Evidence of aging in spin-glass mean-field models

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We study numerically the out-of-equilibrium dynamics of the hypercubic cell spin glass in high dimensionalities. We obtain evidence of aging effects qualitatively similar both to experiments and to simulations of low-dimensional models. This suggests that the Sherrington-Kirkpatrick model as well as other mean-field finite connectivity lattices can be used to study these effects analytically.

There has been a long-standing and mainly unresolved controversy on which kind of models can describe real (experimental) spin glasses, focused on whether the mean-field models (as opposed to low-dimensional models) can or cannot describe, even at a qualitative level, the essential features of them. This controversy has been mainly centered on the nature of the ground-state structure.[1,2]

However, because of the long time scales involved in spin-glass dynamics, most experiments are effectively performed out of equilibrium. Thus, spin-glass physics is usually described as being essentially dynamical. Indeed, the main point concerning the relevance of a given model should be its dynamical behavior—whether or not it resembles the experimental one.

Spin glasses exhibit the striking phenomenon of aging:[3] their dynamical properties depend on their history even after very long times and they continue to evolve long after thermalization at a subcritical temperature.

A fully microscopic description of these effects in realistic spin glasses is still lacking. There have been several phenomenological attempts to describe the physical mechanism of aging;[2,4–7] and some numerical studies of the three-dimensional Edwards-Anderson (3D EA) model have given results showing aging effects in good agreement with experiments.[8,9]

Thus, the present understanding of the problem is at two extremes: on the one hand, there are phenomenological models whose physics is explicit but without a direct reference to the microscopy and, on the other hand, there are simulations of microscopic models close to real materials of which an analytical description seems at present far off.

The interest of studying mean-field models is that they may provide a bridge between the phenomenology and the microscopy of real systems. It was only recently shown that mean-field dynamical models can exhibit aging effects even in the thermodynamic limit.[10] Although the model considered there (spherical spin glass with multispin interactions) is simple enough that even analytic results for the nonequilibrium dynamics could be found, the price paid was that it is quite unrealistic.

The scope of this work is to show that the dynamical behavior of the mean-field models is strikingly similar to the behavior of the low-dimensional models and, moreover, that they mimic very well the experimental observations. With this aim we give evidence of aging phenomena in a model whose behavior is expected to approach, for high dimensionality D, that of high-dimensional spin glasses on a hypercubic lattice and, in particular, that of the Sherrington-Kirkpatrick (SK) model (as $D \to \infty$).

The fact that mean-field models capture the essential characteristics of spin-glass experiments is promising because they are technically much simpler than realistic models. Indeed, it has been argued that mean-field models can be solved analytically whenever their long-term memory is weak; i.e., whenever the response to a constant field applied during a fixed time interval decreases to zero after long enough times.[10] We will show below that this happens in this model.

In principle, one would like to demonstrate aging effects in the SK model, as the archetypical mean-field model. However, the use of Monte Carlo dynamics is strongly limited because of its full connectivity. A whole sweep of the lattice requires a computer time which grows as N^2 (N is the number of spins) and this restricts the sizes that can be analyzed. As we shall discuss below, this implies a strong limitation in the range of times free of finite-size effects. In order to study mean-field dynamics up to large enough sizes it is then convenient to choose lattices which are also mean-field models but with connectivity growing slower than N (see, e.g., Ref. 11).

The model we consider has been introduced in Ref. 12, where its equilibrium properties have been studied. It consists of a single hypercubic cell in D dimensions; on each of its corners there is a ± 1 spin that interacts with its D nearest neighbors. The total number of spins is $N = 2^D$. The Hamiltonian is of the usual type

$$H = -\sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j ,$$

where $\langle ij \rangle$ denotes nearest neighbors and the probability distribution of the couplings is given by

$$P(J_{ij}) = \tfrac{1}{2}\delta(J_{ij} - J) + \tfrac{1}{2}\delta(J_{ij} + J) ,$$

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with $J=1/\sqrt{D}$.

This model is particularly interesting since for large dimension $2D$ it is expected to mimic the full lattice of dimension D (Ref. 12) and to approach in the $D\to\infty$ limit the SK model.[13] As regards numerical simulations, the computation time for a whole sweep of the cell grows as $N\ln N$. Indeed, Monte Carlo simulations of the SK model for more than a few thousand spins become quickly unfeasible while throughout this paper we consider $D=15$, 17, i.e., $N=2^{15},2^{17}$ spins.

Recently, Eissfeller and Opper[14] have devised a very powerful Monte Carlo procedure to solve the mean-field dynamical equations in the limit $N\to\infty$, i.e., with no finite-size effects. Unfortunately, this method requires a computer time that grows faster than the square of the number of time steps. As we discuss below, large times are essential to distinguish aging phenomena from an ordinary nonequilibrium relaxation. Thus, this semianalytical method would have required enormous computer times to arrive at the times here considered.

We perform here a usual heat-bath dynamics. Even if the short-time behavior may depend on the update procedure used, we expect long-time features to be essentially independent of it. The computer time is linear in the simulated time and, as we shall see, finite-size effects are not the limiting factor.

We have simulated "field jump" experiments after a fast cooling to a subcritical temperature which is afterwards kept constant and measured the averaged magnetization:

$$m(\tau)=\frac{1}{N}\sum_i \overline{\langle\sigma_i(\tau)\rangle}$$

and the correlation function

$$C(t_w+t,t_w)=\frac{1}{N}\sum_i \overline{\langle\sigma_i(t_w+t)\sigma_i(t_w)\rangle}\ . \quad (1)$$

The overbar denotes a mean over different realizations of the couplings and $\langle\cdot\rangle$ denotes an average over noise realizations. The number of samples taken will be denoted N_s. We will present in detail the results for $T=0.2T_c$ ($T_c=1$), and at the end we will indicate the dependence of these results with the temperature.

In this kind of simulation one should carefully select the appropriate time window corresponding to the experimental physical situation. For long enough times, a finite system eventually reaches equilibrium and all aging effects disappear. A necessary (though not sufficient) condition for having aging phenomena is that the time needed for the system to achieve the thermodynamic (Gibbs-Boltzmann) distribution be longer than the experimental time. In a computer simulation of the kind we consider here N is finite, and one has to check that the observation times be small enough that the system is not allowed to reach "thermal death" due to finite size.

The method we use to check this is to consider several copies of the system with the same couplings starting from different random configurations and evolving with different realizations of thermal noise. We then calculate the evolution of the square of the overlap between the configurations. This is a quantity that starts from $O(1/N)$ and tends to $\langle q^2\rangle_{\rm eq}$, the mean-square overlap calculated with the equilibrium measure. Roughly, $\langle q^2\rangle_{\rm eq}\simeq 0.7$.[15] Since we only consider times such that the value of $\overline{\langle q^2\rangle(t)}$ remains small ($\overline{\langle q^2\rangle(t)}\lesssim 0.04$), the different copies are not able to cross barriers in their search for the few deepest states.

A more subtle problem is that of small times: once one is satisfied that the dynamics is nonequilibrium one has to check that this is an *asymptotic* aging process, i.e., not an ordinary out of equilibrium transient. In a realistic system the question is ultimately resolved by the actual time scales involved, as compared with the experimental time. The model we are discussing being only qualitatively realistic, we have to content ourselves with some other criterion. We have chosen the following: we consider the asymptotic nonequilibrium regime to start at the time when one-time quantities (energy, magnetization, etc.) are near to their asymptotic values and approach them with their asymptotic power law. Because we are avoiding finite-size effects, "asymptotic" means a large time limit taken *after* the large-N limit.

We have analyzed the relaxation of the energy and performed a separate power-law fit for each time interval [30–100, 100–300, 300–1000, 1000–3000, 3000–10 000 Monte Carlo sweeps (MC's)]. We found time exponents which were roughly consistent for these intervals ($\simeq -0.3$) while the first 30 steps deviate from this behavior.

We have simulated thermoremanent magnetization (TRM) and zero-field-cooling (ZFC) experiments. In the TRM experiments, the sample is rapidly cooled from above the critical temperature down to a temperature T in the spin-glass phase ($T<T_c$) with a small field h applied. Then, the system is allowed to evolve during a "waiting" time t_w at the constant temperature T and field h. After t_w the field is cut off and the relaxation of the magnetization $m(t+t_w)$, i.e., the TRM, is measured as a function of the subsequent time t. In the ZFC experiments, the sample is cooled from above T_c in zero field and after a waiting time t_w a small field is applied. Then, the increase of $m(t+t_w)$, i.e., the ZFC magnetization, is measured.

The starting configuration in the simulations was chosen at random corresponding to a fast quench to a temperature $T<T^c$. This is slightly different from the experimental procedures in that our scheme corresponds to a fast enough quench such that the initial magnetization is still zero even in the TRM case. The difference is, however, small, since the magnetization rapidly grows to its final value. If linear response theory holds (as it should for small fields) one expects that the sum of the magnetizations obtained from the TRM and the ZFC processes with the same t_w and h yield the magnetization associated to a constant field h applied since the temperature quench [the field-cooled magnetization (FCM)]. Because of the particular initial conditions, the sum of the ZFC plus the TRM is not a constant, but a curve for the FCM that saturates very fast. We have analyzed the TRM, the ZFC magnetization, their sum, and the FCM

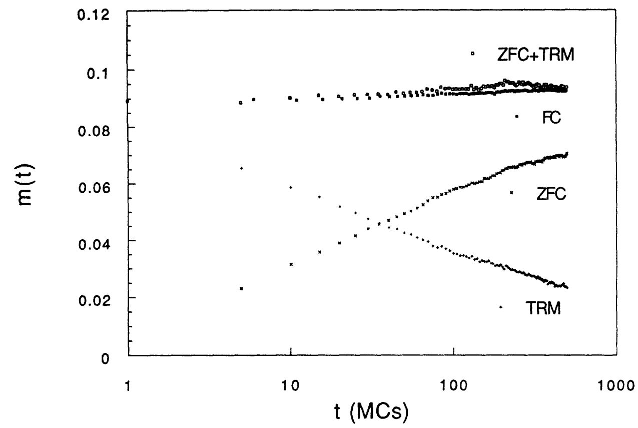

FIG. 1. Zero-field-cooled magnetization and thermoremanent magnetization and their sum for waiting time $t_w=100$, and the magnetization corresponding to a constant field. For all the curves $D=17$, $T=0.2$, and $h=0.1$.

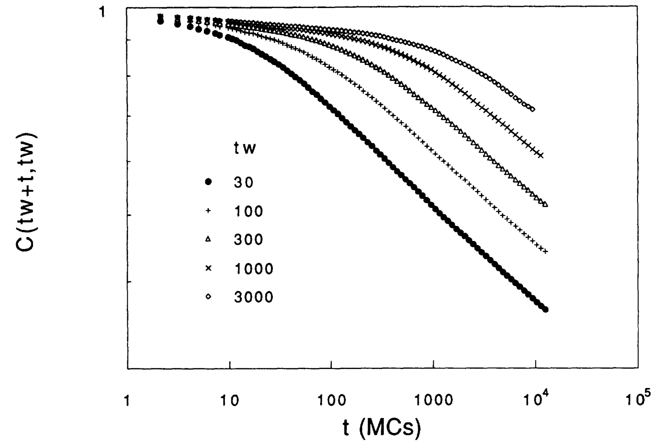

FIG. 3. Autocorrelation function $C(t+t_w,t_w)$ vs time t for waiting times $t_w=30$, 100, 300, 1000, 3000; $D=15$; and $T=0.2$.

for $t_w=100$ and 1000, $D=17$, and $h=0.1$: the linear response theory holds within 5%. In Fig. 1, we show the plot for $t_w=100$. We henceforth concentrate only on TRM simulations.

In Fig. 2, we show the TRM simulations for $D=17$, $h=0.1$ and waiting times $t_w=100$, 300, 1000, 3000, 10 000, with $N_s=10$ for $t_w=100$, 300 and $N_s=5$ for the rest. The curves clearly depend on the waiting time: the response decreases with t_w. These curves are very similar to, e.g., the corresponding experimental curves for the indium-diluted chromium thiospinel of Ref. 16.

In Fig. 3, we show the decay of the correlation [Eq. (1)] vs t for $D=15$, $h=0$, averaged over five samples. We note that the system distances from itself with a speed that decreases with t_w: the phenomenon of "weak ergodicity breaking."[7] These curves are remarkably similar to those of the 3D EA model.[9]

In Fig. 4, we show the fitting for $t \ll t_w$ of the correlation curves. We plot $C(t+t_w,t_w)t^{\alpha}$ in terms of t/t_w. The exponent that makes the curves superpose is $\alpha=0.01$ and thus the departure from a pure function of t/t_w is very small. Similarly there is a small departure from a pure dependence on t/t_w on the sector $t \gg t_w$.

We have also performed simulations at higher temperatures (up to $T=0.8$). As expected, aging effects are still present but they decrease with increasing temperatures and they disappear as it approaches the critical temperature. In addition, we have analyzed the response of the model to changes in the temperature during the waiting time.

First, we have performed "temperature jump" simulations in the manner of the experiments of Ref. 17. The system has been kept at a constant temperature $T-\delta T$ during t_w when the temperature has been suddenly changed to T and afterwards it has been kept constant. These results are presented in Fig. 5. We have measured the correlation function (1) for various values of δT and we have found the following.

If $\delta T>0$ and $t>t_w$ the system behaves as a younger system and the greater the value of δT, the younger the system seems to be.

If $\delta T<0$ and $t<t_w$, the system seems to be older and

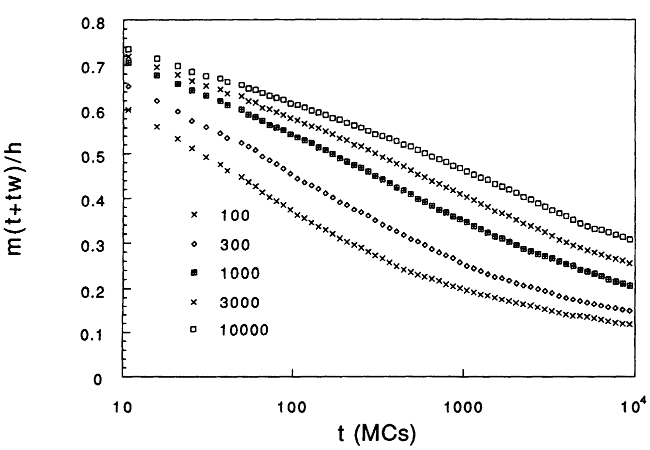

FIG. 2. Thermoremanent magnetization for waiting times $t_w=100$, 300, 1000, 3000, 10 000; $D=17$; $T=0.2$; and $h=0.1$.

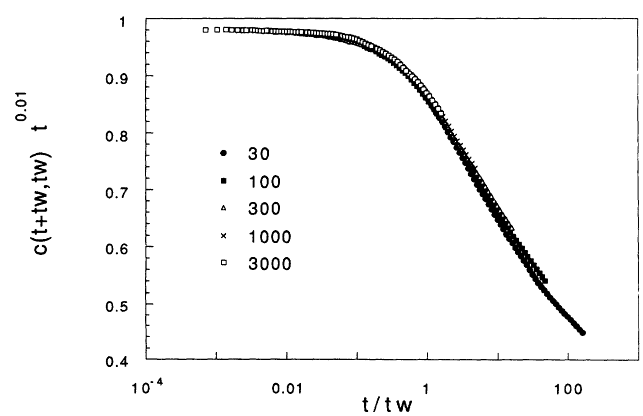

FIG. 4. Fitting (for $t \ll t_w$) of the correlation functions of Fig. 3. The plot shows the autocorrelation functions times $t^{-0.01}$ for each waiting time vs t/t_w.

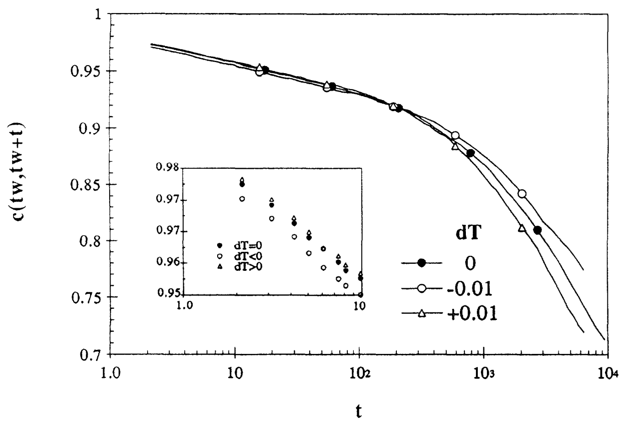

FIG. 5. Autocorrelation function $C(t+t_w,t_w)$ vs time t in a "temperature jump" simulation. $D=15$, $t_w=3000$, $T=0.2$, and $\delta T=\pm 0.01$.

the greater the absolute value of δT the older the system seems to be.

At times smaller than the waiting time ($t<t_w$) the response of the system is the opposite. The $\delta T>0$ ($\delta T<0$) curve is below (above) the reference $\delta T=0$ curve. Furthermore, the responses are in this range of times asymmetric as can be seen in the inset of Fig. 5. We do not have a full understanding of this point and to draw a definite conclusion a more detailed analysis is needed.

Second, we have performed some temperature cycling simulations in the manner of the experiments in Refs. 18 and 19. We have found that the system is very insensitive to short high-temperature pulses during the waiting time in the whole range of subcritical temperatures. This last result seems to indicate that the total waiting times we have been using are rather short: assuming the effect of a high-temperature pulse is somehow proportional to its absolute duration, then a small percentage of a small waiting time necessarily has a small effect. If this is the case one should not draw conclusions on the symmetry or asymmetry of the response to small changes of temperature since one expects asymmetries to be pronounced only at very long times.

The model studied has both frustration and disorder. It is interesting to understand the relative importance of these two features as regards aging effects. With this aim we have studied the fully frustrated model[20] on the hypercubic cell. We have measured the correlation functions (1) at constant temperature for various waiting times and we have not observed any aging effect. However, we have found that a very small amount of disorder introduced changing at random the sign of a fixed small percentage of the bonds suffices to make the system exhibit these effects.

In conclusion, there is at present good evidence of aging effects in mean-field systems with ergodicity breaking in the thermodynamic limit: "weak" and "true" ergodicity breaking coexist. Our numerical results suggest that mean-field models have a qualitatively similar dynamical behavior to that of low-dimensional systems (cf. Refs. 8 and 9).

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