Meissner effect and gauge invariance in anisotropic narrow-band Bloch-electron and hole-type superconductors

Susumu Misawa
Faculty of Human Science, Tokiwa University, Mito, Ibaraki 310, Japan
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Within the framework of the ladder-diagram approximation, the *manifestly* gauge-invariant expression for the static electromagnetic response kernel of the anisotropic narrow-band superconductor consisting of Bloch electrons is derived, on the basis of the charge-conserving current expression for them under the magnetic field. For the two-dimensional square lattice, the magnetic penetration depth $\lambda_L(T)$ is calculated as a function of hole density n_h and temperature T ($<T_c$). The resultant $\lambda_L(0)$ has a symmetrical dependence on the hole and electron densities, and is almost unaffected by the anisotropy of the order parameter. The behavior of $\lambda_L(T)/\lambda_L(0)$ as a function of T/T_c for the d-wave state substantially deviates from those for the extended and usual s-wave states for all $T<T_c$. The obtained results are compared with those of the effective hole- and electron-mass approximations.

I. INTRODUCTION

In spite of a number of experimental and theoretical works, the pairing state of high-T_c superconductors has not been fully understood. The magnetic penetration depth $\lambda_L(T)$ (as a function of temperature T) is one of the most important quantities that give information on the pairing state. In most recent experiments on $\lambda_L(T)$ of single-crystal oxide superconductors,[1-4] the s-wave-like behaviors of the weak- and strong-coupling superconductivities are observed. While, in some studies,[5] the simple power-law behavior $\lambda_L(T)\sim T^2$ at very low temperature is reported.

In the recent hydrodynamical strong-coupling treatment,[6] which involves some phenomenological effective mass approximation, $\lambda_L(T)$ for the d-wave state has been calculated. In most theoretical studies for high-T_c superconductors, however, the pairing interaction is assumed to work over the whole region of the narrow band, and various properties are calculated for Bloch electrons in the narrow tight-binding model. The effect of the van Hove singularity on $\lambda_L(T)$ also should be examined, since these oxide superconductors have the quasi-two-dimensional character. Furthermore, in most high-T_c superconductors, the carriers are found to be holelike. Therefore, we consider it worthwhile to investigate theoretically $\lambda_L(T)$ of the anisotropic superconductor consisting of Bloch electrons and that consisting of holes in detail.

Some studies have already treated this problem. Schneider and Frick[7] have investigated the effect of the anisotropy due to the layered structure on the behavior of $\lambda_L(T)$. They have calculated $\lambda_L(T)$ by making the difference between the paramagnetic term in the superconducting state and that in the normal state. For low hole density n_h, their $\lambda_L(0)$ at $T=0$ K has approximately the London form but with *hole* mass and *hole* density.[7] Hirsch and Mirsiglio[8] have investigated $\lambda_L(T)$ of the layered system with a different mechanism, by using the charge-conserving expression for the current carried by Bloch electrons under the magnetic field. However, they have not explicitly examined the problem of the gauge invariance. By using the charge-conserving expression for the current carried by Bloch electrons under the magnetic field,[9] Ohkawa has derived the expression for $\lambda_L(T)$ of the impure strongly correlated superconductor and has calculated $\lambda_L(T)$ for the d-wave state analytically for low temperature[10] and numerically.[11] However, he has not explicitly examined the gauge invariance of the electromagnetic response kernel of this system. His approximated $\lambda_L(0)$ was written only in the usual London form with heavy *electron* mass and *electron* density.[11]

In a previous paper,[12] by using the above current expression for Bloch electrons,[9] the present author also derived the expression for $\lambda_L(T)$ of the anisotropic narrow-band superconductor; the expression essentially coincides with the pure (and no correlation) limit of Ohkawa's;[10,11] λ_L for the two-dimensional square lattice has been calculated numerically as a function of chemical potential μ and temperature T ($<T_c$). The resultant $\lambda_L(T)$ in Ref. 12, however, is based on the simple pairing approximation, and is not gauge invariant basically. In the present work, in Sec. II we give the slightly corrected expression for the current carried by Bloch electrons under the weak long-wavelength electromagnetic field; by using this expression, we show that the ladder diagram approximation[13,14] is consistent with the gauge invariance for the general anisotropic narrow-band superconductor consisting of Bloch electrons. Within the framework of this approximation, in Sec. III we derive the *manifestly* gauge-invariant expression for the static electromagnetic response kernel, and calculate λ_L for the two-dimensional square lattice as a function of hole density n_h and temperature T ($<T_c$). We also compare the obtained results with those of the effective hole- and electron-mass approximations. A conclusion is given in Sec. IV.

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II. MODEL AND FORMULATION

In this section, we apply the ladder diagram approximation[13,14] to the electromagnetic response of the general narrow-band anisotropic superconductor consisting of Bloch electrons. We consider the following general tight-binding model on a D-dimensional lattice:

$$H=\sum_{\langle \mathbf{r},\mathbf{r}'\rangle\sigma} t(\mathbf{r}-\mathbf{r}')c^{\dagger}_{\mathbf{r}\sigma}c_{\mathbf{r}'\sigma}+\text{H.c.} -\mu\sum_{\mathbf{r},\sigma} n_{\mathbf{r}\sigma}+\sum_{\langle \mathbf{r},\mathbf{r}'\rangle\sigma\sigma'} V(\mathbf{r}-\mathbf{r}')n_{\mathbf{r}\sigma}n_{\mathbf{r}'\sigma'} , \tag{2.1}$$

where $t(\mathbf{r}-\mathbf{r}')$ is the transfer integral and $V(\mathbf{r}-\mathbf{r}')$ the attractive interaction; $c_{\mathbf{r}\sigma}$ and $n_{\mathbf{r}\sigma}$ are, respectively, the annihilation and number operators for an electron of spin σ with the chemical potential μ at the rth site. By using the Nambu notation[13] and by dropping some nonessential terms, we rewrite Eq. (2.1) into the Fourier transformed form

$$H=\sum_{\mathbf{k}}(\epsilon_{\mathbf{k}}-\mu)\Psi^{\dagger}_{\mathbf{k}}\tau_3\Psi_{\mathbf{k}} +\frac{1}{2N}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V(\mathbf{q})\Psi^{\dagger}_{\mathbf{k}+\mathbf{q}/2}\tau_3\Psi_{\mathbf{k}-\mathbf{q}/2}\Psi^{\dagger}_{\mathbf{k}'-\mathbf{q}/2}\tau_3\Psi_{\mathbf{k}'+\mathbf{q}/2} , \tag{2.2}$$

where $\epsilon_{\mathbf{k}}=\sum_{\mathbf{r}}t(\mathbf{r})e^{-i\mathbf{k}\mathbf{r}}$ and $V(\mathbf{q})=\sum_{\mathbf{r}}V(\mathbf{r})e^{-i\mathbf{q}\mathbf{r}}$ with $\mathbf{r}$ relative lattice sites; $\Psi^{\dagger}_{\mathbf{k}}=[c^{\dagger}_{\mathbf{k}\uparrow},c_{-\mathbf{k}\downarrow}]$; τ_i $(i=1,2,3)$ are Pauli matrices, and N is the total number of the lattice sites. The wave-number summations are restricted within the unit cell of the reciprocal lattice.

We consider only the singlet superconducting state. The temperature Green's function in the Hartree-Fock approximation is given by

$$G(\mathbf{k},i\omega_n)\equiv-\int_0^{\beta}d\tau\, e^{i\omega_n\tau}\langle\Psi_{\mathbf{k}}(\tau)\Psi^{\dagger}_{\mathbf{k}}(0)\rangle =\{i\omega_n-(\epsilon_{\mathbf{k}}-\mu)\tau_3-\Delta_{\mathbf{k}}\tau_1\}^{-1} , \tag{2.3}$$

where $\omega_n=(2n+1)\pi T$ with T the temperature and n is an integer, $\beta=1/T$, and

$$\Delta_{\mathbf{k}}\equiv\sum_{\mathbf{q}}V(\mathbf{k}-\mathbf{q})\langle c_{-\mathbf{q}\downarrow}c_{\mathbf{q}\uparrow}\rangle/N$$

is the order parameter chosen to be real and is determined by

$$\Delta_{\mathbf{k}}\tau_1=-\frac{T}{N}\sum_{n\mathbf{q}}V(\mathbf{k}-\mathbf{q})\tau_3G(\mathbf{q},i\omega_n)\tau_3 =-\frac{1}{N}\sum_{\mathbf{q}}\frac{V(\mathbf{k}-\mathbf{q})\Delta_{\mathbf{q}}}{2E_{\mathbf{q}}}\tanh\frac{E_{\mathbf{q}}}{2T}\tau_1 , \tag{2.4}$$

with $E_{\mathbf{k}}=\sqrt{(\epsilon_{\mathbf{k}}-\mu)^2+\Delta^2_{\mathbf{k}}}$. Equation (2.4) is expressed in the form of the self energy (in which we have neglected some contribution in proportion to τ_3, for it is only to give a nonessential correction to $\epsilon_{\mathbf{k}}$); we will use this expression (2.4) to see the generalized Ward's identity below. We determine μ by

$$n_e\equiv 2-n_h=\frac{1}{N}\sum_{\mathbf{k}}n_{\mathbf{k}}\equiv\frac{1}{N}\sum_{\mathbf{k}}\left[1-\frac{\epsilon_{\mathbf{k}}-\mu}{E_{\mathbf{k}}}\tanh\frac{E_{\mathbf{k}}}{2T}\right] , \tag{2.5}$$

where n_e is the electron density.

If the system is under a weak long-wavelength electromagnetic field described by the vector and scalar potentials $A_i(\mathbf{r})(i=1,\ldots,D)$ and $\varphi(\mathbf{r})$ respectively, and in addition the Coulomb potential between electrons is switched on, then the following terms are added to the Hamiltonian (2.2):

$$H'=H_p+H_d+H_C , \tag{2.6}$$

$$H_p=-\frac{1}{cN}\sum_{\mathbf{q}}\sum_{\mu=0}^{D}j^p_{\mu}(-\mathbf{q})A_{\mu}(\mathbf{q}) , \tag{2.7}$$

$$H_d=\frac{e^2}{2c^2N^2}\sum_{\mathbf{k}\mathbf{q}\mathbf{q}'}\sum_{i,j=1}^{D}\frac{\partial^2\epsilon_{\mathbf{k}}}{\partial k_i\partial k_j}\Psi^{\dagger}_{\mathbf{k}+\mathbf{q}/2+\mathbf{q}'/2}\tau_3 \times\Psi_{\mathbf{k}-\mathbf{q}/2-\mathbf{q}'/2}A_i(\mathbf{q})A_j(\mathbf{q}') , \tag{2.8}$$

$$H_C=\frac{1}{2N}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}v(\mathbf{q})\Psi^{\dagger}_{\mathbf{k}+\mathbf{q}/2}\tau_3 \times\Psi_{\mathbf{k}-\mathbf{q}/2}\Psi^{\dagger}_{\mathbf{k}'-\mathbf{q}/2}\tau_3\Psi_{\mathbf{k}'+\mathbf{q}/2} , \tag{2.9}$$

where $v(\mathbf{q})=2\pi e^2/q$ for $D=2$ ($4\pi e^2/q^2$ for $D=3$) is the Coulomb potential. In Eq. (2.7), we have used the $(D+1)$-dimensional metric[14]

$$g_{\mu\nu}=\begin{cases}1 & (\mu,\nu=1,\ldots,D)\\ -1 & (\mu=\nu=0)\\ 0 & (\text{otherwise}).\end{cases} \tag{2.10}$$

The four-component vector potential is given by

$$A_{\mu}(\mathbf{q})=\begin{cases}A_i(\mathbf{q}) & (\mu=i=1,\ldots,D)\\ c\varphi(\mathbf{q}) & (\mu=0).\end{cases} \tag{2.11}$$

Equations (2.7) and (2.8) are the coupling terms to the external field. They are constructed to yield the following current expressions when the derivatives are taken with respect to $-A_{\mu}(-\mathbf{q})/cN$.[15] The current carried by Bloch electrons under the weak long-wavelength field $A_{\mu}(\mathbf{q})$ should be expressed as

$$j_{\mu}(\mathbf{q})=j^p_{\mu}(\mathbf{q})+j^d_{\mu}(\mathbf{q}) , \tag{2.12}$$

where the four-component paramagnetic- and diamagnetic-current densities are given by

$$j^p_{\mu}(\mathbf{q})=\begin{cases}j^p_i(\mathbf{q})=-e\sum_{\mathbf{k}}\Psi^{\dagger}_{\mathbf{k}-\mathbf{q}/2}\dfrac{\partial\epsilon_{\mathbf{k}}}{\partial k_i}\tau_0\Psi_{\mathbf{k}+\mathbf{q}/2} & (\mu=i=1,\ldots,D),\\ \rho(\mathbf{q})=-e\sum_{\mathbf{k}}\Psi^{\dagger}_{\mathbf{k}-\mathbf{q}/2}\tau_3\Psi_{\mathbf{k}+\mathbf{q}/2} & (\mu=0),\end{cases} \tag{2.13}$$

and

$$j_\mu^d(\mathbf{q})=\begin{cases}-\dfrac{e^2}{cN}\displaystyle\sum_{\mathbf{k}\mathbf{q}'}\sum_{j=1}^{D}\dfrac{\partial^2\epsilon_{\mathbf{k}}}{\partial k_i\partial k_j}\Psi^\dagger_{\mathbf{k}-\mathbf{q}/2+\mathbf{q}'/2}\tau_3\Psi_{\mathbf{k}+\mathbf{q}/2-\mathbf{q}'/2}A_j(\mathbf{q}') & (\mu=i=1,\ldots,D),\\ 0 & (\mu=0),\end{cases} \tag{2.14}$$

respectively. In Eq. (2.13), τ_0 is the 2×2 unit matrix; the $(1,\ldots,D)$th and 0th components are the paramagnetic-current and charge densities, respectively. The previous expression[9–12] for $j_\mu^d(\mathbf{q})$ should be slightly corrected into the form (2.14) (the only $\mathbf{q}'=\mathbf{q}$ terms in Eq. (2.14) were taken in the previous expression). Under this correction, and if only terms up to the first order in $A_\mu(\mathbf{q})$ as well as in q are considered, then the following equation of continuity is satisfied for the original Hamiltonian $H+H'$:

$$\dot\rho(\mathbf{q})+i\sum_{i=1}^{D}q_ij_i(\mathbf{q})=0\ , \tag{2.15}$$

where $\dot\rho(\mathbf{q})$ denotes the derivative of $\rho(\mathbf{q})$ with respect to time. If we concentrate on the linear response to the weak field $A_\mu(\mathbf{q})$, the Fourier transform of the expectation value of the current density is expressed as

$$J_\mu(\mathbf{q},\omega)=-\frac{c}{4\pi}\sum_{\nu=0}^{D}K_{\mu\nu}(\mathbf{q},\omega)A_\nu(\mathbf{q},\omega)\ , \tag{2.16}$$

where the electromagnetic response kernel $K_{\mu\nu}(\mathbf{q},\omega)$ is given by

$$K_{\mu\nu}(\mathbf{q},\omega)=K^p_{\mu\nu}(\mathbf{q},\omega)+K^d_{\mu\nu}(\mathbf{q},\omega)\ . \tag{2.17}$$

Here, the paramagnetic term is described by

$$K^p_{\mu\nu}(\mathbf{q},\omega)=\frac{4\pi}{c^2}P_{\mu\nu}(\mathbf{q},\omega+i\delta)\ , \tag{2.18}$$

with

$$P_{\mu\nu}(\mathbf{q},i\omega_n)=-\frac{1}{N}\int_0^\beta d\tau\, e^{i\omega_n\tau}\langle j^p_\mu(\mathbf{q},\tau)j^p_\nu(-\mathbf{q},0)\rangle\ , \tag{2.19}$$

and the diamagnetic term is obtained as

$$K^d_{\mu\nu}(\mathbf{q},\omega)=\begin{cases}\dfrac{4\pi e^2}{c^2N}\displaystyle\sum_{\mathbf{k}}\frac{\partial^2\epsilon_{\mathbf{k}}}{\partial k_i\partial k_j}n_{\mathbf{k}} & (\mu,\nu=i,j=1,\ldots,D)\\ 0 & (\text{otherwise}).\end{cases} \tag{2.20}$$

In Eq. (2.19), $\langle\ \rangle$ denotes the thermal average of the system under no field.

In the ladder diagram approximation,[13,14] as shown diagrammatically in Fig. 1, $P_{\mu\nu}(\mathbf{q},i\omega_m)$ is expressed as

$$P_{\mu\nu}(\mathbf{q},i\omega_m)=\frac{e^2T}{N}\sum_{n,\mathbf{p}}\mathrm{Tr}[\gamma_\mu(p_-,p_+)G(p_+)\times\Gamma_\nu(p_+,p_-)G(p_-)]\ , \tag{2.21}$$

where $p_\pm\equiv(\mathbf{p}\pm\mathbf{q}/2,i\nu_n\pm i\omega_m/2)$; the free vertex $\gamma_\mu(p_-,p_+)$ is given by

$$\gamma_\mu(p_-,p_+)=\begin{cases}\dfrac{\partial\epsilon_{\mathbf{p}}}{\partial p_i}\tau_0 & (\mu=i=1,\ldots,D)\\ \tau_3 & (\mu=0);\end{cases} \tag{2.22}$$

the vertex function $\Gamma_\nu(p_+,p_-)$ satisfies the linear integral equation

$$\Gamma_\nu(p_+,p_-)=\gamma_\nu(p_+,p_-)-\frac{T}{N}\sum_{l,\mathbf{k}}\tau_3G(k_+)\Gamma_\nu(k_+,k_-)G(k_-)\tau_3\tilde V(\mathbf{p}-\mathbf{k})+\tilde V(\mathbf{q})\tau_3\frac{T}{N}\sum_{l,\mathbf{k}}\mathrm{Tr}[\tau_3G(k_+)\Gamma_\nu(k_+,k_-)G(k_-)]\ , \tag{2.23}$$

with $k_\pm\equiv(\mathbf{k}\pm\mathbf{q}/2,i\nu_l\pm i\omega_m/2)$ and $\tilde V(\mathbf{k})=V(\mathbf{k})+v(\mathbf{k})$; the last term on the right-hand side of Eq. (2.23) expresses the vacuum-polarization correction.[14] In our scheme, the effect of the Coulomb interaction $v(\mathbf{p}-\mathbf{k})$ in the second term of Eq. (2.23) is considered to be small. Therefore, below, we will take $\tilde V(\mathbf{p}-\mathbf{k})$ in this term as $V(\mathbf{p}-\mathbf{k})$ (as in the usual non-Bloch treatment[14]) so as to be consistent with Eq. (2.4) [instead of adding $v(\mathbf{k}-\mathbf{q})$ to $V(\mathbf{k}-\mathbf{q})$ in Eq. (2.4)].

This approximation is consistent with the gauge invariance as shown below. With the help of Eq. (2.4), we can show that $\tau_3G^{-1}(p_-)-G^{-1}(p_+)\tau_3$ satisfies the same equation that is constructed for $\sum_{\nu=0}^{D}\Gamma_\nu(p_+,p_-)q_\nu$ [with $q\equiv(\mathbf{q},i\omega_m)$] from Eq. (2.23). Hence, we obtain the generalized Ward identity

$$\sum_{\nu=0}^{D}\Gamma_\nu(p_+,p_-)q_\nu=\tau_3G^{-1}(p_-)-G^{-1}(p_+)\tau_3\ . \tag{2.24}$$

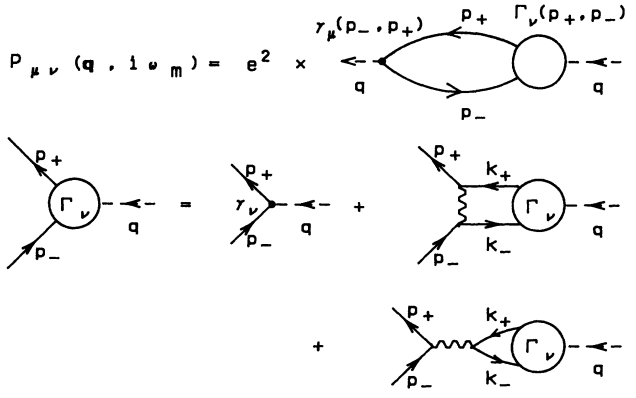

FIG. 1. The equations for the polarizability kernel $P_{\mu\nu}$ and the vertex Γ_ν. Solid and wavy lines represent electron and the sum of the pairing and Coulomb interactions, respectively.

By using this identity and Eq. (2.21), we can easily show (for small q)

$$\frac{4\pi}{c^2}\sum_{\nu=0}^{D}P_{\mu\nu}(\mathbf{q},i\omega_m)q_\nu = -\frac{4\pi e^2}{c^2 N}\sum_{j=1}^{D}\sum_{\mathbf{p}}\frac{\partial^2\epsilon_{\mathbf{p}}}{\partial p_\mu \partial p_j}q_j n_{\mathbf{p}}(1-\delta_{\mu 0}) . \quad (2.25)$$

Finally, from Eqs. (2.17), (2.18), (2.20) and the analytically continued form of Eq. (2.25), we can see that the gauge-invariance condition

$$\sum_{j=1}^{D}K_{\mu j}(\mathbf{q},\omega)q_j - K_{\mu 0}(\mathbf{q},\omega)\omega = 0 , \quad (2.26)$$

is satisfied for the general potential $V(\mathbf{q})$.

III. RESPONSE KERNEL AND PENETRATION DEPTH

Here, by using the ladder diagram approximation examined in Sec. II, we derive the *manifestly* gauge-invariant expression for the electromagnetic response kernel K_{ij} $(i,j=1,\ldots,D)$ at zero frequency, and calculate the penetration depth for the two-dimensional square lattice ($D=2$). We assume that the transfer integral $-t$ and the attractive interaction $-V$ work only between the nearest neighbor sites: $\epsilon_{\mathbf{k}\sigma}=-t\gamma_{\mathbf{k}}$ and $V(\mathbf{k})=-V\gamma_{\mathbf{k}}$ with $\gamma_{\mathbf{k}}=2\,[\cos(k_x a)+\cos(k_y a)]$ and a the lattice constant. The order parameter has the following forms for the possible extended s- and d-wave states: $\Delta_{\mathbf{k}}^{\alpha}=\Delta^{\alpha}w_{\mathbf{k}}^{\alpha}$ $(\alpha=s,d)$ with $w_{\mathbf{k}}^{s}=\cos(k_x a)+\cos(k_y a)$, $w_{\mathbf{k}}^{d}=\cos(k_x a)-\cos(k_y a)$. From Eq. (2.4), we can see that Δ^{α} is determined by

$$1=\frac{1}{N}\sum_{\mathbf{k}}\frac{V(w_{\mathbf{k}}^{\alpha})^2}{2E_{\mathbf{k}}}\tanh\frac{E_{\mathbf{k}}}{2T} \quad (\alpha=s,d) , \quad (3.1)$$

where $E_{\mathbf{k}}=\sqrt{(\epsilon_{\mathbf{k}}-\mu)^2+(\Delta^{\alpha})^2(w_{\mathbf{k}}^{\alpha})^2}$. Here, we note that our potential can be rewritten as

$$V(\mathbf{p}-\mathbf{k})=-V[w_{\mathbf{p}}^{s}w_{\mathbf{k}}^{s}+w_{\mathbf{p}}^{d}w_{\mathbf{k}}^{d}+w_{\mathbf{p}}^{p_x}w_{\mathbf{k}}^{p_x}+w_{\mathbf{p}}^{p_y}w_{\mathbf{k}}^{p_y}] , \quad (3.2)$$

with $w_{\mathbf{k}}^{p_i}=\sqrt{2}\sin(k_i a)$ $(i=x,y)$.

We assume that the solution $\Gamma_i(p_+,p_-)$ $(i=1,\ldots,D)$ to Eq. (2.23) has the form

$$\gamma_i(\mathbf{p})+\textstyle\sum_{j=0}^{3}\sum_{\beta}w_{\mathbf{p}}^{\beta}X_{ij}^{\beta}(\mathbf{q},i\omega_m)\tau_j+Y_{i3}(\mathbf{q},i\omega_m)\tau_3$$

for the superconducting α-wave state ($\alpha=s$ or d). After some manipulation, we can obtain the expressions for X_{ij}^{β} $(\beta=s,d,p_x,p_y)$ and Y_{i3} (the details of these expressions with general $i\omega_m$ and the properties of the collective mode are now in preparation to be published elsewhere). At $\omega=0$, the analytically continued forms of X_{i1}^{β}, X_{i3}^{β}, and Y_{i3} vanish. We expand Eq. (2.23) (written in terms of X_{i2}^{β} and X_{i0}^{β}) up to the order of q^2. We can solve this equation by using Eq. (3.1), integration by parts, and some symmetries. For α-wave state, when we take the limit $q\to 0$, only X_{i2}^{α} has the essential contribution terms of the order $O(q^{-1})$ as

$$X_{i2}^{\alpha}(\mathbf{q},\omega=0)\cong -\frac{2\Delta^{\alpha}\sum_{j=1}^{D}R_{ij}q_j}{\sum_{l,m=1}^{D}q_l R_{lm}q_m}\sqrt{-1} , \quad (3.3)$$

where R_{ij} is given by

$$R_{ij}=\frac{2}{N}\sum_{\mathbf{k}}\frac{\Delta_{\mathbf{k}}^{\alpha}}{E_{\mathbf{k}}^2}\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_i}\left[\Delta_{\mathbf{k}}^{\alpha}\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_j}-(\epsilon_{\mathbf{k}}-\mu)\frac{\partial\Delta_{\mathbf{k}}^{\alpha}}{\partial k_j}\right]\times\left[\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}}+\frac{1}{2E_{\mathbf{k}}}\tanh\frac{E_{\mathbf{k}}}{2T}\right] . \quad (3.4)$$

We have finite values of the order $O(1)$ for $X_{i0}^{p_i}$ $(i=x,y)$. However, these coefficients only correct nonessentially $\gamma_i(\mathbf{k})=\partial\epsilon_{\mathbf{k}}/\partial k_i\tau_0$ in Γ_i, corresponding to that correction of $\epsilon_{\mathbf{k}}$ by the self-energy, which we have neglected in Eq. (2.4). So, we neglect these coefficients, and our solution reduces to that for the pure α-wave potential $V(\mathbf{p}-\mathbf{k})=-Vw_{\mathbf{p}}^{\alpha}w_{\mathbf{k}}^{\alpha}$. [If we retain these corrections, $\partial\epsilon_{\mathbf{k}}/\partial k_j$ in Eqs. (3.4)–(3.6) are replaced by the corrected ones.] It should be noted that the Coulomb potential has no effect on the vertex at $\omega=0$. The rather complicated expression (3.4) can be rewritten as

$$R_{ij}=\frac{1}{N}\sum_{\mathbf{k}}\left[2\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_i}\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_j}\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}}+\frac{\partial^2\epsilon_{\mathbf{k}}}{\partial k_i\partial k_j}n_{\mathbf{k}}\right] . \quad (3.5)$$

Eq. (3.5) can be easily transformed (by integrating the second term by parts) into its original form (3.4). By using Eq. (3.3), we can reduce Eq. (2.21) to

$$P_{ij}(\mathbf{q},0)=\frac{2e^2}{N}\sum_{\mathbf{k}}\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_i}\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_j}\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}} - e^2\frac{(\sum_{l=1}^{D}R_{il}q_l)(\sum_{m=1}^{D}R_{jm}q_m)}{\sum_{l,m=1}^{D}q_l R_{lm}q_m} , \quad (3.6)$$

where the second term on the right-hand side is interpreted as the contribution from the collective mode. Finally, from Eqs. (2.17)–(2.20), (3.5), and (3.6), we obtain the gauge-invariant form for the static total K_{ij} as

$$K_{ij}(\mathbf{q},0)=\frac{4\pi e^2}{c^2}\left[R_{ij}-\frac{(\sum_{l=1}^{D}R_{il}q_l)(\sum_{m=1}^{D}R_{jm}q_m)}{\sum_{l,m=1}^{D}q_l R_{lm}q_m}\right] . \quad (3.7)$$

This form is easily seen to satisfy the gauge-invariance condition at zero frequency: $\sum_{j=1}^{D}K_{ij}q_j=0$ (K_{ij} becomes purely transverse). Furthermore, it is considered to be the generalized form of the hydrodynamical kernel[16] for the singlet superconducting state as seen later; R_{ij} can be interpreted as the superfluid density per effective mass. If R_{ij} has the simple form $R_{ii}\delta_{ij}$ as in our case, it follows that K_{ij} is of the form

$$K_{ij}(\mathbf{q},0)=\frac{4\pi e^2}{c^2}\left[\delta_{ij}-\frac{q_i q_j}{q^2}\right]R_{ii} \ . \tag{3.8}$$

The London penetration depth of our two-dimensional square-lattice system ($D=2$) is obtained from Eq. (3.7) or (3.8) as

$$\lambda_L(T)^{-2}=\frac{4\pi e^2}{c^2}R_{xx}(T)$$
$$=\frac{4\pi e^2}{c^2}\frac{1}{N}\sum_{\mathbf{k}}\left[2\left[\frac{\partial\epsilon_{\mathbf{k}}}{\partial k_x}\right]^2\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}}+\frac{\partial^2\epsilon_{\mathbf{k}}}{\partial k_x^2}n_{\mathbf{k}}\right] . \tag{3.9}$$

This expression coincides with that obtained in the previous report[12] for the transverse field in the Hartree-Fock approximation. Here, however, Eq. (3.7) or (3.8) guarantees that the expression (3.9) is really gauge invariant. The expression (3.9) also essentially coincides with the pure limit of Ohkawa's more general one.[10,11] The gauge invariance, however, has not been examined explicitly in Ohkawa's papers.[10,11]

For general n_h, we calculate $\lambda_L(T)$ numerically. The chemical potential μ is determined by Eq. (2.5). At $T=0$ K, only the second term in Eq. (3.9) contributes, and the resultant $\lambda_L(0)$ as a function of n_h is almost not affected by the anisotropy of $\Delta_{\mathbf{k}}^{\alpha}$ as shown in Fig. 2. We also see the symmetrical dependence on hole and electron densities.

The resultant behavior of $\lambda_L(T)/\lambda_L(0)$ as a function of T/T_c is affected by the anisotropy of $\Delta_{\mathbf{k}}^{\alpha}$ as shown in Fig. 3 (in the figures, the results for the usual s-wave state and the empirical two-fluid model are added for comparison). The extended s-wave behavior is almost equal to the usual s-wave one, while the d-wave one substantially deviates from the s-wave one for all $T<T_c$. These results for the fixed n_h are almost identical with those obtained for the fixed μ in the previous report.[12] The d-wave (extended s-wave) gap has nodes (no node) on the Fermi surface and yields T-linear (exponential) behavior of $\lambda_L(T)$ in the low-temperature region.

For the d-wave state, the similar behavior has already been obtained by the recent hydrodynamical treatment[6] and by the previous works for Bloch electrons.[10-12] The result in Fig. 3 seems to show that the experimental results[1-4] for the high-T_c superconductors have the s-wave character even in the near-T_c region (where Ohkawa[10] asserts the existence of the d-wave state to explain the NMR experiments). If we take the s-wave state for high-T_c superconductivity, however, the experimental simple power-law result for $\lambda_L(T)$ at very low temperature[5] and the NMR experiments[10] remain unresolved.

For different n_h in Fig. 3, $\lambda_L(T)/\lambda_L(0)$ differs by a small amount, since the available part of $\mathbf{k}$ space in Eq. (3.9) differs a little for different μ. However, no special feature due to the van Hove singularity has not been obtained even at $n_h=1$ (where the Fermi level is located at the point of this singularity) as seen in Fig. 3.

We next compare the above results with those of the following effective mass approximation. For holelike (electronlike) carriers with hole (electron) mass $m_h=1/2ta^2$ ($m_e=1/2ta^2$), Eq. (3.5) can be approximately rewritten as

$$R_{ij}\cong\frac{n_{ij}^{\xi}}{m_{\xi}}\equiv\frac{1}{m_{\xi}}\left[\frac{1}{N}\sum_{\mathbf{k}}\frac{2k_ik_j}{m_{\xi}}\frac{\partial f(E_{\mathbf{k}})}{\partial E_{\mathbf{k}}}+n_{\xi}\delta_{ij}\right]$$
$$(\xi=e,h) \ , \tag{3.10}$$

where n_{ij}^{ξ} is the superfluid density tensor for holelike ($\xi=h$) or electronlike ($\xi=e$) carriers; $w_{\mathbf{k}}^{\alpha}$ as well as ϵ_k in $E_{\mathbf{k}}$ are replaced by their suitably expanded forms. For $\xi=h$, we interpret k_i as $k_i'=k_i\pm\pi/a$, and we should make this approximation only after replacing $n_{\mathbf{k}}$ in Eq. (3.5) safely by $-(2-n_{\mathbf{k}})$. Hence, Eqs. (3.7) and (3.9) are approximated by

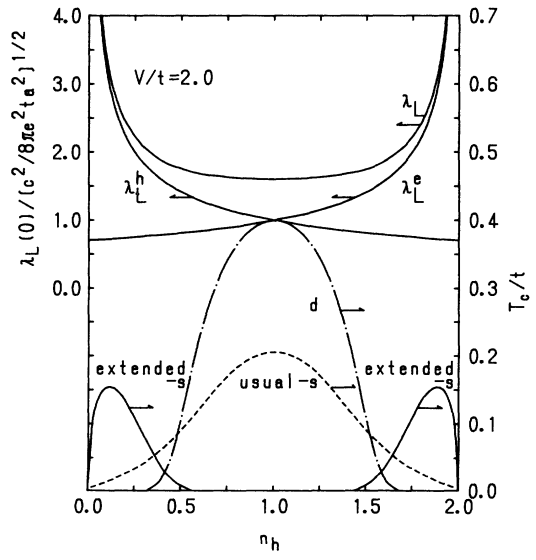

FIG. 2. The penetration depth $\lambda_L(0)$ and the transition temperature T_c as functions of hole density n_h. The result for the effective hole- (electron-) mass approximation $\lambda_L^h(0)$ [$\lambda_L^e(0)$] is also shown.

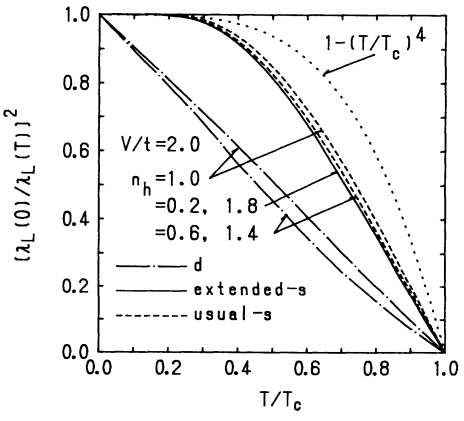

FIG. 3. The reduced penetration depth $\lambda_L(T)/\lambda_L(0)$ as a function of reduced temperature T/T_c.

$$K_{ij}^{\xi}(\mathbf{q},0)\simeq\frac{4\pi e^2}{m_{\xi}c^2}\left[n_{ij}^{\xi}-\frac{(\sum_{l=1}^{D}n_{il}^{\xi}q_l)(\sum_{m=1}^{D}n_{jm}^{\xi}q_m)}{\sum_{l,m=1}^{D}q_l n_{lm}^{\xi}q_m}\right], \tag{3.11}$$

and

$$\lambda_L^{\xi}(T)^{-2}=\frac{4\pi n_{xx}^{\xi}(T)e^2}{m_{\xi}c^2}, \tag{3.12}$$

respectively, for holelike ($\xi=h$) or electronlike ($\xi=e$) carriers. For $\xi=h$ ($\xi=e$), Eq. (3.11) has the hydrodynamical form[16] but with *hole* (electron) mass and superfluid-*hole* density $n_{ij}^{h}(T)$ [superfluid-electron density $n_{ij}^{e}(T)$] given by Eq. (3.10). In view of this, our expression (3.7) for the original kernel is considered to be the generalized form of the hydrodynamical kernel for the singlet superconducting state. Equation (3.12) for $\xi=h$ also has the London form but with *hole* mass m_h and superfluid-*hole* density $n_{xx}^{h}(T)$.

At $T=0$ K, $n_{xx}^{\xi}(T)$ in Eq. (3.12) is equal to n_{ξ} as seen from Eq. (3.10); this result for $T=0$ K coincides with that already obtained by other treatments.[7,8] As seen in Fig. 2, in the holelike-carrier (electronlike-carrier) region of low (high) n_h, the resultant $\lambda_L^{h}(0)$ [$\lambda_L^{e}(0)$] as a function of n_h coincides with the original $\lambda_L(0)$ calculated from Eq. (3.9). These, however, deviate from the original $\lambda_L(0)$ as the carrier densities increase. Especially, for *low* n_h, we *cannot* use the effective *electron*-mass approximation ($\xi=e$) with n_e as far as the absolute value of $\lambda_L(0)$ is concerned.

At finite temperature, the resultant behavior of the ratio $\lambda_L^{h}(T)/\lambda_L^{h}(0)$ as a function of T/T_c in Fig. 4 qualitatively agrees with the original one shown in Fig. 3.

IV. CONCLUSION AND DISCUSSION

In this work, we have examined the Meissner effect in the anisotropic narrow-band superconductor consisting of Bloch electrons, within the framework of the ladder diagram approximation. We have slightly corrected the previous charge-conserving expression[9-12] for the current carried by Bloch electron under the magnetic field (though this correction makes no change in our result). By using this expression, we have shown that the ladder diagram approximation is consistent with the gauge invariance also in our system. The obtained gauge-invariant static electromagnetic response kernel K_{ij} has the generalized form of the hydrodynamical kernel;[16] if the carriers are holelike, the obtained form is approximated by the hydrodynamical one but with *hole* mass m_h and superfluid-*hole* density tensor; the Coulomb potential between electrons has no effect on K_{ij} at zero frequency. The resultant expression for the London penetration depth λ_L coincides with that obtained for the transverse field within the pairing approximation; for low n_h, our λ_L is approximated by the London form but with *hole* mass m_h and superfluid-*hole* density, even at finite T.

We have calculated λ_L for the two-dimensional square lattices as a function of hole density n_h and temperature T. The resultant $\lambda_L(0)$ has the symmetrical dependence on hole and electron densities, and it is almost not affected by the anisotropy of the order parameter. The behavior of $\lambda_L(T)/\lambda_L(0)$ as a function T/T_c for the extended s-wave state is almost equal to the usual s-wave one, while the d-wave one substantially deviates from the s-wave one for all $T<T_c$. The high-T_c superconductors seem to show the s-wave character even in the near-T_c region as far as $\lambda_L(T)$ is concerned. No special feature due to the van Hove singularity has not been obtained in the T dependence of λ_L.

We have compared the above results with those in the effective hole- and electron-mass approximations. The results for $\lambda_L(0)$ in these approximations coincide with our original one in the corresponding low carrier-density regions, but deviate from it as the carrier densities increase. It should be noted that we *cannot* use the effective *electron*-mass approximation (with electron density) when we calculate λ_L for *holelike* carriers, as far as the absolute value is concerned. The T-dependence of $\lambda_L(T)/\lambda_L(0)$ in the effective *hole*-mass approximation qualitatively agrees with that of the original one.

There are some problems in the treatment of this work. It has been assumed that there is no renormalization effect in the band structure (except for the small transfer t) and that two electrons can occupy the same site. This should be altered to explain the properties of the real high-T_c oxides in which the strong Coulomb repulsion exists on Cu sites. The effects of impurities, the interlayer hopping (causing anisotropic mass) and coupling, and the strong coupling to the certain bosons have also been neglected. Including these effects remains a future problem.

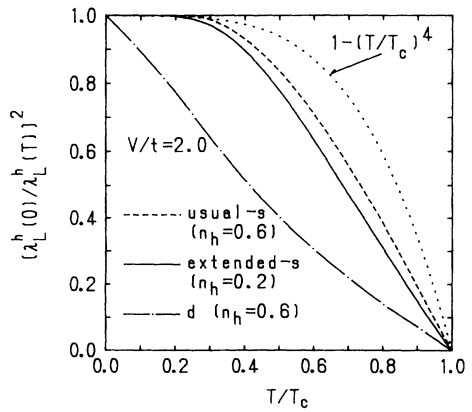

FIG. 4. The reduced penetration depth in the effective hole-mass approximation $\lambda_L^{h}(T)/\lambda_L^{h}(0)$ as a function of reduced temperature T/T_c.

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