Correlations in vector-spin-glass models in a transverse field

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Several publications dealing with the infinite-range quantum XY and Heisenberg spin glasses in an external magnetic field K in the z direction share a common supposition: the correlation between the x and y components of the spins is zero. Using the Matsubara imaginary-time formalism we show that this assumption holds only for those models where $K = 0$. With the help of the Trotter-Suzuki formulation we investigate the phase diagram of the XY model for spin $S = 1/2$, and find that it is different from those previously reported because of the nonzero xy correlation. It is also argued that for a general spin S in the XY model at zero temperature the spin-glass phase persists up to $K_c = 2S$.

I. INTRODUCTION

In quantum spin-glass models with infinite-range interactions[1] attention is increasingly paid to systems with more and more complicated forms of the interactions[2] or symmetries,[3] whereas simpler cases such as the XY or Heisenberg models in a transverse field are not yet properly described. In the present paper we investigate in detail the XY model, but our basic results hold also for the Heisenberg model.

The Hamiltonian of the XY model is given by

$$H = -\frac{1}{2}\sum_{i\neq j}^{N} J_{ij}\left(S_{xi}S_{xj} + S_{yi}S_{yj}\right) - K\sum_{i=1}^{N} S_{zi}, \qquad (1)$$

where $\mathbf{S}_i = (S_{xi}, S_{yi}, S_{zi})$ is the quantum spin operator associated with the spin S at site i. The J_{ij} $(i < j, J_{ij} = J_{ji})$ are quenched random exchange interactions governed by independent symmetric Gaussian distributions with mean zero and variance $1/\sqrt{N}$ (results for variance $J/\sqrt{N}$ can be obtained by an appropriate rescaling of the temperature and of the transverse field).

The phase diagram of the system with the Hamiltonian (1) has been investigated within several approximations,[4–8] including an "exact" Trotter-Suzuki calculation[9] for spin 1/2. The results seem to indicate that the XY spin-glass model behaves qualitatively as the Ising model does, the only difference being that the transition temperatures are lower in the former because of its higher degree of freedom.

In the Ising model, the transverse field acts as a source of a "noise," as does the temperature, and the spin-glass phase seems to have the same properties as the classical spin glass.[10] In the XY model, the transverse field acts differently from a simple "noise:" since the magnetization connected to the field commutes with the Hamiltonian (1), the eigenstates of the Hamiltonian (1) are independent of the field values and only their energies change as the field varies. This means that one cannot say *a priori* that the XY spin glass has a smaller critical temperature than the Ising spin glass for all transverse fields.

This paper is organized as follows. First, in Sec. II, we present arguments to show that at zero temperature in the XY model the spin-glass phase persists up to $K_c = 2S$. This value is higher than that of the Ising model, contrary to previous predictions. We show in Sec. III that the contradiction comes from the fact that previous researchers have overlooked an important correlation between the x and y components of the spin operators. In Sec. IV we present a Trotter-Suzuki calculation of the free energy and some preliminary results for the phase diagram of the XY model. Finally, we discuss the possible extension of our results to the Heisenberg model.

II. THE QUANTUM ZERO-TEMPERATURE PHASE TRANSITION

Although there have been several attempts to describe the zero-temperature behavior of the infinite-range quantum Ising spin-glass model in a transverse field,[11–13] only extrapolations from finite-temperature analysis have been made for its XY counterpart. For $S = 1/2$, estimations of the critical transverse field $K_c(T = 0)$ vary from[8] 1.253/2 to[6,9] 1.44/2 in contrast to the value $K_c = 2/2$ resulting from the static approximation[4,5] (SA) (the factor 1/2 comes from the relation $S_\mu = \frac{1}{2}\sigma_\mu, \mu = x, y, z$, between the spin operators and the Pauli matrices: see Sec. IV).

As we have shown elsewhere,[14] to discuss the ground state properties of the Hamiltonian (1) only two properties of the interaction matrix J_{ij} need be considered: its symmetry with $J_{ii} = 0, \forall i$, and the fact that its largest eigenvalue is finite, i.e., $\lambda_{\max} < \infty$. Since H commutes with the total magnetization in the z direction $M_z = \sum_i S_{zi}$, all eigenstates of H can be characterized by a fixed magnetization M_z. The state $\mid NS\rangle$, where all spins are aligned in the direction of the external field, is the ground state for $K > K_c = S\lambda_{\max}$. For $K < K_c$, the ground state has $M_z < NS$.

For a random Gaussian matrix J_{ij}, $\lambda_{\max} = 2$ in the limit[15] $N \to \infty$ (for N finite $\lambda_{\max}$ has a distribution whose width is proportional to $N^{-2/3}$).[16] This value of $\lambda_{\max}$ gives $K_c = 2S$, implying that for $K > K_c$ the ground state cannot have spin-glass order. For $K < K_c$ and $N \to \infty$ the spin-glass order may be present, as we believe to be the case. For $S = 1/2$ this conjecture is supported by the finite Trotter-Suzuki calculation presented in Sec. IV, where we find that the transition line lies above the one resulting from the SA. On the other hand, the SA at zero temperature gives exactly the same critical value,[4,5] $K_c = 2S$, as this zero-temperature analysis. This means that if the static approximation yields a lower bound for any temperature, the exact critical field for $T = 0$ is $K_c = 2S$. We think that the above-described change in the ground state is the reason that the same critical field is found using different approximations.[4,5,7]

III. MATSUBARA IMAGINARY-TIME FORMULATION

In this section we show that the contradiction between the results of the zero-temperature analysis discussed above and that of previous publications is due to a correlation between the x and y components of the spins, overlooked in these publications.

The derivation of the free energy, using the Matsubara time formalism is given in Ref. 4 and is a straightforward generalization of the work of Bray and Moore.[1] Here we quote only the paramagnetic free energy F_p:

$$F_p = \int_0^1 \int_0^1 d\tau d\tau' \big[R_{xx}^2(\tau,\tau') + R_{yy}^2(\tau,\tau') - 2R_{xy}^2(\tau,\tau')\big] - \ln Q_p, \quad (2)$$

$$Q_p = \mathrm{Tr}\, e^{\beta K S_z} T_\tau \exp\left[\beta \int_0^1 \int_0^1 d\tau d\tau' \Phi(\tau,\tau')\right], \quad (3)$$

$$\begin{aligned}\Phi(\tau,\tau') = {} & R_{xx}(\tau,\tau')S_x(\tau)S_x(\tau') \\ & + R_{yy}(\tau,\tau')S_y(\tau)S_y(\tau') \\ & + 2iR_{xy}(\tau,\tau')S_x(\tau)S_y(\tau'),\end{aligned} \quad (4)$$

where T_τ is the time-ordering operator rearranging the $S_\alpha(\tau)$ operators in the expansion of the exponential of Eq. (3) in order of decreasing time arguments. The operator $S_\alpha(\tau)$ is defined as

$$S_\alpha(\tau) = e^{-\beta\tau K S_z} S_\alpha e^{\beta\tau K S_z}, \quad \alpha = x, y, z\ , \quad (5)$$

and the correlation functions $R_{\alpha\beta}(\tau,\tau')$ are determined from the saddle-point equations:

$$R_{xx}(\tau,\tau') = \frac{\beta}{2}\langle T_\tau S_x(\tau)S_x(\tau')\rangle_p, \quad (6)$$

$$R_{yy}(\tau,\tau') = \frac{\beta}{2}\langle T_\tau S_y(\tau)S_y(\tau')\rangle_p, \quad (7)$$

$$R_{xy}(\tau,\tau') = \frac{\beta}{2i}\langle T_\tau S_x(\tau)S_y(\tau')\rangle_p\ , \quad (8)$$

where $\langle T_\tau \cdots\rangle_p$ means an average with respect to the effective Hamiltonian defined in the exponent of Eq. (3). From Eqs. (6) and (7) it is clear that both R_{xx} and R_{yy} are even functions with respect to the exchange of τ and τ'.

Some authors[8,9] claim that $R_{xy}(\tau,\tau') = 0$ by symmetry, without defining what kind of symmetry they mean. Other authors using the SA neglect $R_{xy}(\tau,\tau')$ without any explanation[4] and give a misleading statement about the symmetry of the paramagnetic phase.[5] Finite Trotter-Suzuki calculations[6,17] also use the $R_{xy}(\tau,\tau') = 0$ supposition for the high-temperature phase. We will show that a nonzero external field K gives a nonzero R_{xy} even in the paramagnetic phase.

The system possesses rotational symmetry about the z axis, so the free energy, Eqs. (2)–(4), and the correlation functions, Eqs. (6)–(8), are invariant under any such rotation, i.e.,

$$R_{\tilde\mu\tilde\nu} = R_{\mu\nu}\ , \quad \mu,\nu = x,y\ ,$$
$$\int_0^1\int_0^1 d\tau d\tau' \tilde\Phi(\tau,\tau') = \int_0^1\int_0^1 d\tau d\tau' \Phi(\tau,\tau')\ ,$$

where the *tilde* designates the new coordinates. To see the consequences of this symmetry let us rotate our system by an angle φ in the x-y plane. Then, from Eqs. (6)–(8) we have the relations

$$\begin{aligned}R_{\tilde x\tilde x}(\tau,\tau') = {} & \cos^2\varphi R_{xx}(\tau,\tau') + \sin^2\varphi R_{yy}(\tau,\tau') \\ & + i\cos\varphi\sin\varphi\,[R_{xy}(\tau,\tau') + R_{xy}(\tau',\tau)]\,,\\ R_{\tilde x\tilde y}(\tau,\tau') = {} & \cos^2\varphi R_{xy}(\tau,\tau') - \sin^2\varphi R_{xy}(\tau',\tau) \\ & + i\cos\varphi\sin\varphi\,[R_{xx}(\tau,\tau') - R_{yy}(\tau,\tau')]\,,\end{aligned}$$

which are fulfilled for any angle φ, if the correlation functions, Eqs. (6)–(8), obey the symmetry relations

$$R_{xx}(\tau,\tau') = R_{yy}(\tau,\tau'), \quad (9)$$

$$R_{xy}(\tau,\tau') = -R_{xy}(\tau',\tau). \quad (10)$$

Equation (9) is a natural consequence of the rotational symmetry. However, Eq. (10) does not imply that $R_{xy}(\tau,\tau')$ is identically zero: it is simply an odd function with respect to the interchange of its arguments. To show explicitly that $R_{xy}(\tau,\tau') \neq 0$ one can calculate the difference between the left- and right-side limits at $\tau = \tau'$ from Eq. (8):

$$R_{xy}(\tau,\tau-0) - R_{xy}(\tau,\tau+0) = \frac{\beta}{2}\langle T_\tau S_z(\tau)\rangle_p. \quad (11)$$

If K is not zero, $\langle T_\tau S_z(\tau)\rangle_p$ is also different from zero, implying that $R_{xy}(\tau,\tau')$ has a discontinuity at $\tau = \tau'$. This discontinuity disappears for $K = 0$, where $R_{xy}(\tau,\tau') \equiv 0$.

If one assumes that the spin-glass order parameters $q_{\alpha\beta}(\tau,\tau')$ $(\alpha,\beta = x,y)$ do not depend on the imaginary times τ and τ', the transition temperature will depend only on the integrals of $R_{\alpha\beta}(\tau,\tau')$.[1] Because $R_{xy}(\tau,\tau') = -R_{xy}(\tau',\tau)$, its integral is zero, i.e., the critical-line equation remains unchanged,

$$2\int_0^1\int_0^1 d\tau d\tau' R_{xx}(\tau,\tau') = 1. \quad (12)$$

To obtain a simple approximation for the free energy

(2) one can neglect the τ dependence of the correlation functions, Eqs. (6)–(8). Within this so-called static approximation (SA), $R_{xx}(\tau,\tau') \approx R_0$ and $R_{xy}(\tau,\tau') \approx 0$. Unfortunately there is no simple way to approximate an odd function like $R_{xy}(\tau,\tau')$, so the form of the SA remains unchanged. Nevertheless, we emphasize that $R_{xy}(\tau,\tau') \approx 0$ is only an approximation, not a relation due to a certain symmetry. For spin 1/2 we have[4,5]

$$F_p^{\rm SA} = -2R_0^2 + \ln Q_0,$$
$$Q_0 = 2\int_0^\infty dx\, e^{-x}\cosh\Gamma(x), \tag{13}$$
$$\Gamma(x) = \sqrt{(\beta K/2)^2 + \beta R_0 x}\ ,$$

where R_0 should be determined from the saddle-point equation $\partial F_p^{\rm SA}/\partial R_0 = 0$. Using Eq. (12) the transition temperature is obtained from the relation $2R_0 = 1$, and the resulting phase diagram is shown in Fig. 1.

We would like to recall here that the SA assumes that the saddle point occurs when $R_{xy}(\tau,\tau')$ vanishes. Within our improved approximation, the saddle point is also located on the R_{xy} axis, but at a finite distance from the origin of the $R_{xx} - R_{xy}$ plane. This means that it is no longer true that the SA, Eq. (13), gives an upper bound for the free energy, as has been claimed by some authors.[4]

Using these simple suppositions of the SA we can calculate the correlation functions to first order.[18] Hence the right-hand sides (RHS) of Eqs. (6) and (8) are

$$g(\tau,\tau') = \langle T_\tau S_x(\tau)S_x(\tau')\rangle_0$$
$$= \frac{1}{8Q_0}\int_0^\infty dx\, e^{-x}\left(\frac{\beta R_0 x}{\Gamma^2(x)}\cosh\Gamma(x) + \left[2 - \frac{\beta R_0 x}{\Gamma^2(x)}\right]\cosh\left[\Gamma(x)\left(1 - 2\mid\tau-\tau'\mid\right)\right]\right), \tag{14}$$
$$\gamma(\tau,\tau') = \langle T_\tau S_x(\tau)S_y(\tau')\rangle_0$$
$$= \frac{i\beta K}{8Q_0}{\rm sgn}(\tau-\tau')\int_0^\infty dx\, e^{-x}\frac{\sinh\left[\Gamma(x)\left(1-2\mid\tau-\tau'\mid\right)\right]}{\Gamma(x)}. \tag{15}$$

Here $\langle\cdots\rangle_0$ denotes the thermal average with respect to the effective Hamiltonian of the SA, Eq. (13). One can see that the cross-correlation function $\gamma(\tau,\tau')$ is identically zero only for $K = 0$ and as K increases, the τ-dependent parts of $\gamma(\tau,\tau')$ and $g(\tau,\tau')$ approach the same order of magnitude.

As it has been shown,[18] using these approximate correlation functions one can make a first-order approximation of the free energy, which goes beyond the SA. Although this first-order calculation gives a good result for the Ising model, we found that in the case of the XY model there is no solution of the saddle-point equations for some values of the transverse field and the temperature. Actually, even for the case $K = 0$, where there is a saddle point giving a paramagnetic-spin-glass transition temperature close to the exact one,[19] this saddle point is unstable. This means that $\gamma(\tau,\tau')$ and $g(\tau,\tau')$ are too large to be used as the basis of a perturbation series. Nevertheless, for higher spin values where the SA is thought to be better, this type of perturbation series might converge.

IV. TROTTER-SUZUKI DISCRETE PATH FORMULATION

In this section we present some preliminary results for the phase diagram of the XY model in a transverse field for spin 1/2, using finite Trotter-Suzuki calculations. This type of calculation has been performed for the system under consideration[9] and for the Heisenberg model[17], but in all of these publications the cross-correlation function R_{xy} was omitted. Our second aim is to discuss the symmetries of the correlation functions in this approximation.

Since in almost all work dealing with $S = 1/2$ the Pauli matrices are used as spin operators without the 1/2 prefactor, we also adopt such a normalization in this section (the previous notation can be easily recovered by scaling the field as $K \to K/2$ and the temperature as $T \to T/4$). Instead of the Hamiltonian (1) we have

$$\hat{H} = -\frac{1}{2}\sum_{i\neq j}^{N} J_{ij}(\hat\sigma_{xi}\hat\sigma_{xj} + \hat\sigma_{yi}\hat\sigma_{yj}) - K\sum_i^N \hat\sigma_{zi}\ , \tag{16}$$

where $\hat\sigma_\alpha$ ($\alpha = x,y,z$) is the αth Pauli matrix.

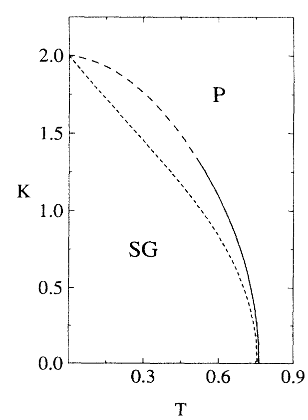

FIG. 1. Phase transition lines for the XY spin-glass model in a transverse field for spin 1/2. T, temperature in units of $J = (\langle J_{ij}^2\rangle/N)^{1/2}$; K, field in units of J; P, paramagnetic phase; SG, spin-glass phase. Solid line: numerical Trotter-Suzuki calculation. Dash-dotted line: an interpolation between the exact phase transition point (0, 2) and the solid line. Dashed line: static approximation.

Let us denote by $\hat{H}_\alpha$ that part of $\hat{H}$ [Eq. (16)] which contains only $\hat{\sigma}_\alpha$ operators ($\alpha = x, y, z$). In order to avoid the difficulties of the noncommutativity of the operators involved one can use a Trotter-Suzuki formula[20]

$$\mathrm{Tr}\exp(-\beta\hat{H}) = \lim_{M\to\infty} \mathrm{Tr}\left[e^{-\frac{\beta}{M}\hat{H}_x}e^{-\frac{\beta}{M}\hat{H}_y}e^{-\frac{\beta}{M}\hat{H}_z}\right]^M \tag{17}$$

and make an extrapolation to $M \to \infty$ from finite M calculations. Although there is no general proof that the finite M approximant of the RHS of Eq. (17) convergences to the $M = \infty$ limit in the same way as M^{-2n} $(n = 1, 2, \ldots)$, we found that because of the similar form of $\hat{H}_x$ and $\hat{H}_y$, this convergence is valid for spin 1/2. On the other hand, there is a general proof of M^{-2n} convergence for the symmetrized formula[21]

$$\mathrm{Tr}\exp(-\beta\hat{H}) = \lim_{M\to\infty} \mathrm{Tr}\left[e^{-\frac{\beta}{2M}\hat{H}_x}e^{-\frac{\beta}{2M}\hat{H}_z}e^{-\frac{\beta}{M}\hat{H}_y}e^{-\frac{\beta}{2M}\hat{H}_z}e^{-\frac{\beta}{2M}\hat{H}_x}\right]^M . \tag{18}$$

We will use Eq. (18), which does not converge better than the form (17), but which, as will be seen, has some other advantages. From Eq. (18) the partition function is

$$Z = \mathrm{Tr}\, e^{-\beta\hat{H}} = \lim_{M\to\infty} \mathrm{Tr}\left[e^{-\frac{\beta}{M}\hat{H}_x}e^{-\frac{\beta}{2M}\hat{H}_z}e^{-\frac{\beta}{M}\hat{H}_y}e^{-\frac{\beta}{2M}\hat{H}_z}\right]^M , \tag{19}$$

where we have rearranged the operators of Eq. (18) cyclicly. Let us denote the RHS of Eq. (19) by Z_M for finite M. If we insert a complete set $| \vec{\sigma}_k^\alpha\rangle = | \sigma_{1k}^\alpha\rangle \cdots | \sigma_{Nk}^\alpha\rangle$ $(\sigma_{ik}^\alpha = \pm 1)$ of eigenstates of the operators $\hat{\sigma}_{\alpha i}$ at the kth operator $\exp(-\frac{\beta}{M}\hat{H}_\alpha)$ in Eq. (19) ($\alpha = x, y;\ k = 1, \ldots, M$), we have

$$Z_M = \mathrm{Tr}_{\vec{\sigma}_k^x\vec{\sigma}_k^y} \exp\left[-\frac{\beta}{M}\sum_k^M (H_{xk} + H_{yk})\right] \prod_k^M \langle\vec{\sigma}_k^x \mid e^{-\frac{\beta}{2M}\hat{H}_z} \mid \vec{\sigma}_k^y\rangle\langle\vec{\sigma}_k^y \mid e^{-\frac{\beta}{2M}\hat{H}_z} \mid \vec{\sigma}_{k+1}^x\rangle , \tag{20}$$

where $H_{\alpha k} = \langle\vec{\sigma}_k^\alpha \mid \hat{H}_\alpha \mid \vec{\sigma}_k^\alpha\rangle$ with the boundary condition $M + 1 \equiv 1$. Since $\hat{H}_z$ is factorized in the real space index $i = 1, \ldots, N$ it is easy to calculate the remaining matrix elements of Eq. (20),[22] yielding

$$Z_M = A^{MN}\mathrm{Tr}_{\sigma_{ik}^x\sigma_{ik}^y} \exp\left[\frac{\beta}{2M}\sum_{ij}^N J_{ij}\sum_k^M(\sigma_{ik}^x\sigma_{jk}^x + \sigma_{ik}^y\sigma_{jk}^y) + i\phi\sum_i^N\sum_k^M(\sigma_{ik}^x\sigma_{ik}^y - \sigma_{ik}^y\sigma_{i,k+1}^x)\right] , \tag{21}$$

where

$$A = \frac{1}{2}\cosh\left(\frac{\beta K}{M}\right), \qquad \phi = \arctan(e^{-\frac{\beta K}{M}}). \tag{22}$$

From Eq. (21) one can calculate the free energy per spin using the standard replica trick. We give here only the paramagnetic free energy

$$f_M = f_0 - \frac{\beta^2}{4M^2}\sum_{kl}^M \left[r_{xx}^2(k,l) + r_{yy}^2(k,l) - 2r_{xy}^2(k,l)\right] + \ln\mathrm{Tr}\exp(H_{\rm eff}), \tag{23}$$

$$H_{\rm eff} = \frac{\beta^2}{2M^2}\sum_{kl}^M\left[r_{xx}(k,l)\sigma_k^x\sigma_l^x + r_{yy}(k,l)\sigma_k^y\sigma_l^y + 2ir_{xy}(k,l)\sigma_k^x\sigma_l^y\right] + i\phi\sum_k^M(\sigma_k^x\sigma_k^y - \sigma_k^y\sigma_{k+1}^x), \tag{24}$$

with the saddle-point equations

$$r_{\alpha\alpha}(k,l) = \langle\sigma_k^\alpha\sigma_l^\alpha\rangle_{\rm eff}, \quad \alpha = x, y , \tag{25}$$

$$r_{xy}(k,l) = \frac{1}{i}\langle\sigma_k^x\sigma_l^y\rangle_{\rm eff}, \tag{26}$$

where $\langle\cdots\rangle_{\rm eff}$ means an average according to $H_{\rm eff}$ (24).

Although in the limit $M \to \infty$ the Matsubara and the Trotter-Suzuki formulas become equivalent, this does not mean that even for M finite, $r_{\alpha\beta}(k,l)$ will have the same symmetries as $R_{\alpha\beta}(\tau,\tau')$ [Eqs. (6)–(8)]. Thus one has to analyze the symmetries of $r_{\alpha\beta}(k,l)$ directly from Eqs. (23)–(26). Because the effective Hamiltonian in Eq. (24) represents a periodic chain, we have

$$r_{\alpha\beta}(k,l) = r_{\alpha\beta}(k - l). \tag{27}$$

From Eq. (25) it is also clear that

$$r_{\alpha\alpha}(k,l) = r_{\alpha\alpha}(l,k), \quad \alpha = x, y. \tag{28}$$

Equation (21) is symmetric with respect to the transformation $\sigma_{ik}^x \to \sigma_{ik}^y$, $\sigma_{ik}^y \to -\sigma_{i,k+1}^x$. The corresponding transformation in the effective Hamiltonian (24) is

$$\sigma_k^x \to \sigma_k^y, \quad \sigma_k^y \to -\sigma_{k+1}^x \tag{29}$$

which is a symmetry transformation if

$$r_{xx}(k,l) = r_{yy}(k,l), \tag{30}$$

$$r_{xy}(k,l) = -r_{xy}(l+1,k). \tag{31}$$

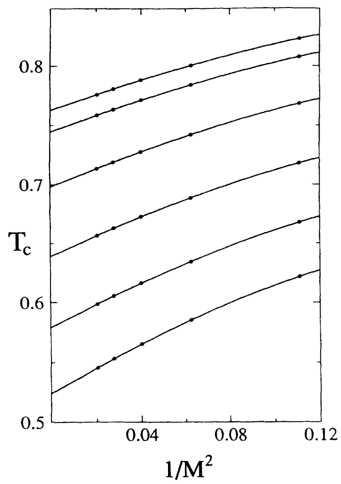

FIG. 2. Phase transition temperature T_c for different field/temperature ratios, $\xi = K/T$, from $\xi = 0.0$ (upper curve) up to $\xi = 2.5$ in steps of 0.5. Points represent results of finite M calculations. The curves are obtained from parabolic fit.

It is easy to check that these relations hold self-consistently in Eqs. (25) and (26). Equation (30) is natural, but Eq. (31) is not trivial. It says that $r_{xy}(l,k)$ is not simply an odd function, but one of its arguments must be shifted after the interchange. Of course, if $M \to \infty$ this small shift becomes negligible and we recover the odd correlation function $R_{xy}(\tau,\tau')$, Eq. (10), obtained with the Matsubara formalism. From Eq. (31) we see that $\sum_{k,l} r_{xy}(k,l) = 0$, which ensures that the critical line equation does not change,

$$\frac{\beta}{M^2}\sum_{k,l}^{M} r_{xx}(k,l) = 1 \tag{32}$$

(note the different normalization of $R_{\alpha\beta}$ and $r_{\alpha\beta}$). If one uses the nonsymmetric formula (17), there is no simple symmetry relation for $r_{xy}(k,l)$ for finite M; even its integral is different from zero, i.e., the critical line equation (32) should be replaced by a more complicated one.

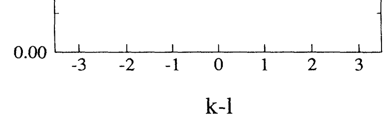

FIG. 3. Correlation function $r_{xx}(k,l)$ obtained from the finite Trotter-Suzuki calculation, Eq. (25), with $M = 7$ for different field/temperature ratios $\xi = K/T$ in the phase-transition point: ⊙, $\xi = 2.5$; △, $\xi = 1.5$; ◇, $\xi = 0.5$. The lines are only visual guides.

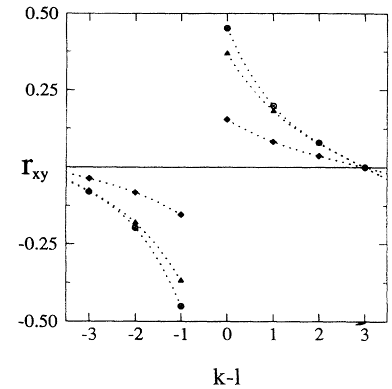

FIG. 4. Correlation function $r_{xy}(k,l)$ obtained from the finite Trotter-Suzuki calculation, Eq. (26), with $M = 7$ for different field/temperature ratios $\xi = K/T$ in the phase-transition point: ⊙, $\xi = 2.5$; △, $\xi = 1.5$; ◇, $\xi = 0.5$. The lines are only visual guides.

Using Eqs. (22)–(32) we calculated the critical temperatures for $M = 3,\ldots,7$ and used a M^{-2n} fit to the $M \to \infty$ extrapolation. [In previous work[6,9] calculations were carried out up to $M = 10$ and $M = 14$, but the nonzero $r_{xy}(k,l)$ makes this type of calculation more complicated.] Up to $K/T = 2.5$ these points fit nicely on a curve $a + bM^{-2} + cM^{-4}$ as is shown in Fig. 2. The correlation functions r_{xx} and r_{xy} for $M = 7$ are presented in Figs. 3 and 4, respectively. To obtain a reliable extrapolation for smaller temperatures and higher fields a larger M should be chosen. Figure 1 shows the phase diagram resulting from an interpolation up to the exact phase transition point ($T = 0$, $K_c = 2$) obtained in Sec. II.

V. DISCUSSION

We have shown that for the XY spin-glass model there is an important cross-correlation function between the x and y spin components due to the external field in the z direction. This function was overlooked in previous publications. We have analyzed the symmetries of this function in the Matsubara and in the Trotter-Suzuki formalism, and calculated the corresponding phase diagram for spin 1/2. We have found that in the XY model at small temperatures the spin-glass phase survives for higher external field values than in the Ising model, due to the fundamental difference of the role played by the external field. This difference can be summarized by the fact that the field does not change the eigenstates of the XY spin-glass Hamiltonian (1), as it does for the Ising model.

Now we discuss briefly the symmetries of the Heisenberg model defined by the Hamiltonian

$$H = -\frac{1}{2}\sum_{i\neq j}^{N} J_{ij}\mathbf{S}_i\mathbf{S}_j - K\sum_{i=1}^{N} S_{zi}. \tag{33}$$

In the Matsubara formalism the paramagnetic free energy is similar to that of the XY model, Eq. (2), but we have three additional correlation functions, R_{xz}, R_{yz} and R_{zz} [see Eqs. (6)–(8)], and a nonzero Edwards-Anderson parameter[23] q_{zz} due to the external field.[4] The rotational symmetry about the z direction (discussed in Sec. III) shows that the new cross correlations R_{xz} and R_{yz} are identically zero, and that R_{xy} remains antisymmetric [see Eq. (10)].

In the Trotter-Suzuki formalism one has to decide between the nonsymmetric [Eq. (17)] and the symmetric [Eq. (18)] formulas. For both, the new cross-correlations r_{xz} and r_{yz} are identically zero. With the symmetric formula one can find a symmetry relation such as Eq. (31) for r_{xy}, but there will be four effective spins for each Trotter index. With the nonsymmetric formula only three effective spins per Trotter index are needed, but the useful relation for r_{xy}, Eq. (31), and the simple form of the critical line equation (32) are lost.

We would like to point out that the XY model is simpler to analyze than the Heisenberg model because (i) a zero-temperature transition can be found analytically (see Sec. II), (ii) the symmetrized Trotter-Suzuki formula can be used without difficulty to carry out finite Trotter calculations (see Sec. IV).

More extensive Trotter-Suzuki calculations for both the XY model (with $M > 7$) and the Heisenberg model are needed to locate precisely the phase-transition line between the paramagnetic and spin-glass phases. The properties of the spin-glass phase, e.g., the time dependence of the order parameters and the nature of the replica symmetry breaking, remain to be investigated.

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[1] A. J. Bray and M. A. Moore, J. Phys. C **13**, L655 (1980).
[2] T. K. Kopeć and G. Büttner, Phys. Rev. B **41**, 10 853 (1991); Y. Y. Goldschmidt, *ibid.* **41**, 4858 (1990), J. Phys. I (France) **2**, 31 (1992).
[3] G. Büttner and K. D. Usadel, Europhys. Lett. **14**, 165 (1991); K. Binder and J. D. Reger, Adv. Phys. **41**, 547 (1992).
[4] Y. Y. Goldschmidt and P.-Y. Lai, Phys. Rev. B **43**, 11 434 (1991).
[5] K. Walasek and K. Lukierska-Walasek, Phys. Lett. A **159**, 97 (1991).
[6] L. De Cesare, K. Lukierska-Walasek, I. Rabuffo, and K. Walasek, Phys. Rev. B **45**, 1041 (1992).
[7] L. De Cesare, K. Lukierska-Walasek, I. Rabuffo, and K. Walasek, Phys. Lett. A **145**, 291 (1990).
[8] G. Büttner, T. K. Kopeć, and K. D. Usadel, Phys. Lett. A **149**, 248 (1990).
[9] G. Büttner and K. D. Usadel, Z. Phys. B **83**, 131 (1991).
[10] G. Büttner and K. D. Usadel, Phys. Rev. B **42**, 6385 (1990); **41**, 428 (1990).
[11] T. Yamamoto and H. Ishii, J. Phys. C **20**, 6053 (1987).
[12] J. Miller and D. A. Huse, Phys. Rev. Lett. **70**, 3147 (1993).
[13] J. Ye, S. Sachdev, and N. Read, Phys. Rev. Lett. **70**, 4011 (1993).
[14] F. Pázmándi and Z. Domański, J. Phys. A **26**, L689 (1993).
[15] M. L. Mehta, *Random Matrices and the Statistical Theory of Energy Levels* (Academic, London, 1967).
[16] The $N^{-1/6}$ tail of the semicircular distribution of the eigenvalues of a Gaussian random matrix reported by B. V. Bronk [J. Math. Phys, **5**, 215 (1964)] should be renormalized by the variance of J_{ij}, i.e., $N^{-1/2}$.
[17] Y. Y. Goldschmidt and P.-Y. Lai, Physica A **177**, 544 (1991).
[18] F. Pázmándi and Z. Domański, J. Phys.: Condens. Matter **5**, L117 (1993).
[19] K. D. Usadel, Nucl. Phys. B (Proc. Suppl.) **5A**, 91 (1988).
[20] M. Suzuki, Prog. Theor. Phys. **56**, 1454 (1976).
[21] M. Suzuki, Phys. Rev. B **31**, 2957 (1985).
[22] V. Dobrosavljević and R. M. Stratt, Phys. Rev. B **36**, 8484 (1987).
[23] S. F. Edwards and P. W. Anderson, J. Phys. F **5**, 965 (1975).