Model for layered superconductors: From Josephson coupling to three-dimensional behavior

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(Received 19 July 1993)

We present a model for layered superconductors, where the order parameter between the layers is nonzero, due to proximity effects. In the limit of decoupled layers, this model gives all the terms of the usual Lawrence-Doniach free energy, as well as new terms arising from the existence of an order parameter between the layers. The corresponding Josephson coupling is found to be field dependent. The upper critical field normal to the layers has positive curvature as a function of T, and decreases with increasing interlayer distance, contrary to the predictions of the Lawrence-Doniach model. In addition, the anisotropy deduced from the critical fields H_{c2} is markedly different from the low-field anisotropy.

I. INTRODUCTION

The high-T_c superconductors are layered compounds made of alternating superconducting (S) layers, the CuO_2 planes, and insulating or metallic layers. They pertain to a larger class of materials including cuprate superconductor artificial multilayers Y 1:2:3/Pr 1:2:3 (Ref. 1), conventional superconductor multilayers with modulated composition (Ref. 2), low-temperature layered compounds such as chalcogenides (Ref. 3), and quasi-two-dimensional (quasi-2D) organic superconductors (Ref. 4). One of the main features of these superconductors is the weak interlayer coupling, leading to electronic anisotropy, which becomes extreme in the Bi 2:2:1:2 (Ref. 5), the Y/Pr multilayers (Ref. 1), and the organic compound $(BEDT\text{-}TTF)_2Cu(SCN)_2$ (Ref. 6). This results in a short effective coherence length $\xi_c^{\rm eff}$ across the layers, which becomes smaller than the interlayer distance d.

The simplest description relies on the three-dimensional Ginzburg-Landau (3DGL) model, and accounts for the anisotropy through an electronic mass anisotropy constant. It is defined as $\Gamma=(M/m)^{1/2}$, where m and M are masses along and across the layers, respectively. This constant is involved in the superconducting properties, such as magnetization, critical fields, critical currents, and resistivity (Ref. 7). However, specific features of weakly coupled layered systems, such as the quasi-2D character of the superconducting properties, lie beyond this description.

The simplest model able to account for such a behavior is the Lawrence-Doniach (LD) model, which retains the GL model for order parameter variations along each layer but couples them through Josephson tunneling (Ref. 8). This model differs from the 3DGL one mainly in its consequences concerning thermal fluctuations in the mixed (vortex) state (melting of the vortex lattice, Kosterlitz-Thouless transition), but also in specific features for fields very close to the layer direction, e.g., kinks, the lock-in transition, and the intrinsic pinning by the layers (Ref. 9). In the weak-field and low temperature regime the LD model does not differ much from the 3DGL model, provided an effective mass anisotropy $\Gamma_{\rm eff}$ is defined from the Josephson coupling. One should emphasize that this coupling is assumed to be temperature independent, so the same is true for the anisotropy constant. Values of $\Gamma_{\rm eff}$ of the order of 10^2 to 10^3, obtained recently by torque magnetometry, point towards a very weak Josephson coupling in compounds like Bi 2:2:1:2 (Refs. 5, 6, and 10).

However, even the LD model, in its usual formulation, misses some important points. First, it assumes infinitely thin layers, since it describes the order parameter as a quantity that is zero between the layers, and equal to a function $\Psi_n(x,y)$ on the nth layer, where x and y are the coordinates in the layers. The discreteness of Ψ across the layers leads to a dimensional crossover at a temperature T^* below T_c, where $\xi_c^{\rm eff}(T^*)$ is of order $d/\sqrt{2}$ (Ref. 3). Below T^*, the parallel orbital critical field $H_{c2}^{\parallel}$ diverges in the LD model, since such a field cannot destroy superconductivity in the layers. Modifying the LD model by accounting for a finite layer thickness d_0 leads indeed to a finite orbital $H_{c2}^{\parallel}$ (Ref. 11). Besides, in lower fields vortices may penetrate the layers if d_0 is large enough, just as in the artificial Y 1:2:3/Pr 1:2:3 superlattices (Ref. 12).

Secondly, the Josephson coupling is taken as a constant. However, it is known from the microscopic derivation of the Josephson tunneling current in a

superconductor-insulator-superconductor (SIS) or superconductor–normal-metal–superconductor (SNS) junction that this coupling occurs through a proximity effect (Ref. 13). The order parameter Ψ extends through the junction within a region depending on the characteristics of the superconducting and junction materials, such as the superconducting coherence length $\xi_S(T)$ or the normal coherence length $\xi_N(T)$. Therefore, the coupling should not be a universal quantity but may depend on field and temperature, especially in the SNS case, where Ψ can extend over a large length. This may be important for high-temperature compounds involving metallic layers that separate the superconducting layers, or in superlattices such as Y 1:2:3/(Y,Pr) 1:2:3.

The main consequence of this should be a temperature and field dependence of the effective anisotropy. Indeed, in the low-field regime the anisotropy directly reflects the Josephson coupling and can therefore be extremely large if the latter is weak. On the other hand, a high enough magnetic field is known to decrease the proximity effect, thus affecting the Josephson coupling and the anisotropy (Ref. 14). Moreover, in copper oxide superconductors or superlattices, the superconducting or normal layers are themselves made of coupled atomic planes, so they are anisotropic, and this should be distinguished from the effective anisotropy of the overall structure. The same kind of distinction has to be made between the effective coherence length ξ_c^{eff} of the layered structure and the coherence lengths ξ_S or ξ_N of the individual layers. For instance, in the Lawrence-Doniach model, below the crossover temperature T^*, ξ_c^{eff} is much smaller than d. In this case ξ_c^{eff} is meaningless and the microscopic variations of the order parameter are indeed governed by the parameters ξ_S or ξ_N.

Thus a more general model is required, which may describe the coupling by proximity effect and may be able to yield both the LD model for weak coupling and a true 3D model for strong coupling, depending on the parameters. Such a model was recently proposed by Theodorakis (Ref. 15). It assumes a z-dependent GL parameter α, and a uniform mass M for electrons along the normal to the layers $\hat{z}$. This model is three dimensional in nature, but it is able to treat the proximity effects in a simplified manner. In Ref. 15, α was taken to be small between the layers, without changing sign. This resulted in the possibility of a LD behavior at low temperature and a 3D behavior close to T_c. In the LD regime, additional terms appeared, which may introduce new physics, such as the alternation of the sign of the order parameter from layer to layer. Note that a similar assumption of spatially dependent Ginzburg-Landau parameters was studied numerically in Ref. 16.

In this paper we extend the model by including the possibility of $\alpha(z)$ changing sign. Thus the coefficient $\alpha(z)$ of the quadratic term is negative for S (superconducting) layers, and positive for N (normal) or I (insulating) layers. As a first stage, we focus on the case of very thin layers. We explore in Sec. II the zero-field behavior of the order parameter, and obtain an effective Josephson coupling. In Sec. III we show explicitly that our model can be mapped onto an effective LD model, with an effective field and temperature-dependent Josephson coupling. Furthermore, we obtain once more the new kinetic term of Ref. 15, whereby the gradients of neighboring order parameters are coupled. The upper critical field is calculated in Sec. IV for both perpendicular and parallel configurations. Finally, Sec. V is concerned with a qualitative summary of our work, and some experimental predictions of our model.

II. THE MODEL AND ZERO-FIELD RESULTS

Let us write the free-energy functional for our model in zero field:

$$\int\int\int dx\,dy\,dz\left[\alpha(z)|\Psi|^2+\beta|\Psi|^4/2+\frac{\hbar^2}{2M}\left|\frac{\partial\Psi}{\partial z}\right|^2+\frac{\hbar^2}{2m}|\nabla_{\parallel}\Psi|^2\right], \tag{2.1}$$

where $\nabla_{\parallel}$ is the gradient parallel to the layers, and $\Gamma_0=(M/m)^{1/2}$ is the bare anisotropy. This bare anisotropy Γ_0 reflects the anisotropy of individual layers and must be carefully distinguished from the effective anisotropy Γ_{eff} that is measured and that we shall encounter later.

One should emphasize that even though we adopt here a continuous description, the gradients in the z direction may be large and they may generate an important amplitude modulation of the order parameter at the scale of the layered structure. This will be shown to be the case, under some conditions concerning the variation of the "source" term $\alpha(z)$.

The parameter $\alpha(z)$ takes a distinctive form, involving δ functions:

$$\alpha(z)=-\alpha_1\sum_n\delta(z/d-n)+\alpha_2\ , \tag{2.2}$$

where α_1 and α_2 are positive temperature-dependent quantities, while d is the interlayer spacing. We note that outside the δ functions $\alpha(z)$ is always positive, and thus the order parameter tends to be very small. In fact, it would be exactly zero, were it not for the α_1 terms, which create superconductivity on the layers at $z=nd$. Physically, this means that the source of superconductivity lies within the layers, but the finite stiffness of the order parameter makes it extend outside the layers. We simply assume here that the electrons have the same masses m along and M across the layers also when crossing the intervening space. The δ-function description is valid, provided we neglect the variation of the order parameter within the layers, as is indeed the case for very thin layers. δ functions have been used in the past as well (Ref. 17), but in the entirely different context of superconducting twinning planes along the layers. In the general case, the sum of δ functions in Eq. (2.2) can be considered, for instance, as the limit of a sum of functions with finite width, like the Gaussian shapes assumed in Ref. 15.

Expression (2.1) can be rewritten in a dimensionless form, by expressing the energy in units of $d^3\alpha_2^2/\beta$, $|\Psi|$ in units of $\sqrt{\alpha_2/\beta}$, and x, y, and z in units of d:

$$\int\int\int dx\,dy\,dz\left\{[1-\alpha(T)\sum_n \delta(z-n)]|\Psi|^2 + |\Psi|^4/2+\nu\Gamma_0^2|\nabla_\parallel\Psi|^2+\nu\left|\frac{\partial\Psi}{\partial z}\right|^2\right\}, \tag{2.3}$$

with $\Gamma_0^2=M/m$, $\alpha(T)=\alpha_1/\alpha_2$, and $\nu=\hbar^2/2M\alpha_2 d^2=\xi^2/d^2$, where ξ is the coherence length, which determines the decay of the order parameter outside the layers. In the limit $\nu\ll 1$ the order parameter is well localized on the layers, giving rise to a quasi-2D layered superconductor while in the opposite limit $\nu\gg 1$ it is nearly uniform, leading to a true 3D superconductor.

The significance of the parameter $\alpha(T)$ can be made clearer by considering for instance d_0 to be the thickness of the superconducting layers, and by making the assumption $\alpha(z)=\alpha_S$ for $nd-d_0/2<z<nd+d_0/2$, while $\alpha(z)=\alpha_N$ for $nd+d_0/2<z<(n+1)d-d_0/2$. If d_0 is much smaller than d, this $\alpha(z)$ variation can be modeled by Eq. (2.2), provided $\alpha_2=\alpha_N$ and $\alpha_1=(\alpha_N-\alpha_S)d_0/d$. This expresses the model parameters α_1 and α_2 in terms of the physical parameters α_N and α_S for "normal" and "superconducting" regions, respectively.

Let us find the state of equilibrium for this free energy, assuming a real periodic order parameter that is maximal on the layers. Afterwards we shall examine the case of a complex order parameter, obtaining a current of the Josephson type, and hence an effective Josephson coupling between the layers.

We first solve the problem assuming a real order parameter $\Psi=\psi(z)$ that varies only along the z axis. The GL equation reads

$$\nu\frac{\partial^2\Psi}{\partial z^2}=\left[1-\sum_n \alpha(T)\delta(z-n)\right]\Psi+|\Psi|^2\Psi\ . \tag{2.4}$$

The solution $\psi(z)$ will be periodic, and subject to the boundary condition

$$-\alpha(T)\Psi(n)=\nu\left[\frac{\partial\Psi}{\partial z}(n+)-\frac{\partial\Psi}{\partial z}(n-)\right]. \tag{2.5}$$

The order parameter attains the same maximum value ψ_0 on all the layers, and the same minimum value a halfway in between. Of course, $\psi(z)$ has a kink on each layer, due to the δ functions.

The first integral of Eq. (2.4), obtained by multiplying it by $\partial\psi/\partial z$ and integrating, when combined with the boundary condition leads to

$$\alpha(T)\psi_0/2=[\nu(\psi_0^2-a^2)(1+\psi_0^2/2+a^2/2)]^{1/2}\ . \tag{2.6}$$

A further integration of that first integral leads to the *exact* solution of Eq. (2.4),

$$a/\psi(z)=\mathrm{cn}\left[\sqrt{(1+a^2)/\nu}|z-n-1/2|,\frac{2+a^2}{2+2a^2}\right], \tag{2.7}$$

for $n\le z\le n+1$, where cn is a Jacobian elliptic function. These expressions are valid for any value of ν, large or small.

In the limit of a well-localized order parameter, or close to T_c, we expect a to be small. Then Eq. (2.7) reduces to

$$\psi(z)\approx a\cosh(|z-n-1/2|/\sqrt{\nu})\ , \tag{2.8}$$

whereby

$$\psi_0\approx a\cosh(1/2\sqrt{\nu})\ . \tag{2.9}$$

This equation is valid for all values of ν, as long as a is small. Hence, it is also valid at T_c. When we combine it with Eq. (2.6) it yields $\alpha(T_c)=2\sqrt{\nu}\tanh(1/2\sqrt{\nu})$, as well as ψ_0^2. This equation defines the effective T_c of the structure. As expected, T_c is less than T_c^0, the intrinsic critical temperature of the superconducting layer, defined by $\alpha_1=0$.

Actually, let us now restrict ourselves to the limit $\nu\ll 1$, corresponding to the case $\psi_0\gg a$. Then Eqs. (2.6) and (2.9) give $\psi_0^2=-2+\alpha^2(T)/2\nu$, as well as

$$a^2\approx 4e^{-1/\sqrt{\nu}}[-2+\alpha^2(T)/2\nu]\ . \tag{2.10}$$

We observe that, since ψ_0 and a are zero at T_c, $\alpha^2(T_c)\approx 4\nu$, for a layered system at small ν. We have thus the full periodic solution for a real Ψ.

Let us now proceed by examining the case of a complex order parameter. We shall consider again the periodic state, and we shall let $\Psi=\psi e^{i\phi}$. Then $\psi(z)$ attains its maximum value ψ_0 on the layers, while its minimum value a occurs halfway between them. Equation (2.5) implies that $\partial\phi/\partial z$ is continuous everywhere, just like ψ, but it has kinks on the layers. In fact, the imaginary part of Eq. (2.4) yields

$$\psi^2\frac{\partial\phi}{\partial z}=j\ , \tag{2.11}$$

where j is the same constant everywhere, due to the periodicity and continuity of ψ. This constant is simply related to the Ginzburg-Landau current, which is equal to $2e\hbar j\alpha_2/M\beta d$. The first integral of the real part of Eq. (2.4) gives the exact relation

$$\int_{a^2/\psi^2}^1 dx(x-x^2)^{-1/2}\left[x+\frac{a^2}{2}(1+x)+\frac{\nu j^2x^2}{a^4}\right]^{-1/2}=\frac{2}{\sqrt{\nu}}|z-n-1/2|\ . \tag{2.12}$$

The boundary condition of Eq. (2.5) becomes

$$\alpha(T)\psi_0=2[\nu(\psi_0^2-a^2)(1+\psi_0^2/2+a^2/2+\nu j^2/a^2\psi_0^2)]^{1/2}\ . \tag{2.13}$$

We shall now make the approximations $\nu\ll 1$, $a\approx 0$, and $\psi_0\gg a$, corresponding to the superconductivity being localized in the layers. Then we may neglect the a^2 terms within the integrand of Eq. (2.12), to obtain

$$\psi(z)=a\left[\cosh^2(|z-n-1/2|/\sqrt{\nu})+\frac{\nu j^2}{a^4}\sinh^2(|z-n-1/2|/\sqrt{\nu})\right]^{1/2}. \tag{2.14}$$

Integrating the relation (2.11) from n to $n+1$, we find

$$j\sqrt{\nu}/a^2=\tan(\Delta\phi/2)\ , \tag{2.15}$$

with $\Delta\phi=\phi_{n+1}-\phi_n$, and

$$a/\psi_0=2e^{-1/2\sqrt{\nu}}\cos(\Delta\phi/2)\ . \tag{2.16}$$

ψ_0^2 can be found from Eq. (2.13) to be $\psi_0^2=-2+\alpha^2(T)/2\nu$, leading thus to the Josephson equation

$$j=\frac{2}{\sqrt{\nu}}e^{-1/\sqrt{\nu}}\psi_0^2\sin\Delta\phi\ . \tag{2.17}$$

The Ginzburg-Landau current is proportional to $\psi^2(\partial\phi/\partial z)$. From Eq. (2.11) then it must be proportional to j, and hence also proportional to $\sin\Delta\phi$, through Eq. (2.17). It is therefore of the Josephson type. In the LD model, such a current is derived from an energy term $\eta|\Psi_{n+1}-\Psi_n|^2$, where η is the Josephson coupling. Therefore, in this context, we may consider the current as being due to precisely such a term in our free energy, where in fact η turns out to be

$$\eta=\frac{\hbar^2}{Md^2}\frac{1}{\sqrt{\nu}}e^{-1/\sqrt{\nu}}\ . \tag{2.18}$$

In terms of our dimensionless quantities, then, this free-energy term is

$$2\sqrt{\nu}e^{-1/\sqrt{\nu}}|\Psi_{n+1}-\Psi_n|^2\ . \tag{2.19}$$

This is valid at any temperature, but in zero magnetic field. Note that the coefficient falls exponentially as the interlayer distance increases. This means that the effective anisotropy parameter Γ_{eff} derived from the Josephson coupling is exponentially large. Such a coupling will be obtained in fact at H_{c2} and, hence, at T_c as well, in Sec. III, where we show that our model is mapped exactly onto an effective Lawrence-Doniach model in the limit $\nu\ll1$, with the novel terms of Ref. 15 appearing as well.

III. THE EXACT MAPPING AT H_{c2}

As a central result of this paper, we present here a model that leads to all the usual LD terms, as well as the novel terms of Ref. 15, at least in the limit $\nu\ll1$. Our order parameter is continuous everywhere and, hence, nonzero between the layers, in contrast to the LD model, and in agreement with Ref. 15. However, our exact results improve the variational results of the latter reference for the $\nu\ll1$ case.

We shall achieve our goal of showing that the usual LD terms are obtained if we can map the free energy of our model onto the LD model. Indeed, since $\Psi(z)$ is determined uniquely everywhere by the value Ψ_n on the layer through the continuity condition $\Psi(n)=\Psi_n$, we can generalize the equilibrium solution to nonequilibrium states, where Ψ_n can take any complex value. This is especially useful in the $\nu\ll1$ limit, since it provides a way of determining the effective Josephson coupling in our model.

It is easiest to obtain the mapping mentioned above at H_{c2}, where Ψ is small, and where the nonlinear free-energy terms can be dropped. Then it is possible to express the free energy as a function of the Ψ_n's.

At H_{c2}, the free energy is

$$\int_{-\infty}^{+\infty}dx\,dy\,dz\left[\left(1-\alpha(T)\sum_n\delta(z-n)\right)|\Psi|^2+\nu\Gamma_0^2|\Pi_\parallel\Psi|^2+\nu|\Pi_z\Psi|^2\right]\ , \tag{3.1}$$

with $\Pi=-i\nabla-A$, where $|\Psi|$ is always expressed in units of $\sqrt{\alpha_2/\beta}$, the free energy in units of $d^3\alpha_2^2/\beta$, the vector potential A in units of $\hbar c/2ed$, and the magnetic field in units of $\hbar c/2ed^2$. In the gauge $A=Hx\hat{y}$, where the field is perpendicular to the layers, the equation that minimizes the free energy is separable. Thus the order parameter is simply a product of a function of x and y, and of a function of z. We shall think of the order parameter then as attaining the value $\Psi_n(x,y)$ on the nth layer, while it achieves a minimum in the z direction at $z=n+b_n$, with b_n a constant in [0,1]. More precisely,

$$\Psi=f_n(x,y)\cosh[\gamma(z-n-b_n)]\ , \tag{3.2}$$

for $n\le z\le n+1$. The actual value of the constant γ will be determined by the x,y part of the equation of motion.

The continuity of Ψ at $z=n$ gives

$$\Psi_n=f_n\cosh(\gamma b_n)=f_{n-1}\cosh[\gamma(1-b_{n-1})]\ . \tag{3.3}$$

This relation, valid for all x and y, leads also to

$$f_n\sinh(\gamma b_n)=(\coth\gamma)\Psi_n-\Psi_{n+1}/\sinh\gamma \tag{3.4}$$

and

$$f_n\sinh[\gamma(1-b_n)]=(\coth\gamma)\Psi_{n+1}-\Psi_n/\sinh\gamma\ . \tag{3.5}$$

These relations also hold if the f_n's and Ψ_n's are replaced by their gradients.

Let us calculate then the resulting free energy, by inserting Eq. (3.2) in Eq. (3.1), and integrating from $z=n$ to $z=n+1$. We shall present this calculation in some greater detail for a particular term, say the kinetic energy along the layers. After the integration over z, this term takes the form

$$\nu\Gamma_0^2\int_{-\infty}^{+\infty}dx\,dy\,|\Pi_\parallel f_n|^2\left[\frac{1}{2}+\frac{1}{2\gamma}\sinh[\gamma(1-b_n)]\cosh[\gamma(1-b_n)]+\frac{1}{2\gamma}\sinh(\gamma b_n)\cosh(\gamma b_n)\right]\ . \tag{3.6}$$

Combining Eqs. (3.3) and (3.4) gives

$$|\Pi_\parallel f_n|^2\sinh(\gamma b_n)\cosh(\gamma b_n)=(\Pi_\parallel\Psi_n)^*[\coth\gamma(\Pi_\parallel\Psi_n)-(\Pi_\parallel\Psi_{n+1})/\sinh\gamma]\ . \tag{3.7}$$

We see already the appearance of the terms $(\Pi_\parallel\Psi_n)(\Pi_\parallel\Psi_{n+1})^*+\text{c.c.}$ of Ref. 15, which couple the gradients of the order parameters of neighboring layers. Similar repeated use of Eqs. (3.3), (3.4), and (3.5) gives then that the total free energy of Eq. (3.1) is exactly equal to

$$\sum_n \int_{-\infty}^{+\infty} dx\,dy \left\{ A(|\Psi_n|^2+|\Psi_{n+1}|^2)+\nu_{\text{eff}}|\Psi_n-\Psi_{n+1}|^2+K_1(|\Pi_\parallel\Psi_n|^2+|\Pi_\parallel\Psi_{n+1}|^2) \right.$$
$$\left. +K_2[(\Pi_\parallel\Psi_n)(\Pi_\parallel\Psi_{n+1})^*+(\Pi_\parallel\Psi_n)^*(\Pi_\parallel\Psi_{n+1})] \right\}, \tag{3.8}$$

where

$$A=-\alpha(T)/2+(\nu\gamma/2+1/2\gamma)\tanh(\gamma/2) + \frac{1-\nu\gamma^2}{4}\Big/\cosh^2(\gamma/2)\,, \tag{3.9}$$

$$\nu_{\text{eff}}=(1/2\gamma+\nu\gamma/2)/\sinh\gamma-\frac{1-\nu\gamma^2}{2}\cosh\gamma/\sinh^2\gamma\,, \tag{3.10}$$

$$K_1=\nu\Gamma_0^2\left[\frac{\coth\gamma}{2\gamma}-\frac{1}{2\sinh^2\gamma}\right], \tag{3.11}$$

$$K_2=\nu\Gamma_0^2(\coth\gamma-\gamma^{-1})\frac{1}{2\sinh\gamma}\,. \tag{3.12}$$

The x,y part of the equation of motion shows that all the f_n's behave like $\exp(-Hx^2/2)$, confirming thus the implication of Eq. (3.3) that f_n/f_{n-1} is independent of x and y. It also gives

$$\gamma=\sqrt{\nu^{-1}+\Gamma_0^2 H}\,. \tag{3.13}$$

This important result shows that ν_{eff}, our effective Josephson coupling, is field dependent at H_{c2}, exactly as proposed in Ref. 18. Furthermore, at $T=T_c$, H_{c2} is zero, and thus

$$\nu_{\text{eff}}(T_c)=2\sqrt{\nu}e^{-1/\sqrt{\nu}}\,, \tag{3.14}$$

if $\nu=\xi^2/d^2$ is taken to be quite small.

However, the effective Josephson coupling was derived earlier for zero fields and for all temperatures. We see that the result of Eq. (3.14) at T_c and zero field is precisely the one given by Eq. (2.19) as well. Thus this is a useful check of our calculation.

For a given temperature T different from T_c, Eqs. (2.19), (3.10), and (3.13) indicate that $\nu_{\text{eff}}(H=0)$ is different from $\nu_{\text{eff}}(H=H_{c2})$. Thus clearly ν_{eff} is field dependent. The interpretation of this field dependence can be seen from Eqs. (3.10) and (3.13): an increasing field reduces the order parameter between the S layers. Indeed, for small ν an increasing field increases γ, and hence the exponential factors in (3.10) tend to *decrease* ν_{eff} from its zero-field value. We should stress that this field dependence can be considered as a T dependence as well, since one moves along the $H_{c2}(T)$ line. Thus ν_{eff} *increases* as T approaches T_c. Furthermore, Eq. (2.19) indicates that ν_{eff} may have an additional temperature dependence as well, if α_2 and thus ν is temperature dependent.

As far as the effective anisotropy Γ_{eff} of the system is concerned, it appears in the coefficient $\nu_{\text{eff}}\Gamma_{\text{eff}}^2$ of the term $|\Pi_\parallel\Psi_n|^2$, and is given hence by the relation $\Gamma_{\text{eff}}^2=K_1/\nu_{\text{eff}}$. So at $H=0$, for small ν,

$$\Gamma_{\text{eff}}\approx\frac{\Gamma_0\sqrt{\nu}}{2}e^{1/2\sqrt{\nu}}\,. \tag{3.15}$$

As the field increases, γ increases. Since γ is quite large for small ν, the exponentials dominate the behavior of Γ_{eff}, and hence Γ_{eff} increases with increasing field (Ref. 18).

Thus models such as ours, or such as that of Ref. 15, indicate that the continuity of Ψ over all space has critical consequences, namely, the explicit dependence of the effective Josephson coupling on H and T. Note that the boundary condition at the layers has not been used at all in Sec. III, so it has no effect on the field dependence of ν_{eff}.

A final remark is that the coefficients (3.9)–(3.12) of the effective free energy were derived explicitly and are thus valid for any value of ν at H_{c2}. We note that we do not obtain terms of the form $\Psi_n\Psi_{n+2}^*$ etc., because of the presence of the δ function in our model, which restricts the layers to interact only with their neighboring ones.

IV. UPPER CRITICAL FIELDS

In this section we shall derive the upper critical field, for magnetic fields both perpendicular and parallel to the layers. This involves solving the equation

$$0=\left[1-\alpha(T)\sum_n\delta(z-n)\right]\Psi+\nu\Gamma_0^2\Pi_\parallel^2\Psi+\nu\Pi_z^2\Psi\,, \tag{4.1}$$

where $\Pi=-i\nabla-A$.

A. Perpendicular fields

Let us first examine the case of a single layer, i.e., a thin S layer embedded in a very thick N layer. We take $A=Hx\hat{y}$ and $H=H\hat{z}$. Then the exact solution is $\Psi=\exp(-\gamma|z|-Hx^2/2)$, where $\gamma^2=H\Gamma_0^2+\nu^{-1}$. The corresponding exact $H_{c2}^\perp$ for this single layer, for any value of ν, is

$$H_{c2}^\perp=[\alpha^2(T)-4\nu]/4\nu^2\Gamma_0^2\,. \tag{4.2}$$

We deduce then that $\alpha(T_c)=2\sqrt{\nu}$ for the single layer. However the meaning of the rescaling of all quantities needs clarification in this case, where d is infinite. Taking

the example (see Sec. II) of an S layer of thickness d_0, one finds that T_c is defined by $1-\alpha_S/\alpha_N=2\xi/d_0$. On the other hand, in terms of the true physical field (not the rescaled one) $H_{\rm ph}$, the order parameter decay length in the N region is $l_N=d/\gamma=\xi(1+H_{\rm ph}/H_0)^{-1/2}$, where $H_0=\hbar c/2e\Gamma_0^2\xi^2$. One finds as well the physical critical field

$$H_{\rm ph}^{c2\perp}=H_0(d_0^2/4\xi^2)[(1-\alpha_S/\alpha_N)^2-4\xi^2/d_0^2]\ . \quad (4.3)$$

One must remember that ξ is here the normal region coherence length and controls the order parameter decay (proximity effect) in zero field. Equation (4.3) shows that if α_S varies linearly with temperature, while α_N and ξ vary slowly with T, then $H_{c2}^{\perp}$ is linear at T_c, but acquires a parabolic upturn behavior away from T_c.

Now we proceed to the case of many layers in a perpendicular field, where the order parameter is periodic along z, and equal to $f(z)\exp(-Hx^2/2)$. We can easily verify that $f(z)=\cosh[\gamma(z-n-1/2)]$, for $n\le z\le n+1$, where γ is the same as the one for the single layer. The boundary condition (2.5) enables us to determine $H_{c2}^{\perp}$ through the equation

$$\alpha(T)/\nu=2\gamma\tanh(\gamma/2)\ , \quad (4.4)$$

valid for all values of ν. In particular, at $H=0$ we obtain again that for the multilayer $\alpha(T_c)=2\sqrt{\nu}\tanh(1/2\sqrt{\nu})$, which is $\alpha(T_c)=1$ for $\nu\gg1$, and $\alpha(T_c)=2\sqrt{\nu}$ for $\nu\ll1$. Furthermore, we can easily ascertain that $H_{c2}^{\perp}$ has always got a positive curvature, at least if ν varies slowly with temperature. We have here another main result of our model, already obtained in other treatments which took into account inequivalent layers and proximity effects (Ref. 14), again distinctly different from the predictions of the usual LD model.

Again, for an $\alpha(z)$ function alternating between α_N and α_S, one obtains at T_c,

$$1-\alpha_S/\alpha_N=2(\xi/d_0)\tanh(d/2\xi)\ ,$$

showing that T_c depends on the interlayer distance d. Starting from T_c^0, the critical temperature for the S layer, T_c decreases with increasing d and saturates at the value given earlier for the single layer case. Comparison with data on Nb/Cu multilayers (Ref. 19) shows that this general trend is verified in this system.

If $\alpha(T)\gg\nu$, then $\gamma\gg1$. The $H_{c2}^{\perp}$ of the multilayer, as given by Eq. (4.4), then reduces to the $H_{c2}^{\perp}$ of the single layer, given in Eq. (4.2). Indeed, if $\nu\ll1$, then the layers are really quite far apart, and hence rather independent. The upper critical field then would be simply that of the single layer. We call the regime $\alpha(T)\gg\nu$ the *quasi-2D regime*, since the layers are almost independent.

But it should be noted that $H_{c2}^{\perp}$ depends on ν, and therefore on the distance between the layers, contrary to the result of the usual LD model. In particular, as the layer spacing d increases, $H_{c2}^{\perp}$ depends on d through the hyperbolic tangent of Eq. (4.4). More precisely, one has

$$1-\alpha_S/\alpha_N=2(\xi^2/d_0l_N)\tanh(d/2l_N)\ . \quad (4.5)$$

From this expression it is easy to show that $H_{\rm ph}^{c2\perp}$ decreases as d increases. This trend is again qualitatively verified in Nb/Cu multilayers (Ref. 19). More experiments are needed on artificial multilayers, and also a more detailed theory, to check the validity of the above treatment of proximity effects.

If, on the other hand, $\nu\gg\alpha(T)$, then $1\gg\gamma$, and $H_{c2}^{\perp}=[\alpha(T)-1]/\nu\Gamma_0^2$. This is precisely the upper critical field that we would have if the δ functions were so close to each other, that they formed a superconducting continuum. In that case, the $\sum_n\delta(z-n)$ would become $\int dn\ \delta(z-n)=1$, and we would obtain thus a "bulk" $H_{c2}^{\perp}$. We call the case $\nu\gg\alpha(T)$ the *quasi-3D* case, since the S layers almost form a superconducting continuum there.

Let us notice that this situation involves a very weak modulation of the order parameter, and thus a genuine 3D superconductor. This is different from the so-called 3D region of the LD model, where $\xi_c^{\rm eff}\gg d$, but the order parameter is still assumed to be discontinuous.

It is also worth noting that, close to T_c, $H_{c2}^{\perp}$ has a linear behavior, both in the case $\nu\gg1$, where the layers are strongly coupled by the proximity effect, as well as in the case $\nu\ll1$, provided ν varies slowly with T. However, the slope of $H_{c2}^{\perp}$ is different in the two regions, and thus there is a crossover in the region $\alpha(T)\approx\nu$, where $H_{c2}^{\perp}$ changes shape.

B. Parallel fields

Let us consider a single layer in a parallel field $H\hat{x}$ and a vector potential $A=-Hz\hat{y}$. We have then to solve the equation

$$0=[1-\alpha(T)\delta(z)]\Psi+\nu\Gamma_0^2H^2z^2\Psi-\nu\frac{\partial^2\Psi}{\partial z^2}\ , \quad (4.6)$$

subject to the boundary condition

$$-\alpha(T)\Psi(0)=2\nu\frac{\partial\Psi}{\partial z}(0+)\ . \quad (4.7)$$

We may assume that Ψ is a function of z only. In that case the solutions of Eq. (4.6) are the parabolic cylinder functions $D_p(\pm z\sqrt{2\Gamma_0H})$, with $p=-(2\Gamma_0H\nu)^{-1}-1/2$. But the parabolic cylinder function $D_p(z)\to0$ if $z\to\infty$, and $D_p(z)\to\infty$ if $z\to-\infty$. Thus we must restrict ourselves to the solution

$$\Psi=D_p(|z|\sqrt{2\Gamma_0H})\ . \quad (4.8)$$

The boundary condition (4.7) is satisfied only if

$$\alpha(T)=\frac{\Gamma(3/4+1/4\Gamma_0H\nu)}{\Gamma(1/4+1/4\Gamma_0H\nu)}4\nu\sqrt{\Gamma_0H}\ , \quad (4.9)$$

a relation that determines $H_{c2}^{\parallel}$ implicitly for any value of ν.

If we assume that $\Gamma_0H\nu$ is small, then we can use a property of the Γ functions:

$$\frac{1}{\sqrt{y}}\frac{\Gamma(y+3/4)}{\Gamma(y+1/4)}\approx1+\frac{1}{64y^2}\quad\text{for } y\to\infty\ . \quad (4.10)$$

Thus we obtain

$$H_{c2}^{\parallel} \approx \frac{2}{\Gamma_0 \nu} \left[\frac{\alpha(T)}{2\sqrt{\nu}} - 1 \right]^{1/2} , \quad (4.11)$$

valid only for a single layer close to T_c, where $\alpha(T_c)=2\sqrt{\nu}$. Note again that the $\alpha(T_c)$ of the single layer is the same as that of the multilayer only if $\nu \ll 1$.

On the other hand, if $\Gamma_0 H \nu$ is large, then we obtain

$$H_{c2}^{\parallel} \approx 0.547 \alpha^2(T)/\Gamma_0 \nu^2 , \quad (4.12)$$

valid for the single layer in the region away from T_c, $\alpha(T) \gg 2\sqrt{\nu}$. Thus, close to T_c the parallel upper critical field for a single layer behaves like the square root of $(1-T/T_c)$, while further away it has a parabolic upturn behavior.

Consider now the multilayer case. We have to solve the equation

$$0 = \left[1-\alpha(T) \sum_n \delta(z-n) \right] \Psi + \nu \Gamma_0^2 H^2 z^2 \Psi - \nu \frac{\partial^2 \Psi}{\partial z^2} , \quad (4.13)$$

subject to the boundary condition (2.5).

In the quasi-2D case of $\nu \ll 1$, the layers are so far apart that they are practically independent, and so $H_{c2}^{\parallel}$ for the multilayer is the same as that of the single layer. In the quasi-3D case of $\nu \gg 1$ though, the δ functions are so close together, that we can replace $\sum_n \delta(z-n)$ by $\int dn\, \delta(z-n)=1$. Then we have

$$0 \approx [1-\alpha(T)] \Psi + \nu \Gamma_0^2 H^2 z^2 \Psi - \nu \frac{\partial^2 \Psi}{\partial z^2} , \quad (4.14)$$

corresponding to $H_{c2}^{\parallel} = [\alpha(T)-1]/\nu\Gamma_0$. If $\alpha(T)$ is linear in T, and ν varies slowly with T, then $H_{c2}^{\parallel}$ is linear in T.

We note that for the multilayer, when $\nu \gg 1$ we have $H_{c2}^{\parallel}/H_{c2}^{\perp} = \Gamma_0$. On the other hand, in the quasi-2D case of $\nu \ll 1$, and for the region of $\alpha(T) \gg \sqrt{\nu}$ away from T_c, Eqs. (4.2) and (4.12) imply that $H_{c2}^{\parallel}/H_{c2}^{\perp} = 2.188\Gamma_0$. Thus the ratio $H_{c2}^{\parallel}/H_{c2}^{\perp}$ is equal to Γ_0 in the quasi-3D case, but it is 2.188 times larger in the quasi-2D regime. Besides this numerical factor, the main result is that at H_{c2} the multilayer has the ratio $H_{c2}^{\parallel}/H_{c2}^{\perp}$ of the bulk S material. This is due to the extension of the order parameter in between the layers. This behavior differs strongly from that predicted by the LD model, where $H_{c2}^{\parallel}$ diverges below the dimensional crossover, and where the ratio $H_{c2}^{\parallel}/H_{c2}^{\perp}$ diverges. Moreover, the ratio $H_{c2}^{\parallel}/H_{c2}^{\perp}$ we find should not be confused with Γ_{eff}, the effective anisotropy of Sec III, which comes from the effective Josephson coupling, and which is in general much greater.

We shall conclude this section by discussing briefly the temperature dependence of ν. It is well known from the theory of the proximity effect in superconductors (Ref. 13) that, in the case of a contact between a superconductor and an insulator, the order parameter decays within a length $\xi \approx \xi_0^2/l$, where l is of the order of an atomic distance, and ξ_0 is the BCS coherence length. In high-T_c superconductors, $\xi_0 \approx l$, and so our ξ is of the order of some interatomic distances, and depends little on T, provided the nonsuperconducting regions between the superconducting layers are insulating. If, on the other hand, the latter are assumed to be metallic, ξ would be of the order of $\hbar v_{\text{FN}}/k_B T$ (Ref. 13), where v_{FN} is the Fermi velocity of the nonsuperconducting regions. One should also mention the possibility of SWS structures, where the weakly superconducting W layers have a lower T_c than the S layers. In this case ν diverges at T_c^W, so the quasi-3D and quasi-2D regimes may be obtained in the same material as T is varied (Ref. 20).

V. CONCLUDING REMARKS

The standard Lawrence-Doniach model assumes that the Josephson coupling is constant, and that the order parameter is precisely zero between the superconducting layers. In this paper, we examined the behavior of the superconducting order parameter for a well-layered superconductor, within a model that allows continuous variations of the order parameter between the layers. Such a model treats the proximity effect in a simplified manner but presents the great advantage of allowing exact calculations. It is able to cover a wide variety of physical situations, generalizing the Lawrence-Doniach model of weakly coupled layers. Although this model can in principle be solved for any thickness of the constituting (S,N) layers, we presented here exact results for the case of thin S layers. In zero field, and in the nonsuperconducting regions between the layers, the order parameter decays within a characteristic length $\xi = d\sqrt{\nu}$, generally smaller than the interlayer spacing d. In this limit, the free energy of our model is mapped onto an effective free energy of the LD type, with additional kinetic terms coupling the gradients, along the layers, of neighboring order parameters. We emphasize that these terms result here from the fact that the order parameter is allowed to spread in the interlayer region. A further consequence is that the coupling between the planes becomes field dependent, as it should be in real physical systems. This feature is an important generalization of the LD model. We also derived the upper critical field, both in the perpendicular and parallel field orientations. Near T_c, the perpendicular upper critical field has the positive curvature that is characteristic of layered superconductors, and which is solely due to the fact that the order parameter is not uniform along the z axis, but varies periodically throughout space. This, after all, is why the inclusion of inequivalent layers in the standard LD model results also in a positive curvature of $H_{c2}^{\perp}$ (Ref. 14).

Furthermore, $H_{c2}^{\parallel}$ does not diverge any more, since the superconductivity is distributed along the z axis, and can therefore be destroyed by a finite magnetic field. Moreover, the anisotropy, which can be deduced from the ratio $H_{c2}^{\parallel}/H_{c2}^{\perp}$ does not reflect the Josephson coupling. It is therefore smaller than the low-field anisotropy deduced, for instance, from torque magnetometry. This is consistent, in general, with experiments on coppper oxide materials. This points towards the fact that the anisotropy of complex multilayered structures is not a universal quantity, as it is assumed in the pure 3D description, but depends on the way it is extracted from experiments, which may probe either the individual layer anisotropy, or the effective interlayer coupling.

Our model predicts that $H_2^{\perp}$ *decreases* with *increasing* interlayer distance, due to the weakening of the supercon-

ductivity by the proximity effect. Such a prediction could be *tested* experimentally in Y,Pr superlattices, by varying the interlayer distance d at fixed temperature, provided one effectively measures H_{c2}. Another testable prediction of our model is the field dependence of the Josephson coupling. Experiments that tested these predictions would offer a means of probing the new physics that lie beyond the LD model. In particular, besides the generalized LD physics found in the case of a weak proximity effect, there is also the possibility of a true 3D superconductivity with nearly homogeneous order parameter. This situation is definitely different from the so called 3D regime of the LD model, and could be obtained with varying temperature.

ACKNOWLEDGMENTS

Two of us (D.F. and A.M.E.) thank warmly the Department of Natural Sciences of the University of Cyprus for its hospitality and financial support during the completion of this work. We also thank A. Buzdin for pointing out to us Ref. 17. Laboratoire d'Etudes is Laboratoire associe a l'Universite Joseph Fourier, Grenoble.

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