNETIC VORTEX RING IN SUPERCONDUCTING CYLINDERS

A closed Abrikosov vortex with a ringlike core region may be described by means of the London equation with a special right-hand side.[12] In view of the symmetry of the problem, the equation is convenient to perform in cylindrical coordinates (ρ,ϑ,z), where the z axis coincides with the axis of a cylindrical sample of radius R. Taking into account that in the toroidal vortex only the azimuthal component of the magnetic field $\mathbf{h}=(0,h(\rho,z),0)$ is present, one gets

$$\frac{\partial^2 h}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial h}{\partial\rho}-\left[\frac{1}{\rho^2}+\frac{1}{\lambda^2}\right]h+\frac{\partial^2 h}{\partial z^2}=-\frac{\Phi_0}{\lambda^2}\delta(\rho-r)\delta(z)\ , \tag{1}$$

where Φ_0 is the unit flux quantum and $r<R$ is the radius of the vortex loop lying in the $z=0$ plane. The solution of Eq. (1) for an infinite bulk superconductor was obtained in Ref. 12.

Looking for a solution of Eq. (1) in an infinite cylinder of radius R, one should require the field $\mathbf{h}$ to be continuous on the boundary of a cylinder $r=R$ and to vanish at $\rho\to\infty$ and $|z|\to\infty$.[3] Boundary conditions may be, though, simplified by taking into account the well-known fact that an ideal solenoid does not create a field in outer space. Being the superposition of ideal toroidal current solenoids, the closed magnetic vortex as a whole does not create a magnetic field out of a superconducting cylinder including its surface.[5,13] Then one may solve Eq. (1) with boundary conditions $h(\rho=R,z)=0$. It will be seen subsequently that the solution obtained is really continuous in all space.

To solve Eq. (1) with zero boundary conditions, it is convenient to make use of the finite Hankel transformation,[14] which presents a solution in the form of a Fourier-Bessel series:

$$h=\frac{\Phi_0 r}{\lambda^2 R}\sum_{k=1}^{\infty}\frac{J_1(\gamma_k\rho/R)J_1(\gamma_k r/R)}{\sqrt{\gamma_k^2+(R/\lambda)^2}J_2^2(\gamma_k)}\times\exp[-|z|\sqrt{\lambda^{-2}+(\gamma_k/R)^2}]\ ,\quad \rho\le R\ , \tag{2}$$

where $J_{1,2}$ are Bessel functions of order 1 and 2 and γ_k are positive solutions of the equation $J_1(x)=0$. It is easy to check that solution (2) vanishes continuously at $\rho=R$.

Let us note that solution (2) cannot be presented as a superposition of the free vortex field[12] and some of its images as is possible in the case of a flat boundary. It is connected to the fact that the three-dimensional (3D) Laplace operator is not invariant with respect to inversion in cylindrical coordinates and the image in the cylindrical mirror does not exist.

From $h(\rho,z)$, various other quantities, such as the current density $\mathbf{j}$, can be calculated directly. Indeed, $\mathbf{j}$ is given by the Maxwell equation $\mathbf{j}=c\,\mathrm{rot}\mathbf{h}/4\pi$ and has both radial and z components,

$$\mathbf{j}=(j_\rho(\rho,z),0,j_z(\rho,z))\ , \tag{3}$$

where

$$j_\rho(\rho,z)=\frac{c\Phi_0}{4\pi\lambda^2}\frac{r}{R^2}\sum_{k=1}^{\infty}\frac{J_1(\gamma_k\rho/R)J_1(\gamma_k r/R)}{J_2^2(\gamma_k)}\times\exp[-|z|\sqrt{\lambda^{-2}+(\gamma_k/R)^2}]\times\mathrm{sgn}(z) \tag{4}$$

and

$$j_z(\rho,z)=\frac{c\Phi_0}{4\pi\lambda^2}\frac{r}{R^2}\sum_{k=1}^{\infty}\frac{J_0(\gamma_k\rho/R)J_1(\gamma_k r/R)\gamma_k}{\sqrt{\gamma_k^2+(R/\lambda)^2}J_2^2(\gamma_k)}\times\exp[-|z|\sqrt{\lambda^{-2}+(\gamma_k/R)^2}]\ . \tag{5}$$

Expressions (2)–(5) suggest an exact and thorough description of the structure of the vortex ring embedded in the coaxial superconducting cylinder.

The free energy of the vortex may be found using the conventional definition[1–3]

$$F=\frac{1}{8\pi}\int[\mathbf{h}^2+\lambda^2(\mathrm{rot}\mathbf{h})^2]dV\ . \tag{6}$$

Although strictly speaking the field (2) diverges logarithmically at the axis of the vortex, $z=0$, $\rho\to r$, in the London approximation, as it takes place in the case of the usual linear vortex too,[3] the actual field saturates at $r=R$ within the scale of ξ.[1–3] Excluding the region of size ξ near the vortex axis from the integration domain, one can find the free energy of the vortex ring in a cylinder (for details, see Appendix A),

$$F=\frac{\Phi_0^2 r^2}{4\lambda^2 R}\sum\frac{J_1(\gamma_k r/R)J_1[\gamma_k(r-\xi)/R]}{\sqrt{\gamma_k^2+(R/\lambda)^2}J_2^2(\gamma_k)}\ . \tag{7}$$

The vortex ring carriers a magnetic flux that may be calculated by the integration of the field (2) over the half-plane (i.e., over z from $-\infty$ to $+\infty$ and over ρ from 0 to ∞) and equals

$$\Phi(r)=\Phi_0\frac{2rR}{\lambda^2}\sum_{k=1}^{\infty}\frac{J_1(\gamma_k r/R)[1-J_0(\gamma_k)]}{[\gamma_k^2+(R/\lambda)^2]J_2^2(\gamma_k)\gamma_k} . \tag{8}$$

Φ vanishes at $r=R$ when the vortex ring merges with its image and at $r=0$ when it contracts down to a point (Fig. 1).

Although expression (2) presents an exact solution of the problem, the summation of the series (2,4,5,7,8) is problematic. Let us find the free energy (7) in the limiting cases of thick ($R \gg \lambda$) and thin ($R<\lambda$) cylinders. The series in (7) converges rather slowly, which allows one to use asymptotic expressions of Bessel functions for summation. Then one can find for $R<\lambda$ (for details, see Appendix A),

$$F=2\varepsilon\pi r\left\{\ln\left[\frac{R}{\pi\xi}\right]-\pi\left[\frac{1}{2}-\frac{r}{R}\right]\cos\left[\frac{\pi r}{2R}\right]\right.$$
$$\left.+\sin\left[\frac{\pi r}{2R}\right]\ln\left[2\sin\left[\frac{\pi r}{R}\right]\right]+\frac{1}{4}\right\} \tag{9}$$

and, for $R \gg \lambda$,

$$F=2\varepsilon\pi r\left[\ln(\kappa)-\frac{\sin[2(R-r)/\lambda]}{2(R-r)/\lambda}\right.$$
$$\left.+\mathrm{Ci}[2(R-r)/\lambda]-C+\frac{1}{4}\right] , \tag{10}$$

where $\varepsilon=(\Phi_0/4\pi\lambda)^2$ is the electromagnetic energy of the linear vortex line per unit length, $\kappa=\lambda/\xi$ is the Ginzburg-Landau parameter, $\mathrm{Ci}(x)$ is the integral cosine,[14] and $C=0.577$ is Euler's constant.[14] The last term in the brackets in formulas (9) and (10) is not contained in (6) and presents an energy of vortex-core creation equal to $(H_c^2/8\pi)\pi\xi^2 2\pi r$.

The logarithmic divergency at $r \to R$, contained in (9) and (10), is of the same nature as the field divergency at the vortex axis, discussed above when deriving the formula (7). It is connected to the vortex interaction with its image close to the cylinder surface, which may be realized at $R-r \ll R$. The energy (7) vanishes at $r=R-\xi/2$, which gives a natural restriction on the validity of the London approximation.

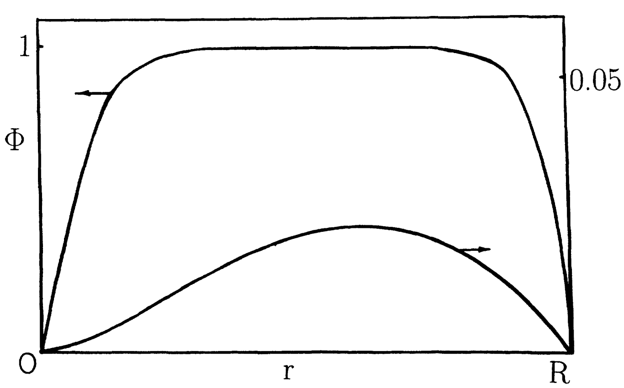

FIG. 1. Magnetic flux (in units of Φ_0) flowing through the vortex ring embedded inside the superconducting cylinder of radius R vs the ring radius r. The upper curve corresponds to the thick cylinder ($R=20\lambda$). The lower curve relating to the thin cylinder ($R=0.5\lambda$) is enlarged by a factor of 20 to make it comparable with the first one.

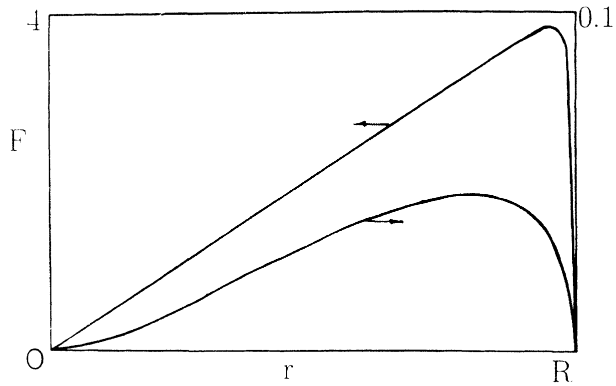

FIG. 2. Free energy of the magnetic vortex ring (in units of Φ_0^2/λ) contained inside the superconducting cylinder of radius R vs vortex ring radius r. Upper and lower curves correspond to the cases of thick ($R=20\lambda$) and thin ($R=0.5\lambda$) cylinders, respectively. To be comparable with the upper curve, the lower one is magnified 40×.

The free-energy dependence on the vortex-ring radius shown in Fig. 2 shows that in a thick wire ($R \gg \lambda$) this function is linear in almost the entire region $0<r<R$, unless the vortex is very close (within a distance λ) to the sample surface, and may be simply estimated as the usual energy of the linear vortex per unit length[3] multiplied by $2\pi r$. The free energy of the vortex ring in a thin cylinder ($R<\lambda$) is not linear in any region and does not contain λ as a characteristic length at all.

The magnetic vortex ring, provided it appears in a cylinder wire, that is to say, due to thermal fluctuations, is not stable at any position. It is attracted to the surface where it annihilates with its image or contracts down to a point on the cylinder axis. From this point of view, there is no lower critical field H_{c1} for the entry of the vortex ring since its entry and consequent contraction is always favorable as well as in the films,[4,5] but the surface Bean-Livingston barrier prevents vortex penetration into the sample. In the absence of a transport current, the width of the barrier is of the order of the cylinder radius as is seen from Fig. 2.

III. GIBBS FREE ENERGY OF A CYLINDER WITH A VORTEX RING

To study the problem of the energy barrier against magnetic vortex entry into current-carrying superconductors, one should evaluate the change in the Gibbs free energy of the system, ΔG, arising from the vortex penetration when an external transport current is applied. The quantity ΔG may be calculated, as in the spirit of Ref. 5, as

$$\Delta G=F-\Delta W_I , \tag{11}$$

where F is the self-energy contribution from inside the superconductor, evaluated in the preceding section, and ΔW_I is the work done by the source of the transport current. Let us note the absence of a self-energy contribution from the magnetic field outside the superconductor. In our linear problem, this field may be found by superimposing the field excited by the transport current and the one determined by the vortex presence. But the latter field equals zero in the case of a circular cross section of a cylinder, as was discussed in the preceding section (see also Ref. 5), and hence the outside field is fixed by the transport current value.

To find the work done by the source of the transport current as the vortex ring is introduced, ΔW_I, let us note that the final result must be independent of the position of the conductor returning to the source of the current I, which is supplied to the specimen. Let us compute ΔW_I by supposing that the return conductor has coordinates $(x,y)=(-\infty,0)$ as is schematically shown in Fig. 3.

When the vortex ring moves toward the cylinder axis, the flux Φ' flowing upward through a source circuit (shown in Fig. 3 by the dashed line) changes with time, producing a back emf of $E_i=(-1/c)(d\Phi'/dt)$. The source of the current I then will have to provide the emf of the same magnitude but opposite sign, $E'=-E_i$, in order to maintain the current I constant.[5]

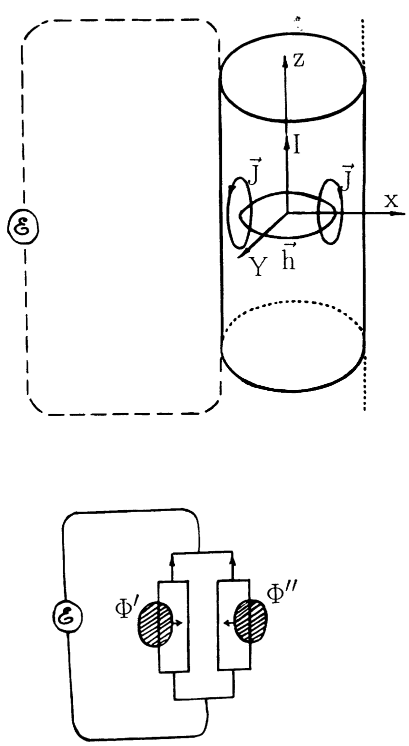

FIG. 3. Schematically shown is the magnetic vortex ring inside an infinitely long superconducting cylinder with the z axis. The ringlike vortex core lies in the plane $z=0$ and has cylindric coordinates $\rho=r$. The current source circuit is indicated by the dashed line. The conductor returning back to the current source has coordinates $(x,y)=(-\infty,0)$. The transport current I flows in the positive direction of the z axis. The inset sketches the equivalent circuit obtained by imaginary division of the cylinder into two parts along the plane $x=0$. Φ' measures the magnetic flux leaving the source circuit by entering the left edge of the cylinder $x=-R$. $\Phi''=-\Phi'$ is the flux entering simultaneously the right edge of the cylinder $x=R$ from the outside. I_1 and I_2 are the currents flowing in the branches of equivalent circuits.

This process in our geometry seems a bit complicated, for when the flux Φ' enters from the left edge of a cylinder moving to the right, from another edge of the cylinder the magnetic flux of the same magnitude but opposite sign, $\Phi''=-\Phi'$, moves to the left, as is shown in Fig. 3. One can circumvent the difficulty as follows.

For the case considered here, the current-flow lines as well as excited electric-field lines never cross the axis of the cylinder (z axis). That makes it possible to imagine the cylinder to be divided along the yz plane and present the source circuit in the equivalent form shown in the inset in Fig. 3. Then the work done by the source of the current reads

$$\Delta W_I=\int dt\,(E'I_1+E''I_2)\ , \tag{12}$$

where the two terms in the integrand stand for the work done in the two branches of equivalent circuits, I_1 and I_2 are the currents flowing in the left and right branches of the circuit, respectively, and integration is carried out over the time of vortex motion from the edge of cylinder to the position with some radius r. In the above expression $E'=c^{-1}d\Phi'/dt$ is the emf applied to the left conductor by the source of the current when the magnetic flux Φ' leaves the contour shown in Fig. 3 by the dashed line, crossing the left edge of the cylinder, and $E''=c^{-1}d\Phi''/dt$ is the emf arising in the right branch of the circuit when the flux Φ'' enters the right edge of the cylinder. Since $\Phi''=-\Phi'$ and moves in a direction opposite to that of Φ', it is obvious that $E'=E''$. Taking into account that $I_1+I_2=I$, one gets

$$\Delta W_I=\int E'I\,dt=\frac{1}{c}I\,\Delta\Phi(r)\ , \tag{13}$$

where the quantity $\Delta\Phi$ measures the magnetic flux leaving the source circuit when the vortex ring moves from the edge of the cylinder to a position with radius r. The latter formula reduces the problem of finding the Gibbs free energy to that of finding the magnetic flux leaving the source circuit in the course of vortex motion.

The value of $\Delta\Phi$ can be defined as the change in the total magnetic flux flowing through the source circuit. The latter is equal to the integral of the magnetic field over the left half-plane,

$$\Phi_t(r)=\int_{-\infty}^{-R}dx\int_{-\infty}^{+\infty}dz\,|h(\rho,z)|=\oint \mathbf{A}\cdot d\mathbf{l}\ , \tag{14}$$

where the integration on right-hand side is over the path shown by the dashed line in Fig. 3.

As we are interested in the change of Φ_t only, the infinite constant contribution to the integral (14) from the transport current field may be omitted. Then one can substitute into (14) the vector potential induced by the vortex ring presence only. The vector potential $\mathbf{A}$ is connected to the current $\mathbf{j}$ by a generalized London equation[3,13]

$$\mathbf{j}=\frac{c}{4\pi\lambda^2}(\mathbf{S}-\mathbf{A}) , \qquad (15)$$

where $\mathbf{S}$ is a source function defined in such a way that rot($\mathbf{S}$) is equal to the right-hand side (RHS) of Eq. (1). Taking into account that $\mathbf{A}$ vanishes as ρ or z goes to infinity, the path of integration in (14) reduces to the left edge of the cylinder $x=-R$. The $\mathbf{S}$ function in (15) does not contribute to the integral in (14) since correspondent δ function in the RHS of Eq. (1) is centered beyond the contour of integration. Finally, one finds

$$\Phi_t(r)=-\frac{4\pi\lambda^2}{c}\int_{-\infty}^{+\infty}dz\, j_z(\rho=R,z) , \qquad (16)$$

where j_z is defined by formula (5). Then the quantity of interest $\Delta\Phi(r)=\Phi_t(R)-\Phi_t(r)$. Upon substitution of Eq. (5) into Eq. (16) and taking the integral, one gets

$$\Delta\Phi(r)=\Phi_0\left[1-2\frac{r}{R}\sum_{k=1}^{\infty}\frac{\gamma_k J_1(\gamma_k r/R)}{J_2(\gamma_k)(\gamma_k^2+R^2/\lambda^2)}\right] . \qquad (17)$$

Let us stress out that the quantity $\Delta\Phi(r)$ has nothing to do with the flux Φ flowing through the vortex itself [Eq. (8)]. $\Delta\Phi(r)=0$ at $r=R$, but goes to Φ_0 at $r=0$, contrary to Φ. That means that the single vortex ring entry and the subsequent contraction down to a point provide an exit out of the source circuit of the single flux quantum.

The Gibbs free energy, resulting from (17), (13), and (11), of the long superconducting cylinder of radius $R=20\lambda$, containing the magnetic vortex loop of radius r, is plotted as function of r for various values of the external transport current I in Fig. 4. The dependence $G(r)$ is of the same character for any sample radius R: At $I=0$, $G=F$ and the edge barrier width is equal to R. Since $F(r=0)=0$, the lower critical field with respect to the entry of the vortex ring, H_{c1}, strictly speaking, is equal to zero as well, as was discussed for thin films.[4,5] At any finite I, it is favorable, in a thermodynamic sense, for the vortex ring to arise deeply in the sample and contract down to a point on the cylinder axis, but the large height of the energy barrier prevents the vortex penetration. When I exceeds some (presumably size-dependent) critical value $I_c(R)$, the barrier vanishes and spontaneous nucleation of the vortex rings occurs.

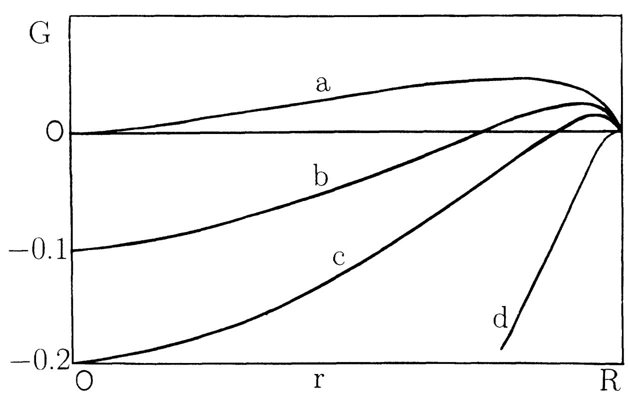

FIG. 4. Gibbs free energy of the cylinder containing the magnetic vortex ring vs the ring radius r when the transport current I is applied. The curves a, b, c, and d correspond to the cases $I=0$, $0.1I_c$, $0.2I_c$, and I_c, respectively, where I_c is the critical current of the spontaneous entry of the vortex rings from the edge, at which the surface barrier vanishes.

Let us estimate the critical current value in the cases of thin ($R<\lambda$) and thick ($R\gg\lambda$) samples. Taking appropriate limits, one finds (for details, see Appendix B), for $R<\lambda$,

$$\Delta\Phi(r)=\Phi_0[1-(r/R)^2] \qquad (18)$$

and, for $R\gg\lambda$,

$$\Delta\Phi(r)=\Phi_0\{1-\exp[-(R-r)/\lambda]\} . \qquad (19)$$

Formulas (18) and (19), upon substituting them into ΔG of expression (11), enable one to estimate the critical current for the spontaneous entry of vortex rings, using the criterion, $\partial G/\partial r|_{r\to R}=0$, corresponding to the vanishing of the edge barrier. Because of the logarithmic divergency of the free energy (6) at the edge of a cylinder, the derivative should be taken at $r=R-\xi/2$, where the free energy (7) vanishes. Then, taking into account that the current is distributed homogeneously over the cross section of a thin conductor, one finds

$$j_c=2\varepsilon c/\Phi_0\xi\cong j_{\mathrm{GL}} . \qquad (20)$$

At this current the self-field on the conductor surface achieves

$$H_s=2j_c\pi R^2/cR\simeq H_c R/\lambda . \qquad (21)$$

In the case of a thick sample, the current flows in the surface layer of thickness λ and one gets for the critical-current density on the surface exactly the same value (20). The self-field on the surface in this case equals

$$H_s=4j_c\pi\lambda/c\cong H_c . \qquad (22)$$

Thus the resistivity arises in thick type-II superconductors with the ideal cylindric surface at such a current at which the thermodynamic critical field is achieved at the surface, in accordance with Silsbee's rule and contrary to statements of Refs. 1–3, where this process was suggested to begin at $H=H_{c1}\cong H_c/\kappa$.

The critical-current density value with regard to the surface barrier effect turns out to be the same for both thick and thin samples and, hence, is size independent. Its value being as great as the critical current of depairing means that, in the ideal cylinder case, the flux-flow mechanism of resistivity may not take place at all, being exceeded by the direct depairing process.[15]

There are, however, some qualitative differences in the process of vortex entry into the thin ($R<\lambda$) and thick ($R\gg\lambda$) samples. Comparing formulas (18) and (19), one sees that the characteristic length of the change of $\Delta\Phi(r)$ is essentially λ in the case of thick cylinders, while in the case of thin samples no characteristic length is present but the radius R. The same is true for the free energy [see formulas (9) and (10)]; therefore, no characteristic current value arises in the current-dependent barrier width of the thin sample [Fig. 5(a)]. On the contrary, in

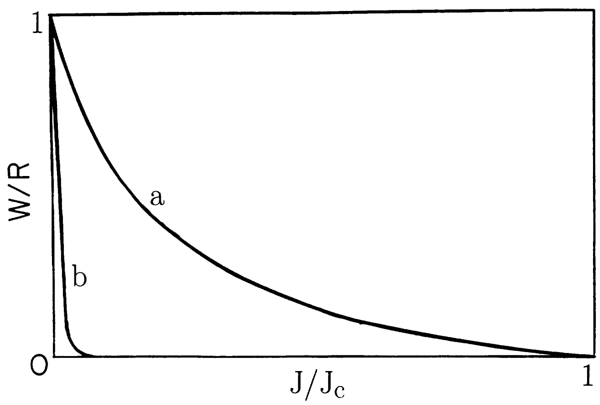

FIG. 5. Current dependence of the width w (in units of the cylinder radius R) of the edge barrier against the vortex ring entry into the superconductor. The upper curve plots the dependence for the case of a thin cylinder ($R = 0.5\lambda$), and the lower curve corresponds to the thick cylinder ($R = 20\lambda$).

the thick sample the value $j_{c1} \cong \varepsilon c / \Phi_0 \lambda \ll j_{GL}$ arises, at which the width of the surface barrier decreases down to a microscopic value of the order of λ [Fig. 5(b)]. At greater currents the barrier width decreases κ times slower to vanish at $j = j_c$. It is easy to check that at j_{c1} the field on the surface is of the order of H_{c1}.

Thus, as well as in films,[4,5] H_{c1} has a meaning of field at which self-field entry on the surface defects of size of λ may occur. We will discuss the implications of the last result for the microbridges of high-T_c superconductors in the next section.

IV. CONCLUSIONS AND SUGGESTIONS

In the two preceding sections, we limited ourselves to the case of isotropic superconductors of circular cross section. Turning back to the high-critical-current observations[9,10] discussed in Sec. I, one can say that the dependence of the edge Bean-Livingston barrier width on current may, at least in principle, explain the high current-carrying capacity of very thin conductors. Really, the reliable bulk-type superconductivity, observed in them, as well as the direct surface study proves that surface defects are, at any rate, much less than the characteristic transverse size of bridges, which are, in their turn, of the order of λ. In accordance with Fig. 5, that means that the critical current of vortex entry should be much more than j_{c1}, characteristics of thick wire. For the samples of highest quality, the current of the onset of the resistivity may be close to the j_c of (20), which is just of the order of 10^9 A/cm^2 for typical 1-2-3 values of λ and ξ, in agreement with experiment.[9,10]

Indeed, in reference to real experiments, the results of the present work may be applied only taking into account at least two factors. The first one is the geometry of bridges, which is, as a rule, far from the circular cross section and close rather to being filmlike. The second one is the high rate of anisotropy typical of high-T_c superconductors. In a curious way, however, these two differences of experiment from the above-studied isotropic symmetrical problem somewhat compensate each other.

Really, though equations for anisotropic superconductors cannot be exactly reduced to the isotropic case by a direct scaling transformation,[16–20] valid for strong magnetic fields, nevertheless the process of closed-loop nucleation as well as vortex-loop entry into a layered superconductor in the presence of a transport current should be very similar to that in the isotropic case. The loop consisted of two Abrikosov-like and two Josephson-like vortices, first discussed by Friedel,[21] and plays the role of the vortex ring in a layered medium. Supposing the in-plane and out-of-plane parts of such a loop to be of the same energy, one finds that the ratio of its in-plane length $l_\parallel$ to the out-of plane length $l_\perp$ should be of the order of $m_\perp / m_\parallel$,[21,22] where $m_\perp$ and $m_\parallel$ are the effective masses of electrons in the respective directions. Some of the samples in the experiments[9,10] had a ratio of sizes $L_\parallel / L_\perp$ that fit well to the $l_\parallel / l_\perp$, which makes the above closed vortex excitations appropriate for entry. In a more filmlike geometry with $L_\parallel / L_\perp \gg l_\parallel / l_\perp$, one should expect other excitations, such as vortex-antivortex pairs in films, to arise. It is interesting that the highest value of the critical current, $\sim 1.3 \times 10^9$ A/cm^2, was observed on the bridge with $L_\parallel / L_\perp \cong 4$, close to $m_\perp / m_\parallel \cong 5$ known for 1-2-3 compounds.

Another remarkable difference between our theory and the treatment of experiment results in Ref. 10 is that theory predicts no size dependence of critical-current density j_c, while in Ref. 10 the latter was stated to rise with a decrease of the transverse size. In our opinion this statement was connected to the manner in which the data were treated. In Ref. 10, j_c was calculated by direct division of the full critical current by the area of the cross section of the sample, regardless of the size of the latter. Correct at the sample size of the order of $\lambda \cong 1400 \times 10^{-8}$ cm, this method gave undoubtedly the understated values of the critical-current density, 5×10^7 A/cm^2, for the sample with a $4000 \times 5000 \times 10^{-16}$ A/cm^2 cross section and 3.6×10^8 A/cm^2 for the sample with a $500 \times 5000 \times 10^{-16}$ cm^2 cross section, since the current in these samples would be distributed mainly near the edges of the samples. Combining the above corrections, one should expect these samples to have actually greater critical-current densities, which may be of the same highest order of 10^9 A/cm^2.

Note added in proof. After the submission of this paper I was informed of the papers (Refs. 25 and 26) where similar results were obtained using another approach.

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APPENDIX A: FREE-ENERGY CALCULATION

To evaluate the free energy (6), let us use the following trick: Making use of the vector identity

$$(\mathrm{curl}\mathbf{h})^2=\mathbf{h}\cdot\mathrm{curl}[\mathrm{curl}(\mathbf{h})]+\mathrm{div}[\mathbf{h}\times\mathrm{curl}(\mathbf{h})]\ , \tag{A1}$$

one can present the energy (6) as a sum of two integrals,

$$F=\frac{1}{8\pi}\int dV\,\mathbf{h}\cdot\{\mathbf{h}+\lambda^2\mathrm{curl}[\mathrm{curl}(\mathbf{h})]\}+\oint\frac{d\sigma}{8\pi}[\mathbf{h}\times\mathrm{curl}\mathbf{h}]\ . \tag{A2}$$

The second term in (A2) is the integral over the surface of a cylinder and is equal to zero because of the boundary condition $h(r=R)=0$. The expression in brackets in the first term of (A2) coincides with the left-hand side of Eq. (1) divided by $(-\lambda^2)$. Then one obtains, from (A2),

$$F=\tfrac{1}{4}r\Phi_0\lim_{\rho\to r}h(\rho,z=0)\ . \tag{A3}$$

Taking into account that $h(\rho)$ saturates at the axis of the vortex within the scale of ξ, the coherence length, one may take $\rho\to r-\xi$ and get (7).

To find the approximations of the exact formula (7), let us note that the series in (7) converges very slow and, hence, the terms with large k make an essential contribution to the result. Then one can use asymptotic representations for the Bessel functions in (7),[23]

$$J_\nu(z)\simeq\sqrt{2/\pi z}\,\cos(z-\pi/4-\pi\nu/2)\ , \tag{A4}$$

and obtain

$$F=\frac{\Phi_0^2 r}{8\lambda^2}\sum_{k=1}^{\infty}\frac{\cos(\gamma_k\xi/R)-\sin(\gamma_k 2r/R)}{\sqrt{\gamma_k^2+(R/\lambda)^2}}\ , \tag{A5}$$

which with a good accuracy approximates the exact expression (7) at all R and r.

Let us note that the lowest zero of the Bessel function, $\gamma_1=3.81$.[23] That allows one to omit $(R/\lambda)^2$ in the denominator of (A5) at $R<\lambda$. Then, using approximately $\gamma_k\cong\pi(k+\frac{1}{4})$,[23] one finds, for the first term in (A5),

$$\pi\sum_{k=1}^{\infty}\frac{\cos(\gamma_k\xi/R)}{\gamma_k}\cong\ln(R/\pi\xi) \tag{A6}$$

and, for the second term,

$$\pi\sum_{k=1}^{\infty}\frac{\sin(\gamma_k 2r/R)}{\gamma_k}\cong\left[\frac{\pi}{2}-\frac{\pi r}{R}\right]\cos\left[\frac{\pi}{2}-\frac{r}{R}\right]-\sin\left[\frac{\pi r}{2R}\right]\ln\left[2\sin\left[\frac{\pi r}{R}\right]\right]\ . \tag{A7}$$

Then, upon substitution of (A6) and (A7) into (A5), one gets (9).

In the case of $R\gg\lambda$, let us divide the series in (A5) into two parts with $k\le R/\pi\lambda$ and $k>R/\pi\lambda$. In the first part, one may neglect γ_k in the denominator if (A5), and in the second one may omit R/λ. Then one obtains

$$\sum_{k=1}^{\infty}\frac{\cos(\gamma_k\xi/R)}{\sqrt{\gamma_k^2+(R/\lambda)^2}}\cong\frac{1}{\pi}[1-C+\ln(\kappa)]\ , \tag{A8}$$

where $C\simeq0.58$ is the Euler constant,[23] and

$$\pi\sum_{k=1}^{\infty}\frac{\sin(\gamma_k 2r/R)}{\sqrt{\gamma_k^2+(R/\lambda)^2}}\cong\frac{1}{\pi}\left\{\frac{\sin[2(R-r)]}{2(R-r)}-\sin\left[\frac{\pi(R-r)}{2R}\right]\left[\frac{\pi}{2}-\mathrm{Si}[2(R-r)]\right]-\cos\left[\frac{\pi(R-r)}{2R}\right]\mathrm{Ci}[2(R-r)]\right\}\ , \tag{A9}$$

where $\mathrm{Ci}(x)$ and $\mathrm{Si}(x)$ are the integral cosine and integral sine, respectively.[23] The validity of the approximation for the energy [Eq. (10)] in a wide range of vortex ring radii, $\lambda\ll R-r<R$ follows from (A9). The obtained expressions (9) and (10) exhibit proper behavior in the limiting cases $r\ll R$ and $r\to R$ and provide a precision of 10% in the intermediate range of r.

APPENDIX B: MAGNETIC-FLUX EVALUATION

In Eq. (17), the series to be evaluated is

$$\sum_{k=1}^{\infty}\frac{\gamma_k J_1(\gamma_k r/R)}{J_2(\gamma_k)(\gamma_k^2+R^2/\lambda^2)}\ . \tag{B1}$$

Making use of the exact formula[24]

$$\sum_{k=1}^{\infty}\frac{\gamma_k J_1(\gamma_k x)}{J_2(\gamma_k)(\gamma_k^2-a^2)}=\frac{J_1(ax)}{J_1(a)} \tag{B2}$$

for $a=iR/\lambda$, $x=r/R$ one can find

$$\Delta\Phi(r)=\Phi_0\left[1-\frac{rI_1(r/\lambda)}{RI_1(R/\lambda)}\right] \tag{B3}$$

that delivers limits (18) and (19) for the cases $R\ll\lambda$ and $R,r\gg\lambda$, respectively. In fact, the exact formula (B3) is well fitted by expression (18) at $R<\lambda$ for the $(R/\lambda)^2$ as compared with γ_k^2 values, the lowest of which $\gamma_1=3.81$.[23]

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