Critical current of a one-dimensional superconductor

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We solve the Bogoliubov-de Gennes equations for a clean, one-dimensional superconductor in the presence of superfluid flow. The maximum electrical current occurs when the superfluid velocity v_s equals the Landau depairing velocity Δ/p_F, where Δ is the pairing potential of the superconductor and p_F is the Fermi momentum. The resulting critical current is approximately $2/\pi$ times smaller than the value $e\Delta/\hbar$ recently predicted for a superconducting point contact. The "discretized" critical current of $(2/\pi)(e\Delta/\hbar)$ arises when all the conducting electrons are forced to drift at the Landau depairing velocity.

I. INTRODUCTION

All the phase boundaries of superconductors are set by "pair-breaking." If electronic paths in the superconductor are no longer effectively coupled to their time-reversed paths, the individual Cooper pairs reduce to ordinary electrons and the material becomes normal. For example, the critical temperature of an ordinary, weak-coupling, clean superconductor[1] can be viewed as a competition between the thermal dephasing length $L_T = v_F(\hbar/k_BT)$ and the pair size $\xi_0 = v_F(\hbar/2\Delta)$. When the temperature is large enough so that thermal dephasing occurs before the pair orbit can be completed, the material becomes normal at $k_BT_c \simeq \Delta$.

Pair breaking also places an upper theoretical limit on the critical current of a superconductor.[2–7] If the electron wave vector is k and the wave vector for the collective drift motion (superfluid motion) of the electrons is q, the ordinary k and $-k$ pairing must be generalized to pair the states $(k+q)$ and $(-k+q)$. This new pairing introduces an oscillation frequency $\omega_r = \hbar kq/m$ into the relative motion of the free electron two-particle wave function.[8] If this wave function changes sign over the pairing time $\hbar/\Delta$, so that $\omega_r(\hbar/\Delta) \simeq 1$, there is essentially destructive interference of the pair at a velocity

$$v_d = \hbar q_d/m = \Delta/p_F \ , \tag{1}$$

where $p_F = \hbar k_F = \sqrt{2m\mu}$ is the Fermi momentum and μ is the Fermi energy. Cooper pairs therefore "depair" if forced to drift too rapidly.

The Landau depairing velocity v_d bears on the recently predicted[9] "discretization" of the critical current in a superconducting point contact to $e\Delta/\hbar$. Reference 10 pointed out that a critical current of magnitude $e\Delta/\hbar$ follows naturally from Landau depairing in a one-dimensional superconductor. The critical current of a narrow superconductor is simply $I_c \simeq env_d$, where the quasiparticle density is $n \simeq 2k_F/\pi$ in one dimension. Using (1) we have

$$I_c \simeq e\left(\frac{2k_F}{\pi}\right)\left(\frac{\Delta}{\hbar k_F}\right) = \left(\frac{2}{\pi}\right)\left(\frac{e\Delta}{\hbar}\right) . \tag{2}$$

The factor of $2/\pi$ in this heuristic argument was dismissed in Ref. 10 both as inconsequential and likely to move closer to unity in a more detailed calculation.[6] However, we show here that the correct numerical factor is indeed $2/\pi$. The actual numerical prefactor in Eq. (2) can probably be distinguished in future experiments on narrow superconductors and point contacts.[11]

Our calculation also impacts the study of mesoscopic superconductor-normal-superconductor (SNS) junctions. In a short SNS junction, where the length L of the normal segment obeys $L \ll \xi_0$, the critical current is supposedly[9] $ev_F/2\xi_0 = e\Delta/\hbar$. However, if the length of the normal segment is not negligible, the critical current is presumably suppressed to $ev_F/(L+2\xi_0)$; for example, see Ref. 10. Inserting a normal metal region into a superconductor enhances the depairing of ordinary and time-reversed electronic paths, suppressing the critical current. Therefore, a short SNS junction cannot permit a 50% larger critical current than a uniform superconductor. Transport calculations applying the Bogoliubov–de Gennes (BdG) or Gorkov equations to SNS junctions[9,10] or NS interfaces[12–14] may need to be modified in some way to obtain physically reasonable results.

II. ELECTRICAL CURRENT FLOW

The BdG equations are[3]

$$\begin{pmatrix} H(x)-\mu & \Delta(x) \\ \Delta^*(x) & -[H^*(x)-\mu] \end{pmatrix}\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = E\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} , \tag{3}$$

where the one-electron Hamiltonian $H(x)$ is

$$H(x) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) , \tag{4}$$

with $V(x) = 0$. Following Ref. 3, we take the pairing potential $\Delta(x)$ to be

$$\Delta(x) = \Delta e^{2iqx}e^{i\phi} , \tag{5}$$

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where Δ is a real number. The traveling wave in the pairing potential $\Delta(x)$ imposes a superfluid drift velocity $v_s = \hbar q/m$ on the quasiparticles.

Solutions of Eq. (3) have the form[3]

$$\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \begin{pmatrix} Ae^{iqx}e^{i\phi/2} \\ Be^{-iqx}e^{-i\phi/2} \end{pmatrix} e^{ikx} , \tag{6}$$

where the A and B depend on k and q. The resulting energy level spectrum is[3]

$$E^{\pm}_{k,q} = (\hbar k)(\hbar q/m) \pm \sqrt{E_A^2 + \Delta^2} , \tag{7}$$

where the "average" or "center of mass"[8] energy E_A is

$$E_A(k,q) = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2 q^2}{2m} - \mu . \tag{8}$$

The spectrum from Eq. (7) is plotted in Fig. 1. (The pairing potential is taken to be $\Delta = 10$ meV, the Fermi energy is $\mu = E_F = 100$ meV, and m is the free electron mass.) The main effect of superfluid flow is to shift the quasiparticle levels by an amount $(\hbar k)(\hbar q/m)$. Figure 1 also shows that the depairing condition occurs when the quasiparticle energy gap is reduced to zero.

The quasiparticle current density J_P and electrical current density J_Q are shown in the Appendix to be

$$J_P(x) = \sum_k f(k)\{J_{u_k}(x) - J_{v_k}(x)\} \tag{9}$$

and

$$J_Q(x) = e\sum_k \{f(k)J_{u_k}(x) - [1-f(k)]J_{v_k}(x)\} . \tag{10}$$

The sum in Eq. (10) can run over either the electronlike branch, the holelike branch, the upper + branch, or the lower − branch of the dispersion law. The sum in Eq. (9) runs over the electronlike branch. The $J_{u_k}(x)$ and $J_{v_k}(x)$ in Eqs. (9) and (10) are the Schrödinger currents carried by the waves $u_k(x)$ and $v_k(x)$, namely,

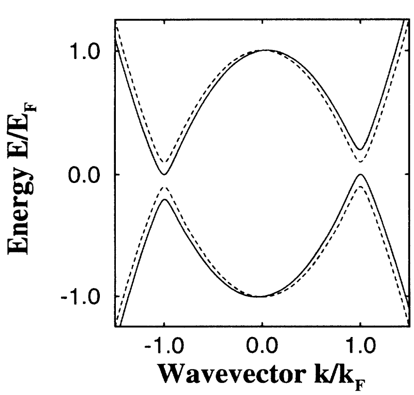

FIG. 1. Quasiparticle energy level spectrum for a clean superconductor subject to a superfluid flow of velocity $v_s = v_d = \Delta/p_F$ (solid line) and $v_s = 0$ (dashed line). The quasiparticle energy gap is reduced to zero at the depairing velocity $v_s = v_d$.

$$J_{u_k}(x) = \frac{\hbar}{m}\mathrm{Im}\,\{u_k^*(x)[\nabla u_k(x)]\} = \frac{\hbar(k+q)}{m}|A|^2 \tag{11}$$

and

$$J_{v_k}(x) = \frac{\hbar}{m}\mathrm{Im}\,\{v_k^*(x)[\nabla v_k(x)]\} = \frac{\hbar(k-q)}{m}|B|^2 . \tag{12}$$

The equilibrium Fermi occupation factor for the state k is

$$f(k) = \frac{1}{1+e^{(E_{k,q}/k_BT)}} . \tag{13}$$

The total particle current I_P and electrical current I_Q carried by the states (6) we therefore find as

$$I_P = 2\left(\frac{\hbar q}{m}\right)\int\frac{dk}{2\pi}f(k) + 2\int\frac{dk}{2\pi}f(k)\left(\frac{\hbar k}{m}\right)(|A|^2 - |B|^2) \tag{14}$$

and

$$I_Q = 2e\left(\frac{\hbar q}{m}\right)\int\frac{dk}{2\pi}\{f(k)|A|^2 + [1-f(k)]|B|^2\} + 2e\int\frac{dk}{2\pi}f(k)\left(\frac{\hbar k}{m}\right) . \tag{15}$$

For the electronlike branch the factors $|A|^2$ and $|B|^2$ are

$$|A_e|^2 = \frac{\sqrt{1+(\Delta/E_A)^2}+1}{2\sqrt{1+(\Delta/E_A)^2}} \tag{16}$$

and

$$|B_e|^2 = \frac{\sqrt{1+(\Delta/E_A)^2}-1}{2\sqrt{1+(\Delta/E_A)^2}} . \tag{17}$$

The factor $|A_e|^2 - |B_e|^2 = 1/\sqrt{1+(\Delta/E_A)^2}$ is therefore zero at the Fermi wave vector, and rises rapidly to nearly unity away from the Fermi wave vector. Note $|A|^2 + |B|^2 = 1$.

We graph the particle current I_P and the electrical current I_Q versus superfluid velocity $v_s = \hbar q/m$ at zero temperature in Fig. 2. The particle current I_P increases linearly with the superfluid velocity when $v_s \le v_d$, saturating at $(2/\pi)(\Delta/\hbar)$ when $v_s \ge v_d$. The electrical current I_Q increases nearly linearly with superfluid velocity until $v_s = v_d$, reaches a maximum value slightly smaller than $(2/\pi)(e\Delta/\hbar)$ when $v_s = v_d$, and decreases abruptly when $v_s \ge v_d$.

We can understand the behavior of I_P and I_Q in Fig. 2 by examining Eqs. (14) – (17). When $v_s \le v_d$, the electron distribution is symmetric such that $f(k) = f(-k)$. In that case, only the first term in Eqs. (14) – (15) contribute to I_P and I_Q. Thus, I_P is simply the superfluid velocity $\hbar q/m$ times the quasiparticle density $P \simeq 2k_F/\pi$, while I_Q is the superfluid velocity times the electrical charge density Q. (P and Q are defined in the Appendix). Figure 2 asserts that $Q \simeq eP$. The heuristic argument in the introduction leading to Eq. (2) therefore applies perfectly to the particle current flow,

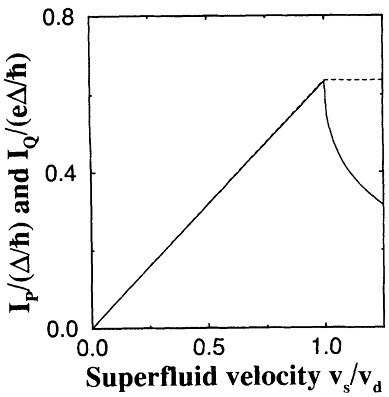

FIG. 2. Electrical current I_Q (solid line) and particle current I_P (dashed line) versus superfluid velocity v_s. The particle current saturates at $(2/\pi)(\Delta/\hbar)$ above the depairing velocity $v_s \geq v_d$, while the electrical reaches a maximum slightly less than $(2/\pi)(e\Delta/\hbar)$ at $v_s = v_d$.

and also seems to apply quite well to the electrical current flow.

For velocities greater than the depairing velocity, some k states near the Fermi wave vector $k = \tilde{k}_F$ are forced above the Fermi level μ and become unoccupied, while new occupied states are added below μ near $k = -\tilde{k}_F$. The last term in Eqs. (14) and (15) then becomes important, as $f(k) \neq f(-k)$. The particle current I_P does not suffer from the occupation of reverse moving electron states near $k = -\tilde{k}_F$, since their contribution to I_P is suppressed by the factor $|A|^2 - |B|^2 \simeq 0$ in the second term of Eq. (14). The first term in Eq. (14) still supports the particle current $I_P = (2/\pi)(\Delta/\hbar)$. In contrast, the electrical current I_Q is drastically reduced when additional states near $k = -\tilde{k}_F$ become occupied, as the second term in Eq. (15) produces a large and negative contribution to I_Q.

III. SELF-CONSISTENT PAIRING POTENTIAL

If the pairing potential Δ remains finite above the depairing velocity, Fig. 2 indicates that a supercurrent can still flow for $v_s > v_d$. To see if such a "gapless" superconductor is possible, we examine the self-consistency relation[3] for the pairing potential $\Delta(x)$, namely,

$$\Delta(x) = |g| \sum_k [1 - 2f(E^+_{k,q})] v^*_k(x) u_k(x) \, . \tag{18}$$

In Eq. (18) $|g|$ is the pairing interaction strength,[1] $E^+_{k,q}$ is the quasiparticle energy from Eq. (7), and the summation over wave numbers k includes only energies on the upper branch of the dispersion curve in Fig. 1. Using the states (6) we have

$$\Delta(x) = |g| \sum_k [1 - 2f(E^+_{k,q})] B^* A e^{2iqx} e^{i\phi} \, . \tag{19}$$

For the upper branch of the dispersion curve we find

$$2B^* A = \frac{\Delta}{\sqrt{E_A^2 + \Delta^2}} \, , \tag{20}$$

so that the self-consistency condition (18) for $\Delta \equiv \Delta(q)$, applying Eq. (5), is

$$1 = |g| \int \frac{dk}{2\pi} \frac{1}{\sqrt{E_A^2(k,q) + \Delta^2(q)}} [1 - 2f(E^+_{k,q})] \, . \tag{21}$$

The pair potential $\Delta(q)$ versus superfluid velocity $v_s/v_d = q/q_d$ is shown in Fig. 3. For $v_s < v_d$, and at zero temperature, Δ is basically unaffected by the superfluid flow. Despite their energy shift, all regions of the energy band continue to support the pairing potential with essentially the same weight as in the absence of a superfluid flow. At the depairing velocity, the states near $k \simeq -\tilde{k}_F$ oppose the contribution to the integral in (21) from the rest of the band. The large density of quasiparticle states near $k \simeq -\tilde{k}_F$ makes their contribution outweigh that from the rest of the energy band, so that at $T = 0$ Eq. (21) has no solution for a superfluid flow faster than the depairing velocity. The decrease in Δ at a finite temperature ($T = 0$, 25, 50, and 75 K) is also shown in Fig. 3.

Rogers [5,6] uses Eq. (21) to show that the superfluid velocity can slightly exceed the depairing velocity in a bulk superconductor. Rogers therefore finds gapless superconductivity is possible for a three-dimensional superconductor. Gapless superconductivity does not occur in a two-dimensional layer.[15]

In Fig. 4 we plot the currents I_P and I_Q versus superfluid velocity $\hbar q/m$ at a finite temperature, with the pairing potential $\Delta(q,T)$ determined self-consistently from Eq. (21). The temperature dependence of the electrical current I_Q versus phase gradient $\xi_0 \nabla\phi(x)$, where the position-dependent phase is $\phi(x) = \phi + 2qx$, is similar to the temperature dependence of the Josephson current versus the phase difference between the two superconductors in an SNS junction. [9,10] The particle current flow J_P

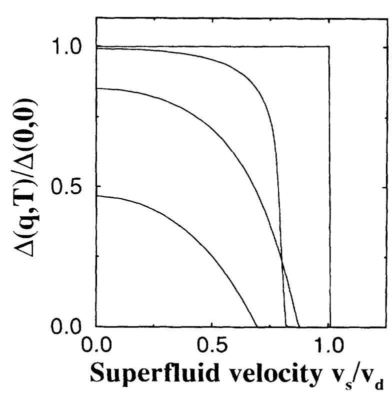

FIG. 3. Pairing potential $\Delta(q,T)$ versus superfluid velocity at temperatures $T = 0$, 25, 50, and 75 K. $\Delta(q,0)$ remains constant up to the Landau depairing velocity, then drops abruptly to zero. This "depairing" transition in $\Delta(q,0)$ sets the critical current phase boundary.

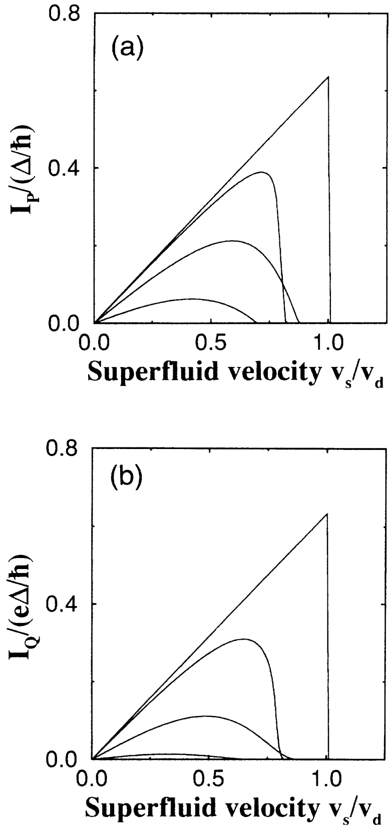

FIG. 4. (a) Particle current $I_P(q,T)$ and (b) electrical current $I_Q(q,T)$ versus superfluid velocity $\hbar q/m$, calculated using the self-consistent pairing potential $\Delta(q,T)$ from Fig. 3. The critical currents are still $I_Q = (2/\pi)(e\Delta/\hbar)$ and $I_P = (2/\pi)(\Delta/\hbar)$ for zero temperature, but are degraded at a finite temperature.

again remains larger than the electrical current flow J_Q for all temperatures.

IV. HELMHOLTZ FREE ENERGY

For the superconducting state to persist, the Helmholtz free energy $F = U - TS$ under a superfluid flow must be less than that of the normal state. If not, the free energy constraint will determine the phase boundary, rather than Eq. (21). The expectation value of the internal energy U we obtain directly from the second quantized Hamiltonian $\mathcal{H}_{\rm eff}$ of de Gennes.[3] By computing $U = \langle \mathcal{H}_{\rm eff} \rangle$, we find

$$U = \sum_k f_k E_k \int |u_k(x)|^2 dx - \sum_k (1-f_k) E_k \int |v_k(x)|^2 dx \, . \tag{22}$$

Equation (22) is evaluated at a fixed superfluid flow velocity q. The energies E_k are given in Eq. (7). S is the usual entropy of independent Fermi particles,

$$-S/k_B = \sum_k [f_k \ln f_k + (1-f_k)\ln(1-f_k)] \, . \tag{23}$$

We take the sum $\sum_k$ in Eqs. (22) and (23) to run over the electronlike states. Minimizing the free energy F with respect to the occupation factor f_k gives Eq. (13), proving that the standard equilibrium Fermi occupation factor with the energies from Eq. (7) is the correct occupation factor for the quasiparticle states.

The Helmholtz free energy of both normal (F_n) and superconducting (F_s) states is shown in Fig. 5. The free energy is normalized to $F_0 \equiv |F_n^0|$, where $F_n^0 = -2n\mu/3$ is the free energy of the normal state electron gas at zero temperature. At low temperature, F_n decreases as the temperature rises due to the increase in entropy S. The free energy of the normal state is also independent of the superfluid flow velocity q, since $\Delta(q) = 0$ in the normal state. Thus, the free energy of a drifting superfluid is being compared to that of a stationary normal electron gas.

The drifting superfluid still maintains a lower free energy than the stationary normal state for all flow velocities where the pairing potential $\Delta(q)$ exists. Our computation is therefore internally consistent. Figure 5 also shows that the free energy F_s has a discontinuous jump at the phase boundary for $T = 0$, indicating a first order phase transition. F_s smoothly approaches F_n as the superfluid velocity increases for any finite temperature T, so that the phase transition is second order at any finite temperature.

V. CONCLUSIONS

We have solved the Bogoliubov–de Gennes equations self-consistently for a one-dimensional superconductor in the presence of superfluid flow. Using the resulting self-consistent order parameter, we have computed the electrical current, quasiparticle current, and Helmholtz free energy subject to the superfluid flow. The calculation confirms that coupled electrons and time-reversed electrons in a superconductor "depair" at a critical velocity $v_d = \Delta/p_F$.

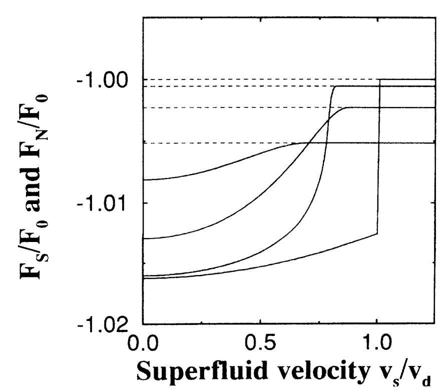

FIG. 5. Helmholtz free energy $F_n(T)$ of the normal state (dashed line) and $F_s(q,T)$ for the superconducting state (solid line), calculated using the self-consistent pairing potential $\Delta(q,T)$ from Fig. 3. The phase transition is first order at $T = 0$, and second order at any finite temperature.

The idea of a superfluid depairing velocity, originated by Landau in the 1940s, qualitatively explains the "discretization" of the critical current carried by a one-dimensional superconductor to $(2/\pi)(e\Delta/\hbar)$. The numerical factor of $(2/\pi)$ is easily understood by noting that all the quasiparticles are drifting at the Landau depairing velocity on the critical current phase boundary.

ACKNOWLEDGMENTS

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APPENDIX: CONSERVATION LAWS

The BdG Eqs. (3) imply the conservation laws [12–14]

$$\frac{\partial}{\partial t}|u_n|^2 + \nabla \cdot J_{u_n} = S_n/2 \tag{A1}$$

and

$$\frac{\partial}{\partial t}|v_n|^2 - \nabla \cdot J_{v_n} = -S_n/2 , \tag{A2}$$

where the "source term" S_n is

$$S_n(x) = \frac{4}{\hbar}\mathrm{Im}[u_n^*(x)\Delta(x)v_n(x)] . \tag{A3}$$

We wish to use Eqs. (A1) and (A2) to construct one conservation law for the quasiparticle current J_P of the form

$$\frac{\partial}{\partial t}P + \nabla \cdot J_P = 0 \tag{A4}$$

and a second conservation law for the electrical current J_Q with

$$\frac{\partial}{\partial t}Q + \nabla \cdot J_Q = 0 . \tag{A5}$$

We obtain Eq. (A4) by multiplying both Eq. (A1) and Eq. (A2) by the Fermi factor f and adding, yielding a quasiparticle density P of

$$P(x) = \sum_n f_n\{|u_n(x)|^2 + |v_n(x)|^2\} \tag{A6}$$

and a quasiparticle current J_P, where

$$J_P(x) = \sum_n f_n\{J_{u_n}(x) - J_{v_n}(x)\} . \tag{A7}$$

To obtain Eq. (A5), we multiply Eq. (A1) by f, Eq. (A2) by $(1-f)$, and add to find

$$\frac{\partial}{\partial t}Q + \nabla \cdot J_Q = -\frac{e}{2}\sum_n S_n(1-2f_n) . \tag{A8}$$

In Eq. (A8) the electrical charge density Q is

$$Q(x) = e\sum_n\{f_n|u_n(x)|^2 + (1-f_n)|v_n(x)|^2\} \tag{A9}$$

and the electrical current J_Q is identified as

$$J_Q(x) = e\sum_n\{f_n J_{u_n}(x) - [1-f_n]J_{v_n}(x)\} . \tag{A10}$$

Note that Eq. (A8) has a new source term on the right-hand side. However, if the pairing potential $\Delta(x)$ obeys the self-consistency condition Eq. (18), namely,

$$\Delta(x) = |g|\sum_n u_n(x)v_n^*(x)(1-2f_n) , \tag{A11}$$

we can write

$$\begin{aligned}\frac{1}{2}\sum_n S_n(1-2f_n) &= \frac{2}{\hbar}\sum_n \mathrm{Im}[\Delta(x)u_n^*(x)v_n(x)(1-2f_n)] \\ &= \frac{2}{|g|\hbar}\mathrm{Im}[\Delta(x)\Delta^*(x)] = 0 .\end{aligned} \tag{A12}$$

Therefore, if the self-consistency condition Eq. (18) is satisfied, there is a conserved electrical current given by Eq. (A10). We have also obtained Eqs. (A8)–(A10) by constructing the conservation laws from the second quantized Hamiltonian of de Gennes.[3] However, we have not been able to derive Eqs. (A6) and (A7) by this method.

A possible alternate definition of the electrical current is found by multiplying both Eq. (A1) and Eq. (A2) by the Fermi factor f and subtracting, yielding the conservation law

$$\frac{\partial}{\partial t}Q' + \nabla \cdot J_Q' = S' . \tag{A13}$$

Here the possible electrical charge is

$$Q'(x) = e\sum_n f_n\{|u_n(x)|^2 - |v_n(x)|^2\} \tag{A14}$$

and the electrical current is

$$J_Q'(x) = e\sum_n f_n\{J_{u_n}(x) + J_{v_n}(x)\} , \tag{A15}$$

with a source term

$$S'(x) = e\sum_n f_n S_n . \tag{A16}$$

Equations (A14) – (A16) are summed over the electron-like branch.

Although the current $J_Q'(x)$ is appealing from the viewpoint of the conservation laws discussed in Refs. 12–14, the source term $S'(x)$ is not obviously zero unless $S_n = 0$ for all n. Therefore, $J_Q'(x)$ might not be a conserved electrical current in general. In this paper, however, we indeed have $S_n = 0$ for all n, making Eq. (A15) a possible candidate for the electrical current. However, Eq. (A15) is very sensitive to which branch of the dispersion law is chosen to carry out the $\sum_k$. [For example, $J'_Q(x) = 0$ if the + branch is chosen at zero temperature.] Further, the current $J_Q(x)$ from Eq. (10) seems to be independent of the branch of the dispersion law chosen to carry out the summation over wave numbers, making it an attractive choice for the electrical current. For the actual numerical computations in Figs. 2 and 4 there is no qualitative difference in the dependence of the currents $J_Q(x)$ and $J'_Q(x)$ on the superfluid flow; however, $J_Q(x)$ is slightly larger than $J'_Q(x)$.

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$$\begin{aligned}\psi(x_1,t_1;x_2,t_2) = {} & e^{i(k+q)x_1}e^{-i(\hbar^2/2m)(k+q)^2(t_1/\hbar)}e^{i(-k+q)x_2} \\ & \times e^{-i(\hbar^2/2m)(-k+q)^2(t_2/\hbar)}e^{i(k+q)x_2} \\ & \times e^{-i(\hbar^2/2m)(k+q)^2(t_2/\hbar)}e^{i(-k+q)x_1} \\ & \times e^{-i(\hbar^2/2m)(-k+q)^2(t_1/\hbar)} \; . \end{aligned} \tag{a}$$

Equivalently, in center of mass coordinates ($2R = x_1 + x_2, 2T = t_1 + t_2$) and relative coordinates ($r = x_1 - x_2, t = t_1 - t_2$), this two-particle wave function is

$$\begin{aligned}\psi(R,T;r,t) = {} & 2ie^{iqR}e^{-i(\hbar^2/2m)(k^2+q^2)(T/\hbar)} \\ & \times \sin[k(r - \{\hbar q/m\}t)] \; . \end{aligned} \tag{b}$$

The plane waves in Eq. (A17) describe the center of mass drift motion, while the sine wave describes the relative motion of the pair. The relative motion of the pair, describing its internal structure, is important to the depairing argument. If the pair drifts too rapidly, so that q is large, the sine wave (at a fixed position r) will change sign over the pairing time scale $t = \hbar/\Delta$. Because of this destructive interference, we then cannot really say a pair exists. The approximate condition for destruction of the pair then becomes

$$1 \simeq k(\hbar q/m)(\hbar/\Delta) \; , \tag{c}$$

the same as Eq. (1) when $k = k_F$.

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