Kinetic inductance and penetration depth of thin superconducting films measured by THz-pulse spectroscopy

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(Received 24 September 1993)

We measure the transmission of THz pulses through thin films of YBa₂Cu₃O_{7- δ} at temperatures between 10 and 300 K. The pulses possess a usable bandwidth extending from ~0.1 to 1.5 THz (3.3-50 cm⁻¹). Below T_c we observe pulse reshaping caused by the kinetic inductance of the superconducting charge carriers. From transmission data, we extract values of the London penetration depth, as a function of temperature, and find that they agree well with a functional form $[\lambda(0)/\lambda(T)]^2 = 1 - (T/T_c)^{\alpha}$, where $\lambda(0) = 148$ nm and $\alpha = 2$.

The unique electrodynamic properties of superconductors are a consequence of the Meissner effect: magnetic fields attempting to penetrate a superconductor are screened out by the flow of supercurrent. The screening takes place within a distance given by the London penetration depth λ . It follows¹ that the frequency-dependent conductivity of a BCS-like superconductor for frequencies far below the gap takes the form

$$\sigma(\omega) = \sigma_1 + i\sigma_2 = \sigma_n + + [i/(\mu_0 \lambda^2)][1/\omega - i\pi\delta(\omega)],$$
(1)

where σ_n is the contribution of the normal carriers, μ_0 is the inductance of free space, and $i = \sqrt{-1}$. The two terms within square brackets represent the contribution of the superconducting carriers. At exactly $\omega = 0$, the conductivity is divergent, as signified by the delta function $\delta(\omega)$. For $\omega \neq 0$, the superconducting part of σ is purely imaginary, and varies as ω^{-1} . Accordingly, the supercurrent responds to the application of a time-varying field like an ideal inductor, a property referred to as "kinetic inductance." The magnitude of the kinetic inductance depends upon λ .

Because of its importance for discriminating between different theories, the temperature dependence of λ in the high- T_c materials has become an object of active investigation. Bonn *et al.*² have recently pointed out that lowfrequency microwave (1–10 GHz),³ muon spin rotation,² as well as low-field magnetization measurements⁴ often reveal a temperature dependence of the form

$$[\lambda(0)/\lambda(T)]^2 = 1 - (T/T_c)^{\alpha},$$
(2)

where $\alpha = 2$. This behavior is incompatible with BCS theory,^{2,5} as well as the Gorter-Casimir two-fluid model for which $\alpha = 4$ in the neighborhood of T_c .⁶ Contemporary theories positing *d*-wave coupling for the Cooper pairs usually find $\alpha = 1$ in the $T \rightarrow 0$ limit, although $\alpha = 2$ can be obtained if enough scattering is built into the theory.⁷ Low-frequency microwave measurements showing $\alpha = 1$ for single crystals and high-quality thin films below 20 K have been recently published.^{8,9}

In this paper, we describe transmission measurements

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is that it permits us to measure the transmitted *electric field* E(t), thereby enabling us to obtain the complex frequency-domain conductivity $\sigma(\omega)$, from which we extract λ . Similar experiments have been previously performed to measure the so-called anomalous coherence peak in σ_1 in YBCO.¹² The experiments are carried out using a THz spectrometer similar to that described in Ref. 11. The system has a usable bandwidth extending from ~0.1 to 1.5 THz. We use a 30 μ m transmitting antenna fabricated on low-temperature-grown GaAs,¹³ biased to 50 V, and driven with optical pulses from a colliding-pulse modellocked

on thin films of $YBa_2Cu_3O_{7-\delta}$ (YBCO) in the less-

investigated high-frequency range 0.1-1.5 THz. Thin-film

samples were employed because suitable single crystals

do not exist for transmission measurements in this frequency regime. Because our sample quality is high, we

do not anticipate problems associated with defects, al-

though effects associated with weak links at grain boundaries cannot be ruled out.^{8,10} The transmission measure-

ments were made using wide-bandwidth electromagnetic pulses generated and detected by microfabricated antenna structures.¹¹ The special feature of this technique

with optical pulses from a colliding-pulse modellocked (CPM) dye laser. The receiver is a $30-\mu m$ antenna fabricated on ion-implanted silicon on sapphire. It was gated with a second pulse from the CPM laser, and the photocurrent is detected with a current-sensitive lock-in amplifier.

The 155 nm YBCO film was grown on a 500 μ m NdGaO₃ substrate by laser ablation.¹⁴ We have found NdGaO₃ to be the ideal substrate for our purposes because it remains transparent and nondispersive over the entire spectral bandwidth of our pulses, as well as over the entire range of temperatures investigated here. The film was grown in the (100) orientation, so that the polarization of our THz pulses was parallel to the $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ plane. The normal-state resistivity of the film, measured with a four-point probe, is linear in *T*, extrapolating to 0 $\mu\Omega$ cm at ~10 K. The 100 $\mu\Omega$ cm resistivity at 100 K indicates high sample quality. The dc superconducting transition is at 92 K, with a width of 0.5 K.

During the experiment, the sample is held in a

continuous-flow He cryostat which has been modified for transmission experiments in the far infrared by the installation of 2-cm-thick polyethylene windows. The temperature is stabilized to better than 1 K. The sample is mounted on a Cu block having a 6 mm hole for transmission; a bare substrate was mounted onto a similar hole on the same block 1 cm away. The THz beam is focused onto the sample and recollimated with a pair of plastic lenses. Care is taken to avoid leakage of THz radiation through the hole not in use. At every temperature, a scan of the YBCO-film/substrate combination is followed by a scan of the bare substrate. The sample and reference temporal scans are Fourier transformed and divided, giving the complex frequency-domain transmission of the YBCO film itself.

The time-domain transmission data shown in Fig. 1 show two noteworthy features. First, the initial pulse is followed at regular intervals by multiple reflections from within the NdGaO₃ substrate [inset to Fig. 1(a)]. The reflected pulses emerge without a (dispersive) shape change, but with alternating signs, i.e., +, -, +, -, and so on. The signs alternate because the NdGaO₃/vacuum interface is a dielectric interface, which has a positive field reflection coefficient, $\Gamma > 0$, while the YBCO/NdGaO₃ interface is a metallic (shorting) interface for which $\Gamma \approx -1.^{15}$

The second noteworthy feature of Fig. 1 is the pulse reshaping that occurs upon cooling the sample from 100 to 85 K. This reshaping, a direct result of the superconductor's kinetic inductance, can be understood with the aid of a transmission line analogy (Fig. 2). If a transmission line is shorted in the middle by an inductor having an impedance $Z = -i\omega L$, it acts as a high-pass filter: the inductor is a short circuit for dc, whereas extremely high frequencies pass by with undiminished magnitude¹⁶ The transfer function $t(\omega)$, relating the input and output voltages $V_i(\omega)$ and $V_0(\omega)$, is



FIG. 1. Time-dependent transmission of THz pulses through YBCO. (a) $T > T_c$, (b) $T < T_c$, showing pulse reshaping. In both cases, trailing pulses with alternating signs resulting from reflections at the NdGaO₃/YBa₂Cu₃O_{7- δ} interface are visible (inset).



FIG. 2. Transmission line analog of THz pulse experiment. The incident voltage pulse $V_i(t)$ enters from the left, propagates past the shunt inductor L which models the kinetic inductance of the superconductor, and emerges on the right as $V_0(t)$, reshaped by the transfer characteristic $t(\omega)$ from Eq. (3).

$$t(\omega) = \frac{V_0(\omega)}{V_i(\omega)} = \frac{2}{2 + Z_0/Z},$$
(3)

where Z_0 is the characteristic impedance of the transmission line. This transfer function, with Z_0 set equal to the vacuum impedance, directly corresponds to the complex field transmission coefficient measured by our apparatus.

We have tested the applicability of the transmission line analog by calculating the effect of this transfer function using a measured THz pulse as the input $V_i(t)$. In the THz regime, $Z_0 \gg Z$ giving $t(\omega) \propto -i\omega$. Depicted in Fig. 3(a) is the leading pulse transmitted through the sample at 100 K [from Fig. 1(a)] plotted on an expanded time scale. Because this pulse propagates through normal-state YBCO, it follows the same optical path as that in Fig. 1(b), but is not modified by the superconducting transfer function (3). We Fourier-transform this pulse, multiply by $-i\omega$, and inverse-transform to obtain $V_0(t)$. This simulation [Fig. 3(b)] agrees well with the experimentally obtained pulse which propagates through the sample in its superconducting state [Fig. 3(c)].



FIG. 3. (a) The leading pulse from Fig. 1(a) on an expanded scale. (b) The calculated pulse shape when the pulse in (a) is acted upon by $t(\omega)$. (c) The measured leading pulse from Fig. 1(b) (expanded scale) for comparison.



FIG. 4. London penetration depth λ vs T determined from the transmission data via Eqs. (4) and (5) (solid circles). Also shown are theoretical curves of Eq. (2) with $\alpha = 2$ (solid line), $\alpha = 4$ (dash-dotted line), and BCS theory (dashed line).

Because we observe clearly the effects of the kinetic inductance, we are able to determine λ from our transmission data. In the case of a superconducting film on a dielectric substrate of index n_s , the transmission function is given by

$$t(\omega) = \frac{1+n_s}{1+n_s+Z_0/Z_{\text{eff}}},\tag{4}$$

where Z_{eff} is the effective surface impedance of the superconducting film.¹² Equation (4) is clearly similar to (3). In general, the surface impedance of an *infinite half-space* of a superconductor is $Z_s = R_s + iX_s = \sqrt{-i\omega\mu_0/\sigma}$, where σ is the complex conductivity given by (1).¹⁷ In the case of *finite* film thickness, the effective surface reactance X_{eff} (the imaginary part of Z_{eff}) must be corrected as $X_{\text{eff}} = X_s \coth(d/\lambda)$, where d is the superconducting film thickness.¹⁸ Recalling that $\sigma_1 \ll \sigma_2$ in the supercon-

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ducting state yields

$$X_{\text{eff}} = \mu_0 \omega \lambda \coth(d/\lambda). \tag{5}$$

Thus by measuring $t(\omega)$, we can determine X_{eff} . Then under the assumption that λ remains constant over our frequency range, we can invert (5) and extract λ .

Using this procedure, we have obtained λ values for a sequence of temperatures ranging from 10 K up to T_c . A least-squares fit of $\lambda(T)$ to a linear function in T for T < 50 K yields an extrapolated $\lambda(0) = 148$ nm. A plot of $[\lambda(0)/\lambda(T)]^2$ versus the reduced temperature is shown in Fig. 4 taking the fitted $\lambda(0)$ and $T_c = 92$ K determined from the dc resistivity measurements. Comparing the data to theoretical curves of the functional form (2)we find the best agreement for $\alpha = 2$ (Fig. 4, solid line). Similar results have also been obtained by several other investigators using other measurement techniques as well as in other frequency ranges.^{3,2,4} Curves corresponding to the Gorter-Casimir two-fluid model⁶ (for which $\alpha = 4$), as well as standard BCS theory⁵ are also shown (Fig. 4, dashed, dash-dotted lines). The fit of the latter two curves to our experimental data is poorer, although a fit to the BCS curve cannot be ruled out due to scatter.

In conclusion, we observe the reshaping of THz pulses upon transmission through thin films of superconducting YBCO. The pulse reshaping is a direct time-domain demonstration of the kinetic inductance of the superconductor. The reshaping may be described using a transmission line analog. From measurements of the pulse transmission, we have extracted values for the temperature-dependent penetration depth $\lambda(T)$.

We acknowledge the expert technical assistance of S. Tippmann and A. Schulz. S.D.B. and J.O.W. acknowledge support by the Alexander von Humboldt Foundation. J.O.W. was supported by Hughes Research Laboratories during part of this work.

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