Magnetointersubband oscillations of conductivity in a two-dimensional electronic system

M. E. Raikh and T. V. Shahbazyan
Department of Physics, University of Utah, Salt Lake City, Utah 84112
(Received 4 May 1993)

Magnetoconductivity of the two-dimensional electron gas occupying two size-quantization subbands is studied theoretically. When the bottoms of subbands are separated by an integer number of Landau levels, the staircases of Landau levels in both subbands are completely aligned. For such values of magnetic field the intersubband scattering is enhanced. As it was pointed out by Polyanovsky, this results in additional Shubnikov–de Haas oscillations of conductivity with magnetic field, with period depending on subband separation, and amplitude depending weakly on temperature, provided that a large number of Landau levels in each subband are occupied. In the calculation of conductivity we make use of the self-consistent Born approximation generalized to the case of two subbands. An analytical theory for the case of strong disorder and numerical results for the case of weak disorder are presented.

I. INTRODUCTION

Magnetotransport in quantum wells and superlattices has been studied extensively in recent years. In a typical GaAs/$Al_xGa_{1-x}As$ well electrons are confined to the interface plane, so that the electronic states represent a set of size-quantized subbands. For large concentrations of electrons several subbands can be populated. Figure 1 schematically illustrates this situation for the case of two occupied subbands with their bottoms at E_1 and E_2 and the Fermi energy E_F lying above E_2. The position of the Fermi level can be controlled by changing the gate voltage[1] or by in-plane magnetic field.[2]

When E_F intersects E_2 peculiarities in thermodynamic (such as specific heat and magnetic susceptibility) as well as in kinetic (such as conductivity and thermopower) characteristics are expected due to the change in the Fermi surface topology (topological transition) and, consequently, the opening of a new scattering channel. In bulk metal and zero magnetic field the peculiarities of the thermodynamic properties in the vicinity of the topological transition were studied by Lifshitz.[3] Anomalies of kinetic characteristics were recently studied by Varlamov and Pantsulaya.[4]

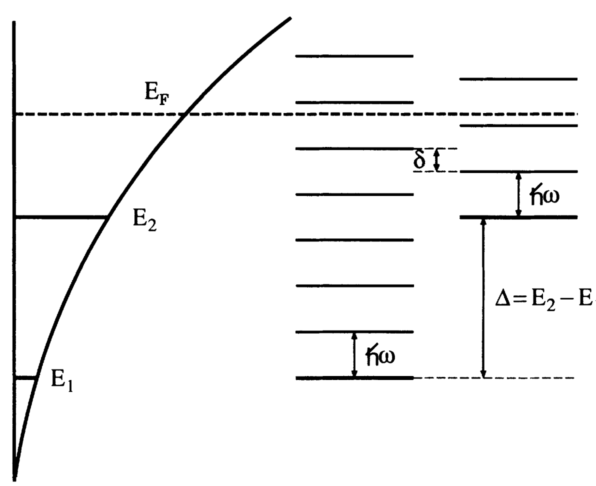

FIG. 1. Schematic illustration of the magnetointersubband oscillations of conductivity. The energy spectrum of the first (second) subband represents a staircase of Landau levels on the top of E_1 (E_2). When $\Delta/\hbar\omega$ is an integer (i.e., $\delta = 0$), two staircases of Landau levels are completely aligned, which results in a peak in the intersubband scattering.

For two-dimensional (2D) systems in zero field, the anomalous behavior of transport coefficients has been studied for a long time both theoretically[5–8] and experimentally[1,2,9–11] (for a review of early works see Ref. 12). From now on we will be considering only the case of two occupied subbands. The total conductivity σ represents a sum $\sigma = \sigma_1 + \sigma_2$ of "partial" conductivities σ_1 and σ_2 corresponding to the first and second subbands, respectively.[6,12] The usual relation $\sigma_i \propto \tau_i$ ($i = 1,2$) still holds; however, the "partial" scattering times, τ_1 and τ_2, are determined by a system of equations and both depend on intersubband scattering. At the transition point (i.e., when E_F intersects E_2) the theory[6,12] predicts a discontinuity in the conductivity and, hence, the divergence of the thermopower. Experimental data, however, show a well pronounced but rounded peak in thermopower[11] and a drop in the mobility.[9,10] The scattering-induced smearing out of these anomalies was considered in Ref. 7. In Ref. 8 the exact solution of the problem in the vicinity of the transition point was found.

In a perpendicular magnetic field B the anomalies in thermodynamic and transport properties, caused by the occupation of several subbands, are even more pronounced. The double minimum structure in the density of states due to the interplay of Landau levels of two occupied subbands was reported in Ref. 13. In parabolic quantum wells, with[14] and without[15,16] superlattices, the suppression of weak quantum Hall plateaus has been observed as a result of the mixing of different series of Landau levels corresponding to different subbands. In a weak

0163-1829/94/49(8)/5531(10)/$06.00

magnetic field, $B \leq 1$ T, the conductivity exhibits multiply periodic Shubnikov–de Haas (SdH) oscillations at low temperatures. Each of the "partial" conductivities σ_i is periodic in inverse magnetic field with the period $e\hbar/mc(E_F - E_i)$, m being electron mass. The resulting superposition of SdH oscillations has been widely studied (see, e.g., Refs. 14–21) and the correspondence between the power spectrum and the occupied subband energies was found to be very accurate.

In the present paper we investigate another mechanism for oscillations of the conductivity in a weak magnetic field, when two or more subbands are occupied. This mechanism is illustrated in Fig. 1. In a magnetic field each subband represents a staircase of Landau levels associated with E_i. The energy spectrum of electronic states is then given by

$$E_{in} = E_i + \hbar\omega(n + 1/2), \tag{1}$$

where ω is the cyclotron frequency. We assume that the subband separation Δ is much larger than the Landau level separation, i.e., $\Delta = E_2 - E_1 \gg \hbar\omega$. Now we note that if $\Delta = \hbar\omega p$, p being an integer, then the two staircases of Landau levels are completely aligned. Clearly, for such particular values of magnetic field the intersubband scattering is enhanced. As a result, the conductivity oscillates with magnetic field with period

$$\delta\left(\frac{1}{\hbar\omega}\right) = \frac{1}{E_2 - E_1} = \frac{1}{\Delta}. \tag{2}$$

Remarkably, unlike the Shubnikov–de Haas oscillations, these "magnetointersubband" (MIS) oscillations should exhibit an anomalously small sensitivity to the temperature. This idea was put forward by Polyanovsky in Ref. 22. The reason is that the period of oscillations, as it is seen from Eq. (2), does *not* depend on the Fermi energy. On the other hand, with increasing temperature the number of Landau levels contributing to the conductivity increases. Since for $\hbar\omega = \Delta/p$ the alignment occurs for an entire staircase of Landau levels, the averaging of the conductivity with the Fermi distribution function does *not* affect the magnetointersubband oscillations.[22]

It should be emphasized that the calculation of the magnetoconductivity in the 2D case requires the smearing out of the Landau levels.[12] Such a smearing occurs due to both intrasubband and intersubband scattering processes. In the case when the widths of Landau levels are much smaller than $\hbar\omega$, the intersubband scattering modifies strongly the shape of the aligned Landau levels in the vicinity of resonance.

To develop the theory of the MIS oscillations we employ the self-consistent Born approximation[12,23] (SCBA), generalized to the case of two subbands. The SCBA approach has been shown[24,25] to be accurate for the short-range random potential and high Landau levels. This is exactly what we need since the intersubband scattering that we consider requires a large momentum transfer and, therefore, can *only* be caused by short-range harmonics of the random potential. Note also that the long-range harmonics of the random potential do *not* affect the MIS oscillations, since they just modulate the Landau levels without destroying the alignment of two Landau staircases.

The calculations of the conductivity are carried out both in the cases of weak $\omega\tau_i \gg 1$ and strong $\omega\tau_i \ll 1$ disorder. The paper is organized as follows. In Sec. II we generalize the self-consistent Born approximation to the case of two subbands. In Sec. III we derive the expressions for the magnetointersubband contribution to the conductivity. The results of numerical calculations for the case of weak disorder are presented in Sec. IV. In Sec. V we discuss the experimental data of Refs. 19–21, which can be accounted for by our theory.

II. TWO-SUBBAND GENERALIZATION OF THE SELF-CONSISTENT BORN APPROXIMATION

Consider an electronic system in a perpendicular magnetic field B, and a random Gaussian potential $V(\mathbf{r})$ with the correlation function

$$\langle V(\mathbf{r})V(\mathbf{r}')\rangle = \gamma\delta(\mathbf{r} - \mathbf{r}'), \tag{3}$$

where γ is the strength of the potential. The unperturbed energy spectrum is given by Eq. (1) and the unperturbed wave functions in the Landau gauge have the form

$$\psi_{ink_y}(\mathbf{r}) = \chi_i(z)\varphi_{nk_y}(\boldsymbol{\rho}), \tag{4}$$

where $\chi_i(z)$ (i=1,2) are the wave functions corresponding to the quantized energy levels E_i along the magnetic field and $\varphi_{nk_y}(\boldsymbol{\rho})$ are the Landau wave functions, $\boldsymbol{\rho}$ being in-plane radius-vector and k_y being the y-component of the momentum. In the presence of a random potential each Landau level acquires a finite width, Γ_i, for the ith subband, which we assume to be much smaller than the subband separation

$$\Gamma_i \ll \Delta = E_2 - E_1. \tag{5}$$

In the case of two subbands both the Green function $G^{ij}(E)$ and the self-energy $\Sigma^{ij}(E)$ represent 2×2 matrixes in the subband indexes i and j, defined in the usual way,

$$G(E) = \left[E - \mathcal{E} - \Sigma(E)\right]^{-1}, \tag{6}$$

where E is the energy and the matrix $\mathcal{E}^{ij}_{nn'} = \delta^{ij}\delta_{nn'}E_{in}$ is diagonal both in the subband and in the Landau level indexes.

In the framework of the self-consistent Born approximation (SCBA) all diagrams with intersecting impurity lines are neglected. The corresponding Dyson equation for the self-energy is shown in Fig. 2. In the one-subband SCBA approach[12,23] the Green function (6) is diagonal in the Landau level indexes n and n', while the self-energy does not depend on n. This same feature appears to be true in the case of two subbands. Indeed, according to Eq. (4), the $\boldsymbol{\rho}$ and z dependences in the wave functions are separated. Therefore, the in-plane variables $\boldsymbol{\rho}$ and k_y in the Dyson equation can be integrated out in the same way as in the one-subband case. The two-subband

SCBA Dyson equation then takes the form

$$\Sigma^{ij}(E) = \Gamma_A^2 \sum_n \sum_{kl} Q_{kl}^{ij} [E - \mathcal{E}_n - \Sigma(E)]_{kl}^{-1} = \Gamma_A^2 \sum_n \sum_{kl} Q_{kl}^{ij} G_n^{kl}(E), \quad (7)$$

where $\mathcal{E}_n^{ij} = \delta^{ij} E_{in}$ and $\Gamma_A^2 = \gamma/2\pi l^2$, $l = (c\hbar/eB)^{1/2}$ being the magnetic length. In deriving Eq. (7) we used Eq. (3) and introduced the notation

$$Q_{kl}^{ij} = \int dz \chi_i(z) \chi_j(z) \chi_k(z) \chi_l(z). \quad (8)$$

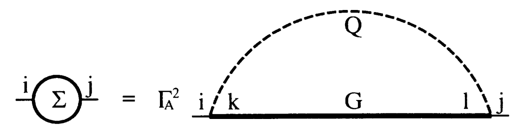

FIG. 2. Graphical representation of the Dyson equation (7) for the SCBA in the case of two subbands. The self-energy Σ^{ij} and the Green function G^{kl} represent matrixes in the subband indexes and Q_{kl}^{ij} is defined by Eq. (8).

It follows from the obvious symmetry properties of Q_{kl}^{ij} that $G^{12} = G^{21}$ and $\Sigma^{12} = \Sigma^{21}$. To analyze the Dyson equation (7), we rewrite it in the component form

$$\Sigma^{12} = \Gamma_A^2 \sum_n \left\{ \frac{Q_{11}^{12}}{E - E_{1n} - \Sigma^{11} - \dfrac{(\Sigma^{12})^2}{E - E_{2n} - \Sigma^{22}}} + \frac{Q_{22}^{12}}{E - E_{2n} - \Sigma^{22} - \dfrac{(\Sigma^{12})^2}{E - E_{1n} - \Sigma^{11}}} - \frac{2Q_{12}^{12}\Sigma^{12}}{(E - E_{1n} - \Sigma^{11})(E - E_{2n} - \Sigma^{22}) - (\Sigma^{12})^2} \right\}, \quad (9)$$

and

$$\Sigma^{11} = \Gamma_A^2 \sum_n \left\{ \frac{Q_{11}^{11}}{E - E_{1n} - \Sigma^{11} - \dfrac{(\Sigma^{12})^2}{E - E_{2n} - \Sigma^{22}}} + \frac{Q_{22}^{11}}{E - E_{2n} - \Sigma^{22} - \dfrac{(\Sigma^{12})^2}{E - E_{1n} - \Sigma^{11}}} - \frac{2Q_{12}^{11}\Sigma^{12}}{(E - E_{1n} - \Sigma^{11})(E - E_{2n} - \Sigma^{22}) - (\Sigma^{12})^2} \right\}. \quad (10)$$

The third equation can be obtained from (10) by the replacement $1 \leftrightarrow 2$.

Let us demonstrate that despite the fact that the diagonal Σ^{ii} and nondiagonal Σ^{12} components of the self-energy are of the same order of magnitude, Σ^{12} does not contribute to the diagonal components G_n^{ii} of the Green function. Consider the first term in the right-hand side (rhs) of Eq. (10). The main contribution to the sum over n comes from such values of n for which the magnitude of the denominator

$$D_1 = E - E_{1n} - \Sigma^{11} - \frac{(\Sigma^{12})^2}{E - E_{2n} - \Sigma^{22}} \quad (11)$$

is of the order of the width Γ_1 of the corresponding Landau level that belongs to the first subband. On the other hand, the denominator of the last term in the rhs of Eq. (10) can be rewritten as

$$E - E_{2n} - \Sigma^{22} = (E - E_{1n} - \Sigma^{11}) - (\Sigma^{22} - \Sigma^{11}) - \Delta, \quad (12)$$

where the first term in the rhs of Eq. (12) is again of the order of Γ_1. Since the Σ^{ii} themselves are of the order of Γ_i, the magnitude of the rhs of Eq. (12) is of the order of $\Delta \gg \Gamma_i$. Thus, we can write

$$D_1 \simeq E - E_{1n} - \Sigma^{11} + \Sigma^{12}\left(\frac{\Sigma^{12}}{\Delta}\right) \simeq E - E_{1n} - \Sigma^{11}, \quad (13)$$

where we again used the condition $\Sigma^{12}/\Delta \ll 1$. Similarly, for the relevant values of n, one can estimate the denominator of the second term in the rhs of Eq. (10) as

$$D_2 = E - E_{2n} - \Sigma^{22} - \frac{(\Sigma^{12})^2}{E - E_{1n} - \Sigma^{11}} \simeq E - E_{2n} - \Sigma^{22}. \quad (14)$$

Consider now the third term in the rhs of Eq. (10). Applying similar arguments one can estimate this term as

$$\frac{\Sigma^{12}}{\Delta}\left(\frac{1}{\Gamma_1} + \frac{1}{\Gamma_2}\right) \ll \frac{1}{\Gamma_1} + \frac{1}{\Gamma_2}. \quad (15)$$

Since the sum of first two terms in the rhs of Eq. (10) is of the same order of magnitude as the rhs of Eq. (15), the third term can be neglected.

Thus, the two-subband SCBA Dyson equation for the diagonal components of the self-energy reduces to the following system of equations

$$\Sigma^{11} = \Gamma_A^2 \sum_n \left(\frac{Q_{11}^{11}}{E - E_{1n} - \Sigma^{11}} + \frac{Q_{22}^{11}}{E - E_{2n} - \Sigma^{22}} \right) = \Gamma_A^2 \sum_n (Q_{11}^{11} G_n^{11} + Q_{22}^{11} G_n^{22}), \quad (16a)$$

$$\Sigma^{22} = \Gamma_A^2 \sum_n \left(\frac{Q_{11}^{22}}{E - E_{1n} - \Sigma^{11}} + \frac{Q_{22}^{22}}{E - E_{2n} - \Sigma^{22}} \right) = \Gamma_A^2 \sum_n (Q_{11}^{22} G_n^{11} + Q_{22}^{22} G_n^{22}), \quad (16b)$$

or, in matrix form,

$$\Sigma_i(E) = \Gamma_A^2 \sum_n \sum_j \frac{Q_{ij}}{E - E_{jn} - \Sigma_j(E)} = \Gamma_A^2 \sum_n \sum_j Q_{ij} G_{jn}(E), \quad (17)$$

where we introduced the notations

$$G_{in} = G_n^{ii}, \;\; \Sigma_i = \Sigma^{ii}, \quad (18a)$$

$$Q_{11} = Q_{11}^{11}, \;\; Q_{12} = Q_{21} = Q_{22}^{11} = Q_{11}^{22}, \;\; Q_{22} = Q_{22}^{22}. \quad (18b)$$

We see from Eqs. (16) that the diagonal components of the Green function depend only on the diagonal components of the self-energy and vice-versa. As we will see in the next section, the nondiagonal component of the Green function G^{12} does not enter in the expression for the conductivity and, therefore, is of no interest. Note also that for the symmetric quantum well $Q_{11}^{12} = Q_{22}^{12} = 0$, and according to Eq. (9), both G^{12} and Σ^{12} are equal to zero identically.

Concluding this section, the following remark is in order. The sum over n in the rhs of Eq. (17) for $\Sigma_i(E)$ diverges at large n. However, in calculating the conductivity we will need only the *imaginary* part of the self-energy, which is finite. The divergence of the real part of $\Sigma_i(E)$ implies that the fluctuations of the random potential cause an infinite shift of the bottom of the ith subband. This divergency is not related to the magnetic field. It is just a consequence of the zero correlation length of the white-noise potential. The careful treatment of the divergency can be found in Ref. 26 for the 3D case (see, also, the review in Ref. 27). This treatment reduces to the introduction of the "physical" (renormalized) energy, measured from the shifted bottom of the band. Particularly, in the paper by Thouless and Elzain[28] the renormalization was carried out for the 2D case in the framework of the coherent-potential approximation—the zero-field analog of the SCBA, which we employ for the calculation of the conductivity.

Since the renormalization, in general, can depend on energy, one might think that it could destroy the Landau levels equidistance, which is crucial for our consideration. Indeed, in the 3D case the energy shift increases for negative energies as[27] $\sqrt{|E|}$. However, in the 2D case the divergence is logarithmical so that the energy dependence vanishes as the correlation length of the random potential tends to zero. In the rest of the paper, calculating the conductivity, we will assume that the difference $E - E_i - \Sigma_i(E)$ is already renormalized in the same way as in Ref. 28.

III. CALCULATION OF THE MAGNETOINTERSUBBAND CONTRIBUTION TO THE CONDUCTIVITY

We start from the conventional Kubo formula for the longitudinal conductivity in a magnetic field,

$$\sigma = \int_{-\infty}^{\infty} dE \left[-\frac{\partial f_0(E)}{\partial E} \right] \sigma(E) \quad (19)$$

with

$$\sigma(E) = \frac{\pi e^2 \hbar}{\Omega} \langle \mathrm{Tr}\left[v_x \delta(E - H) v_x \delta(E - H) \right] \rangle. \quad (20)$$

Here $f_0(E)$ is the Fermi distribution function, H is the Hamiltonian, $\langle \cdots \rangle$ denotes the average over the random potential $V(\mathbf{r})$, v_x is the x-component of the velocity operator, and Ω is the 2D normalization volume (the normalization length along the z-axis is taken to be unity). The trace in Eq. (20) is taken over the complete set of the wave functions, which we choose to be the unperturbed wave functions (4). In this basis the matrix element of the velocity operator is given by

$$\langle jnk_y | v_x | j'n'k_y' \rangle = il\omega \left[\delta_{n',n+1} \left(\frac{n+1}{2} \right)^{1/2} - \delta_{n',n-1} \left(\frac{n}{2} \right)^{1/2} \right] \delta_{jj'} \delta_{k_y k_y'}. \quad (21)$$

In the framework of the SCBA approach, the δ functions in the rhs of Eq. (20) should be averaged independently.[12,23] Replacing $\langle \delta(E - H) \rangle$ by $\mathrm{Im}G(E)/\pi$ and using Eq. (21) we get

$$\sigma(E) = \frac{e^2 \hbar \omega^2}{4\pi^2} \sum_{in} (2n + 1) G_{in}''(E) G_{i,n+1}''(E), \quad (22)$$

with $G_{in}''(E) = \mathrm{Im}G_{in}(E) = \mathrm{Im}\left[E - E_{in} - \Sigma_i(E) \right]^{-1}$. It is seen from Eq. (22) that G^{12} and, hence, Σ^{12} do not contribute to the conductivity. The reason is that the matrix element (21) is diagonal in the subband indexes.

The density of states (DOS) in the two-subband case is given by

$$g(E) = \frac{1}{2\pi^2 l^2} \sum_{in} G_{in}''(E) = \frac{1}{2\pi^2 l^2 \Gamma_A^2} \sum_{ij} Q_{ij}^{-1} \Sigma_j''(E) \quad (23)$$

where $\Sigma_j''(E) = \mathrm{Im}\Sigma_j(E)$ and Q^{-1} is the inverse of the matrix Q defined by Eq. (18b). In deriving Eq. (23), we expressed G'' through Σ'' via the SCBA equation (17).

A. Strong disorder

Consider the case of strong disorder, i.e., $\Sigma_i'' \gg \hbar\omega$. To solve the SCBA equation (17) for $\Sigma''(E)$ we rewrite it using the Poisson summation formula. We have

$$\Sigma_j'' = -\frac{\Gamma_A^2}{\hbar\omega}\sum_k Q_{jk}\mathrm{Im}\sum_{p=-\infty}^{\infty}\exp[2\pi i p(E-E_k)/\hbar\omega] \times\int_{-(E-E_k)}^{\infty}\frac{\exp(2\pi i p x/\hbar\omega)}{x+i\Sigma_k''}dx, \quad (24)$$

where we neglected the exponentially small real part of Σ. We assume that E_F lies well above E_2, so that $E_F - E_2 \sim E_2$. On the other hand, according to Eq. (5), $\Sigma_j'' \ll E_2$. Therefore in the relevant range of energies one can extend the lower limit of the integral over x to $-\infty$. Iterating Eq. (24) and keeping only the first harmonics in the sum over p, we obtain

$$\Sigma_i'' = \frac{\pi\Gamma_A^2}{\hbar\omega}\sum_j Q_{ij}F_j, \quad (25)$$

where

$$F_i(E) = 1 + 2\exp(-2\pi\Gamma_i/\hbar\omega)\cos[2\pi(E-E_i)/\hbar\omega] \quad (26)$$

and

$$\Gamma_i = \frac{\pi\Gamma_A^2}{\hbar\omega}\sum_j Q_{ij} = \frac{\hbar}{2\tau_i}. \quad (27)$$

It is seen from Eq. (27) that the width Γ_i of the Landau level, belonging to the ith subband, does not depend on n and represents the sum $\Gamma_i = \Gamma_{ii}^{\rm intra} + \Gamma_{ij}^{\rm inter}$ of the intrasubband $\Gamma_{ii}^{\rm intra} = Q_{ii}\pi\Gamma_A^2/\hbar\omega$ and the intersubband $\Gamma_{ij}^{\rm inter} = Q_{ij}\pi\Gamma_A^2/\hbar\omega$ contributions. The latter vanishes as the intersubband coupling Q_{ij} $(i \neq j)$ goes to zero. The density of states, calculated with the use of Eq. (25), has the form

$$g(E) = \sum_i g_i(E) = \frac{1}{2\pi l^2\hbar\omega}\sum_i F_i(E) = \frac{m}{\pi\hbar^2}\left\{1 + \sum_i \exp\left(-\frac{\pi}{\omega\tau_i}\right)\cos[2\pi(E-E_i)/\hbar\omega]\right\}. \quad (28)$$

It represents a simple superposition of the "partial" densities of states $g_i(E)$ corresponding to each subband. Note that each τ_i includes the contribution from the intersubband scattering.

To calculate the conductivity we apply the Poisson summation formula to Eq. (22),

$$\sigma(E) = \frac{e^2}{2\pi^2\hbar}\sum_j\sum_{p=-\infty}^{\infty}\frac{E-E_j}{\Sigma_j''}\exp[2\pi i p(E-E_j)/\hbar\omega] \times\int_{-\infty}^{\infty}\frac{dx\,\exp(2\pi i p x\Sigma_j''/\hbar\omega)}{(x^2+1)[(x+\hbar\omega/\Sigma_j'')^2+1]}, \quad (29)$$

where we again used the condition $\Sigma''/E \ll 1$. The integral over x in the rhs of Eq. (29) is equal to $(\pi/2)[1 + (\hbar\omega/2\Sigma_j'')^2]^{-1}\exp(-2\pi|p|\Sigma_j''/\hbar\omega)$. Keeping only the first harmonics in the sum over p we get

$$\sigma(E) = \frac{e^2}{4\pi\hbar}\sum_i\frac{F_i(E)}{\Sigma_i''(E)}\frac{E-E_i}{1+[\hbar\omega/2\Sigma_i''(E)]^2}, \quad (30)$$

with Σ_i'' and F_i given by Eqs. (25) and (26). Let us trace the transition to the one-subband case in Eq. (30). When the subbands are uncoupled (i.e., $Q_{12} = 0$), the factor

$$\frac{F_i}{\Sigma_i''} = \frac{\hbar\omega}{\pi\Gamma_A^2}\frac{F_i}{Q_{ii}F_i + Q_{ij}F_j} \quad (31)$$

in the rhs of Eq. (30) reduces to a constant equal to $\hbar\omega/\pi\Gamma_A^2 Q_{ii}$ for each subband. Then the total conductivity represents a superposition of one-subband conductivities with no intersubband scattering contributions [see Eqs. (25)–(27)].

In the presence of intersubband scattering (i.e., $Q_{12} \neq 0$), the rhs of Eq. (31) oscillates with magnetic field, causing the magnetointersubband oscillations of the conductivity. After simple calculations we finally obtain

$$\sigma(E) = \sigma_1(E) + \sigma_2(E), \quad \sigma_i(E) = \sigma_i^0(E) + \sigma_i^{\rm SdH}(E) + \sigma_i^{\rm MIS}(E), \quad (32a)$$

$$\sigma_i^0(E) = (E-E_i)\frac{e^2}{2\pi\hbar^2}\frac{\tau_i}{1+\omega^2\tau_i^2}, \quad (32b)$$

$$\frac{\delta\sigma_i^{\rm SdH}(E)}{\sigma_i^0(E)} = \frac{4\omega^2\tau_i^2}{1+\omega^2\tau_i^2}\exp\left(-\frac{\pi}{\omega\tau_i}\right)\cos[2\pi(E-E_i)/\hbar\omega], \quad (32c)$$

$$\frac{\delta\sigma_i^{\rm MIS}(E)}{\sigma_i^0(E)} = -2\frac{\tau_i}{\tau_{ij}^{\rm inter}}\left(1-\frac{2\omega^2\tau_i^2}{1+\omega^2\tau_i^2}\right)\left\{\exp\left[-\frac{\pi}{\omega}\left(\frac{1}{\tau_i}+\frac{1}{\tau_j}\right)\right]\cos[2\pi(E_i-E_j)/\hbar\omega] - \exp\left(-\frac{\pi}{\omega\tau_i}\right)\cos[2\pi(E-E_i)/\hbar\omega] + \exp\left(-\frac{\pi}{\omega\tau_j}\right)\cos[2\pi(E-E_j)/\hbar\omega]\right\}, \quad (32d)$$

where

$$\frac{1}{\tau_i} = \frac{1}{\tau_{ii}^{\rm intra}} + \frac{1}{\tau_{ij}^{\rm inter}}, \quad \tau_{ii}^{\rm intra} = \frac{\hbar}{2\Gamma_{ii}^{\rm intra}} = \frac{1}{Q_{ii}}\frac{\hbar^3}{\gamma m}, \quad \tau_{ij}^{\rm inter} = \frac{\hbar}{2\Gamma_{ij}^{\rm inter}} = \frac{1}{Q_{ij}}\frac{\hbar^3}{\gamma m}. \quad (33)$$

Equation (32b) represents the classical contribution to the conductivity, while Eq. (32c) describes the usual SdH oscillations.[12,29] The MIS contribution to the conductivity is given by Eq. (32d). The last two terms describe oscillations caused by the difference in the widths of Landau levels belonging to different subbands. Indeed, it can be easily seen that for $\tau_1 = \tau_2$ the contributions of these terms, from different subbands, to the total conductivity cancel each other. These terms as well as the SdH oscillations disappear with increasing temperature since their period depends on energy.

The first term in the rhs of Eq. (32d) is the one we are looking for. The period of oscillations of this term, given by Eq. (2), depends *only* on the subband separation $\Delta = E_2 - E_1$. Therefore, averaging with the Fermi distribution function does *not* affect MIS oscillations as far as $T \ll E_F$. (We neglect for now the acoustic-phonon-assisted broadening of Landau levels which we will be discussing in Sec. V.) Thus, the magnetointersubband correction to the conductivity, which survives at high temperatures, can be written as

$$\frac{\delta\sigma_i^{\rm MIS}}{\sigma_i^0} = -2\frac{\tau_i}{\tau_{ij}^{\rm inter}}\left(1 - \frac{2\omega^2\tau_i^2}{1+\omega^2\tau_i^2}\right) \times \exp\left[-\frac{\pi}{\omega}\left(\frac{1}{\tau_i} + \frac{1}{\tau_j}\right)\right]\cos(2\pi\Delta/\hbar\omega), \quad (34)$$

where σ_i^0 is given by Eq. (32b) with E replaced by E_F.

Equation (34) was derived under the condition $\omega\tau_i \ll 1$. In accordance to the qualitative consideration of Sec. I, the MIS correction has a minimum, when $\Delta/\hbar\omega$ is an integer, which corresponds to completely aligned staircases of Landau levels. It is also seen that the rhs of Eq. (34) changes sign at $\omega\tau_i = 1$. This can be explained as follows. The case $\omega\tau_i \gg 1$ corresponds to a weak disorder (or strong magnetic field). In this case the classical conductivity σ_i^0 is proportional to $1/\tau_i$ [see, e.g., Eq. (32a)]. Then it is clear that the peak in intersubband scattering results in a maximum in the conductivity. Thus, despite the fact that Eq. (34) does not provide the precise shape of oscillations at $\omega\tau_i \gg 1$, it gives a correct qualitative description of the transition from a strong to a weak disorder.

In contrast to the SdH term (32c), the MIS correction to the conductivity contains the product of two Dingle factors, corresponding to each subband. As a result, at low temperatures the MIS oscillations are not visible on the background of the SdH ones. This situation changes with an increasing temperature. The averaging of Eq. (32c) with the Fermi distribution function yields[12,29]

$$\frac{\delta\sigma_i^{\rm SdH}}{\sigma_i^0} = \frac{4\omega^2\tau_i^2}{1+\omega^2\tau_i^2}A\left(\frac{2\pi^2T}{\hbar\omega}\right)\exp\left(-\frac{\pi}{\omega\tau_i}\right) \times \cos[2\pi(E_F - E_i)/\hbar\omega], \quad (35)$$

where $A(x) = x/\sinh(x)$. At $T > \hbar\omega$ the SdH oscillations are damped by the factor $\exp(-2\pi^2T/\hbar\omega)$, while the MIS ones are not. Therefore, at temperatures $T > \hbar/2\pi\tau_i = \Gamma_i/\pi$ the MIS oscillations should dominate.

For a fixed total concentration N_s of electrons, the Fermi level also exhibits oscillations with magnetic field. In the case of two subbands the period of these oscillations depends on the subband separation Δ. However, these oscillations of Fermi level do not contribute to the conductivity at high temperatures. Indeed, with the help of Eq. (28) the concentration can be written as

$$N_s = \sum_i N_{si} = \frac{m}{2\pi\hbar^2}\sum_i\left\{E_F - E_i + \frac{\hbar\omega}{2\pi}A\left(\frac{2\pi^2T}{\hbar\omega}\right)\exp\left(-\frac{\pi}{\omega\tau_i}\right) \times \sin[2\pi(E_F - E_i)/\hbar\omega]\right\}. \quad (36)$$

The Fermi energy, expressed through the concentration, has the form

$$E_F = \frac{N_s}{N_0}\hbar\omega + \frac{1}{2}\sum_i\left\{E_i - \frac{\hbar\omega}{2\pi}A\left(\frac{2\pi^2T}{\hbar\omega}\right)\exp\left(-\frac{\pi}{\omega\tau_i}\right) \times \sin\left[2\pi\left(\frac{N_s}{N_0} \mp \frac{\Delta}{\hbar\omega}\right)\right]\right\}, \quad (37)$$

where $N_0 = 1/\pi l^2$ and the $-$ $(+)$ sign corresponds to $i = 1$ (2). It is seen from Eq. (37) that the oscillating part of E_F vanishes exponentially at high temperatures.

B. Weak disorder

Consider the case of weak disorder, $\Sigma_i'' \ll \hbar\omega$. Then the MIS oscillations of the conductivity represent a system of isolated peaks at integer values of $\Delta/\hbar\omega$. In other words, $\sigma^{\rm MIS}$ is a function of "deviation" δ from the resonance (see Fig. 1)

$$\delta = \left\{\frac{\Delta}{\hbar\omega}\right\}\hbar\omega - \Delta, \quad (38)$$

where here $\{\cdots\}$ denotes the integer part. For $\Sigma_i'' \ll \hbar\omega$ and $\delta \ll \hbar\omega$, the relevant values of energy lie in the vicinity of aligned Landau levels E_{1n} and $E_{1n} - \delta$. Then in SCBA equations (16) only two terms, corresponding to these levels, contribute to the sums over n in the rhs. All other terms are small in parameter $\Sigma_i''/\hbar\omega \ll 1$. The SCBA equations then take the form

$$\Sigma_1(\mathcal{E},\delta) = \Gamma_A^2\left[\frac{Q_{11}}{\mathcal{E} - \Sigma_1(\mathcal{E},\delta)} + \frac{Q_{12}}{\mathcal{E} + \delta - \Sigma_2(\mathcal{E},\delta)}\right], \quad (39a)$$

$$\Sigma_2(\mathcal{E},\delta) = \Gamma_A^2\left[\frac{Q_{12}}{\mathcal{E} - \Sigma_1(\mathcal{E},\delta)} + \frac{Q_{22}}{\mathcal{E} + \delta - \Sigma_2(\mathcal{E},\delta)}\right], \quad (39b)$$

where $\mathcal{E} = E - E_{1n}$, or, in matrix form,

$$\Sigma_i(\mathcal{E},\delta) = \Gamma_A^2\sum_j Q_{ij}G_j(\mathcal{E},\delta), \quad (40)$$

with

$$G_1(\mathcal{E},\delta) = \frac{1}{\mathcal{E} - \Sigma_1(\mathcal{E},\delta)}, \quad G_2(\mathcal{E},\delta) = \frac{1}{\mathcal{E} + \delta - \Sigma_2(\mathcal{E},\delta)}. \quad (41)$$

It should be emphasized that, for $Q_{12} \neq 0$, each of the Σ_i depends on both $\mathcal{E}$ and δ. The system (39) then reduces to quartic equations for each of the Σ_i.

To obtain an expression for the conductivity we note again that in the sum over n in the rhs of Eq. (22), one should keep only the terms corresponding to aligned Landau levels. Since $\Sigma_i'' \ll \hbar\omega$ and $\delta \ll \hbar\omega$, the Green functions $G''_{i,n\pm1}$ can be simplified as

$$G''_{1,n\pm1} = \mathrm{Im}\frac{1}{\mathcal{E} - \Sigma_1 \mp \hbar\omega} \simeq \mathrm{Im}\frac{1}{-\Sigma_1 \mp \hbar\omega} \simeq \frac{\Sigma_1''}{(\hbar\omega)^2}, \tag{42}$$

and similarly for G_2''. Using Eqs. (42) and (40) we get

$$\sigma(E) = \frac{e^2}{\pi^2\hbar^2\omega}\sum_i (E - E_i) G_i'' \Sigma_i'' = \frac{e^2}{\pi^2\hbar^2\omega\Gamma_A^2}\sum_{ij}(E - E_i)\Sigma_i'' Q_{ij}^{-1}\Sigma_j'', \tag{43}$$

where we replaced the numbers of aligned Landau levels by $E - E_i$ for each subband.

The expression (43) for the conductivity should be averaged with the Fermi distribution function. Note that $\Sigma_i''(E)$ is a periodic function of energy, representing the system of sharp peaks. Therefore the integral over the energy in the rhs of Eq. (19) can be rewritten as a sum over all Landau levels and an integral over the width of the particular level. For $T \ll E_F$ we have

$$\sigma = \frac{e^2}{\pi^2\hbar^2\omega\Gamma_A^2}\sum_{ij} Q_{ij}^{-1}(E_F - E_i)\int d\mathcal{E}\,\Sigma_i''(\mathcal{E},\delta)\Sigma_j''(\mathcal{E},\delta)\sum_n\left[-\frac{\partial f_0(\mathcal{E} + E_{1n})}{\partial E_{1n}}\right], \tag{44}$$

where we replaced E by E_F in the prefactor and substituted $E = \mathcal{E} + E_{1n}$ into the argument of the Fermi function. The sum over n in the rhs of Eq. (44) represents a system of smooth peaks with a width of the order of $T \gg \hbar\omega$, so that one can replace the sum by the integral. Neglecting the terms of the order of T/E_F and $\mathcal{E}/E_F$ we finally obtain

$$\sigma(\delta) = \sum_i \sigma_i(\delta) = \frac{e^2}{\pi^2\hbar^3\omega^2\Gamma_A^2}\sum_{ij} Q_{ij}^{-1}(E_F - E_i)\int d\mathcal{E}\,\Sigma_i''(\mathcal{E},\delta)\Sigma_j''(\mathcal{E},\delta), \tag{45}$$

where $\Sigma_i''(\mathcal{E},\delta)$ $(i = 1,2)$ are solutions of the system (39). Let us again trace the transition to the one-subband case. According to Eqs. (39), in the absence of intersubband scattering (i.e., at $Q_{12} = 0$) Σ_1 and Σ_2 are decoupled. On the other hand, the coefficient in front of the overlap integral between Σ_1 and Σ_2 in the rhs of Eq. (45) vanishes at $Q_{12} = 0$, while the integrals of Σ_i^2 do not depend on the "deviation" δ. Thus, each of the "partial" conductivities in the rhs of Eq. (45) reduces to the constant σ_i^0 independent of δ. With switching the subband coupling on (i.e., $Q_{12} \neq 0$), each of Σ_i acquires the nontrivial dependence on δ, determined by Eqs. (39), and, besides, an additional contribution to the rhs of Eq. (45) comes from the overlap integral between Σ_1 and Σ_2. As a result, the MIS contribution to the conductivity $\delta\sigma_i^{\mathrm{MIS}}/\sigma_i^0$ represents a sharp function of δ. Such a sharp dependence emerges from the fact that at resonance values of magnetic field the alignment occurs for entire staircases of Landau levels. The conductivity has a maximum at $\delta = 0$ and decreases with increasing $|\delta|$. We see that $\sigma(\delta)$ does not depend on temperature as it was discussed in Sec. I. The results of numerical calculations of the conductivity as well as of the DOS are presented in the next section.

IV. NUMERICAL RESULTS

To perform numerical calculations it is convenient to introduce the parameters

$$\Gamma = \Gamma_A\sqrt{Q}, \qquad a = Q_{12}/Q, \tag{46a}$$

$$\varepsilon = \mathcal{E}/\Gamma, \qquad z_i = \Sigma_i/\Gamma, \qquad x = \delta/\Gamma, \tag{46b}$$

where $Q = \sqrt{Q_{11}Q_{22}}$. For numerical calculations we assumed that $Q_{11} = Q_{22} = Q$, i.e., the widths of Landau levels belonging to different subbands are equal. The meaning of Γ in Eq. (46) is the half-width of a Landau level in the absence of intersubband scattering. In this notation the system (39) takes the form

$$z_1 = \frac{1}{\varepsilon - z_1} + \frac{a}{\varepsilon + x - z_2}, \tag{47a}$$

$$z_2 = \frac{a}{\varepsilon - z_1} + \frac{1}{\varepsilon + x - z_2}, \tag{47b}$$

and the density of states is

$$g(\varepsilon,x) = g_1(\varepsilon,x) + g_2(\varepsilon,x) = \frac{g_0}{1+a}[z_1''(\varepsilon,x) + z_2''(\varepsilon,x)], \qquad g_0 = \frac{1}{2\pi^2 l^2\Gamma}, \tag{48}$$

where $z_i'' = \mathrm{Im} z_i$. The MIS correction to the conductivity for the ith subband reads

$$\frac{\delta\sigma_i^{\mathrm{MIS}}}{\sigma_i^0} = \frac{1}{1-a^2}\frac{3}{8}\int d\varepsilon\left[(z_i'')^2 - a z_i'' z_j''\right] - 1, \tag{49}$$

where σ_i^0 is the conductivity without the intersubband scattering contribution (i.e., $a = 0$). The factor 3/8 comes from the integral over $\mathcal{E}$ in the rhs of Eq. (45) for $a = 0$. Note that for $Q_{11} = Q_{22}$ the correction (49) is the same for both subbands, so that in this case the MIS contribution to the total conductivity is also given by Eq. (49).

In Fig. 3 we show the evolution of the DOS, $g_1(\mathcal{E},\delta)$, for the first subband at $a = 0.7$ with increasing "deviation" from resonance $\delta = x\Gamma$, together with the DOS without the intersubband scattering contribution.[12] The DOS g_2 for the second subband can be obtained from g_1 by the replacement $\delta \leftrightarrow -\delta$. The value $a = 0.7$ corresponds to the case of the infinite square well. At $\delta = 0$ [Fig. 3(a)], the DOS (for a given Landau level) represents a semiellipse which is $\sqrt{1+a}$ times broader than Ando's semiellipse shown by the dashed line. At the value $\delta \simeq 2\Gamma$ (which is the width of Landau level in the absence of intersubband scattering) two semiellipses, corresponding to different subbands, begin to depart from each other [Figs. 3(b) and 3(c)], until at the value $\delta \simeq 3.5\Gamma$ the complete separation occurs. We see that the intersubband scattering results in an additional peak in the DOS, when the alignment of Landau staircases is destroyed. The stronger peak corresponds to intrasubband scattering, while the intersubband scattering causes the weaker one. The ratio of the heights of these peaks, when they are well separated, is $a = Q_{12}/Q$—the strength of the subband coupling. The evolution of the total DOS is shown in Fig. 4 for the same values of δ and a as in Fig. 3.

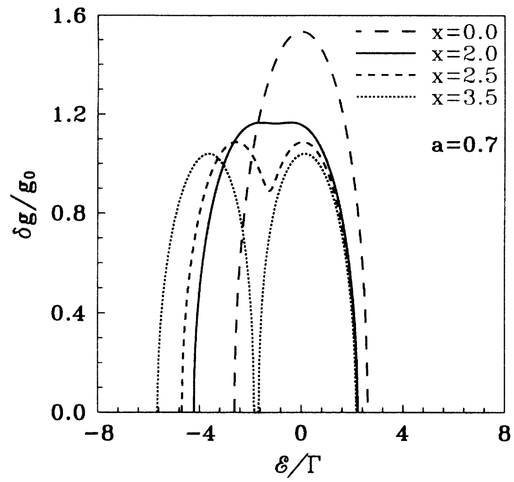

FIG. 4. The evolution of the total DOS at $a = 0.7$ with increasing x is shown for the same values of x as in Fig. 3.

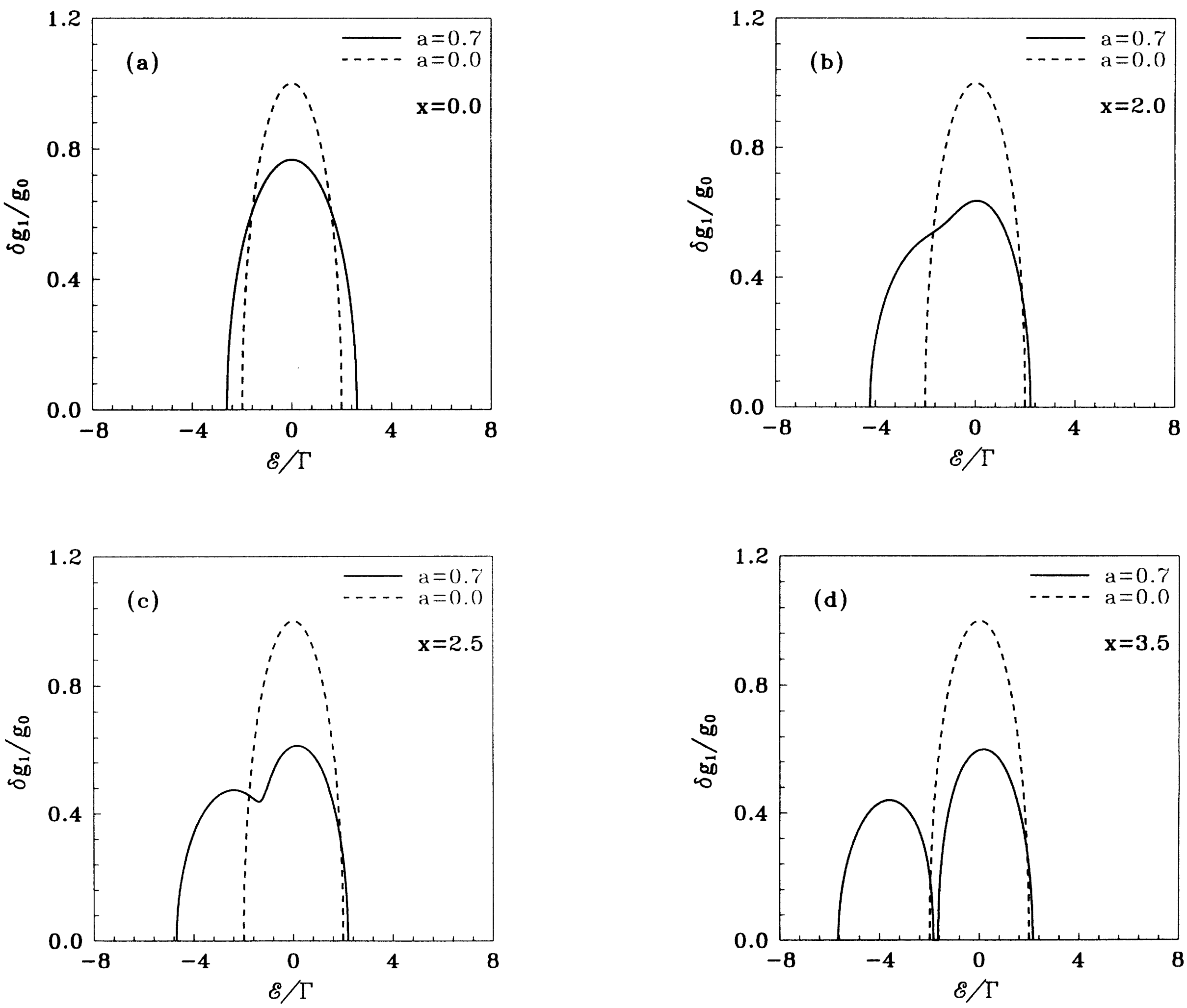

FIG. 3. The DOS for the first subband at $a = 0.7$ is shown for (a) $x = 0.0$; (b) $x = 0.2$; (c) $x = 2.5$, and (d) $x = 3.5$. Dashed line represents Ando's semiellipse at $a = 0$. The parameters a and x are defined by Eq. (46).

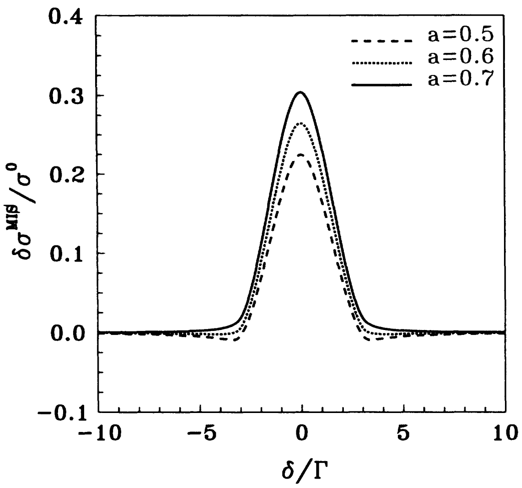

FIG. 5. The MIS contribution to the conductivity as a function of the deviation δ [Eq. (38)] from resonance. Numerical results are shown for the case of a weak disorder at a=0.5, 0.6, and 0.7.

The MIS correction to the conductivity near the resonance versus δ is shown in Fig. 5, for different values of a. It represents an isolated peak at $\delta = 0$ with a width of about $2\Gamma\sqrt{1+a}$. The height of the peak increases with a, reaching about 30% of the background at $a = 0.7$. With increasing $|\delta|$, the MIS correction rapidly falls down and vanishes at $\delta \simeq 3.5\Gamma$—the same value at which complete separation of two semiellipses, constituting the DOS (see Fig. 4), occurs. Note that the values of a chosen in Fig. 5 are close to those for the infinite square well ($a = 0.67$), parabolic ($a = 0.58$), and semiparabolic ($a = 0.52$) potentials.

V. CONCLUSION

In this paper we developed the theory of magnetointersubband (MIS) oscillations of the conductivity in the case when two subbands are occupied. We calculated the MIS contribution to the conductivity using the self-consistent Born approximation, which we generalized to the two-subband case. Calculations were performed both for the cases of weak and strong disorder, which we assumed to be short-range. Physically, MIS oscillations result from the resonant intersubband scattering associated with the alignment of staircases of Landau levels, corresponding to different subbands. We have shown that MIS oscillations survive with increasing temperature, the reason being that their period, given by Eq. (2), does not depend on the position of the Fermi level.

We emphasize that the intersubband scattering requires a large momentum transfer and, hence, occurs in samples with short-range disorder, caused, say, by interface roughness or by impurities close to the 2D electron gas. In quantum Hall samples with a large spacer the random potential is long-ranged and intersubband scattering plays a little role. On the other hand, in the presence of short-range disorder the long-range harmonics of the random potential do not affect the MIS oscillations since they just modulate the bottoms of the subbands, and both staircases of Landau levels follow this modulation.[25] Therefore a smooth potential does not destroy the alignment of these staircases.

In our consideration, we assumed that the energy spectrum of each subband represents a staircase of even-spaced Landau levels. The main reason why this assumption can be violated in real samples is nonparabolicity of the conduction band, which results in a decrease in the interlevel spacing with n. The theoretical and experimental studies performed in Refs. 30 and 31 indicate that the relative decrease in $\hbar\omega$ can be estimated for large n as

$$\frac{\delta E_n}{\hbar\omega} \simeq -\frac{\hbar\omega}{E_c} n, \qquad (50)$$

with $E_c \simeq 1.4$ eV for GaAs. For $B \simeq 0.5$ T and $n = 50$ the ratio (50) is about 0.03, i.e., the alignment of staircases remains unaffected even for considerably large subband separation $\Delta \simeq 40$ meV.

At low temperatures the effect of the alignment of Landau levels, belonging to different subbands, was discussed in the experimental papers of Refs. 13, 15, and 16 in connection with observed anomalies in the DOS and the conductivity in the quantum Hall regime. Such anomalies were caused by a double minimum structure in the total DOS, resulting from the superposition of the DOS for each subband. When the Fermi energy lies in this double-minimum region, a peak in magnetoresistance occurs and the corresponding Hall plateau disappears. The suppressed plateau recovers with increasing electron concentration, when the Fermi level moves up. When the Fermi level reaches the next double-minimum region in the DOS, the corresponding plateau disappears again. The suppression and recovery of quantum Hall states was observed[15,16] for the values of filling factors $\nu = 4$, 8, and 12. Such an observation is possible in the low-temperature regime (the ratio $T/\hbar\omega$ was about 0.07 in experiment) when only Landau levels, closest to the Fermi level, contribute to the conductivity.

The low-field oscillations of the conductivity with an anomalous dependence on temperature were reported by Leadley *et al.* in Ref. 19. They performed Shubnikov–de Haas measurements on a GaAs-$Ga_{1-x}Al_xAs$ heterojunction with two occupied subbands. At $T < 1$ K, the oscillations of conductivity represented a simple addition of sinusoidal oscillations resulting from different subbands. At temperatures above 1 K, two series appeared to be no longer additive but rather multiplicative, i.e., the amplitude of the first-subband series became modulated by the period of oscillations from the second subband. Within the interval $1 < T < 4$ K the degree of modulation was increasing with temperature. Such an increase was first attributed to acoustic-phonon-assisted increase in the intersubband scattering rate. However, the later measurements by the same authors,[20] and also by Coleridge[21] (who observed a similar effect), have shown that acoustic phonons do not, in fact, contribute to the intersubband scattering up to $T \simeq 20$ K.

The rise in intermodulation with temperature can be viewed as an onset of the MIS oscillations. Indeed, the energy distances $E_F - E_1$ and $E_F - E_2$ in experiments[19–21] differed strongly (about 15 times). In this case the sum of the Shubnikov–de Haas term $\delta\sigma_1^{\rm SdH}/\sigma_1^0 \propto \cos[2\pi(E_F - E_1)/\hbar\omega]$ and the magneto-intersubband term $\delta\sigma_1^{\rm MIS}/\sigma_1^0 \propto \cos[2\pi(E_2 - E_1)/\hbar\omega]$ in Eq. (32) can approximately be presented as the SdH oscillations in the first subband modulated by $\cos[2\pi(E_F - E_2)/\hbar\omega]$. Then the temperature damping of SdH oscillations results in an increase in the degree of modulation. The characteristic temperature for such a raise $kT \simeq \Gamma/\pi$ (k being the Boltzmann constant) seems also to be reasonable. Indeed, the width Γ is equal to $\Gamma = \hbar/2\tau$, τ being the scattering time. This time can be estimated from the mobility $\mu = e\tau_{\rm tr}/m$, where $\tau_{\rm tr}$ is the transport relaxation time. For narrow spacer samples used in Refs. 19–21, with the mobility $\mu \simeq 10^5$ cm^2/V^{-1}s^{-1}, a reasonable estimate for the ratio $\tau_{\rm tr}/\tau$ is about 10. Then, for characteristic temperature, we obtain $T \simeq 5\hbar e/\pi k m\mu \simeq 2.2$ K with $m = 0.067m_0$.

It should be pointed out, however, that at $T > 4$ K all the oscillations in Refs. 19– 21 were washed out completely, while our theory predicts that the MIS term should survive at higher temperatures. The possible reason for such a discrepancy is that in our derivation of Eq. (32) it was assumed that a large number of Landau levels are occupied in each subband. This was not the case in Refs. 19– 21. At typical magnetic fields $B \simeq 0.4$ T, only two to three Landau levels in the second subband were occupied. For low Landau levels the modulation of the DOS cannot be presented simply as $\cos[2\pi(E - E_2)/\hbar\omega]$. Consequently, the product $\delta g_1(E)\delta g_2(E)$ cannot be split into the sum of an oscillating term and a term independent of energy. Then it can be shown that this results in the damping of MIS oscillations with temperature. However, such a damping is weaker than $\exp(-2\pi^2 T/\hbar\omega)$ for SdH oscillations.

ACKNOWLEDGMENTS

We are grateful to B. L. Altshuler, Yu. M. Galperin, and D. G. Polyakov for interesting discussions, and to K. Ensslin for informing us about Refs. 13 and 16. We are also grateful to V. Burnett for reading the manuscript.

[1] H. L. Stormer, A. C. Gossard, and W. Wiegman, Solid State Commun. **41**, 707 (1982).
[2] J. J. Harris, D. E. Lacklison, C. T. Foxon, A. M. Suckling, R. J. Nicholas, and K. W. J. Barnhan, Semicond. Sci. Technol. **2**, 783 (1987).
[3] I. M. Lifshitz, Zh. Eksp. Teor. Fiz. **33**, 1569 (1960) [Sov. Phys. JETP **11**, 1130 (1960)].
[4] A. A. Varlamov and A. V. Pantsulaya, Adv. Phys. **38**, 469 (1989).
[5] E. D. Siggia and P. C. Kwok, Phys. Rev. B **2**, 1024 (1970).
[6] S. Mori and T. Ando, Phys. Rev. B **19**, 6433 (1979).
[7] D. G. Cantrell and P. N. Butcher, J. Phys. C **18**, 5111 (1985).
[8] N. N. Ablyazov, M. Yu. Kuchiev, and M. E. Raikh, Phys. Rev. B **44**, 8802 (1991).
[9] R. Fletcher, E. Zaremba, M. D'Iorio, C. T. Foxon, and J. J. Harris, Phys. Rev. B **38**, 7866 (1988).
[10] R. Fletcher, E. Zaremba, M. D'Iorio, C. T. Foxon, and J. J. Harris, Phys. Rev. B **41**, 10 649 (1990).
[11] R. Fletcher, J. J. Harris, and C. T. Foxon, Semicond. Sci. Technol. **6**, 54 (1991).
[12] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[13] K. Ensslin, D. Heitmann, R. R. Gerhardts, and K. Ploog, Phys. Rev. B **39**, 12 993 (1989).
[14] J. H. Baskey, A. J. Rimberg, S. Yang, R. M. Westervelt, P. F. Hopkins, and A. C. Gossard, Appl. Phys. Lett. **61**, 1573 (1992).
[15] K. Ensslin, N. Sundaram, A. Wixforth, J. N. English, and A. C. Gossard, Phys. Rev. B **43**, 9988 (1991).
[16] K. Ensslin, A. Wixforth, N. Sundaram, P. F. Hopkins, J. N. English, and A. C. Gossard, Phys. Rev. B **47**, 1366 (1993).
[17] T. Sajoto, J. Jo, M. Santos, and M. Shaygan, Appl. Phys. Lett. **55**, 1430 (1989).
[18] M. Shaygan, J. Jo, Y. W. Suen, M. Santos, and V. J. Goldman, Phys. Rev. Lett. **65**, 2916 (1990).
[19] D. R. Leadley, R. J. Nicholas, J. J. Harris, and C. T. Foxon, Semicond. Sci. Technol. **4**, 885 (1989).
[20] D. R. Leadley, R. J. Nicholas, J. J. Harris, and C. T. Foxon, Semicond. Sci. Technol. **5**, 1081 (1990).
[21] P. T. Coleridge, Semicond. Sci. Technol. **5**, 961 (1990).
[22] V. Polyanovsky, Fiz. Tekh. Poluprovodn. **22**, 2230 (1988) [Sov. Phys. Semicond. **22**, 1408 (1988)].
[23] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **36**, 959 (1974).
[24] P. Carra, J. T. Chalker, and K. Benedict, Ann. Phys. (NY) **194**, 1 (1989).
[25] M. E. Raikh and T. V. Shahbazyan, Phys. Rev. B **47**, 1522 (1993).
[26] E. Brezin and G. Parisi, J. Phys. C **13**, L307 (1980).
[27] A. L. Efros and M. E. Raikh, in *Optical Properties of Mixed Crystals*, edited by R. J. Elliot and I. P. Ipatova (Elsevier Science Publishers B.V., Amsterdam, 1988), p. 133.
[28] D. J. Thouless and M. E. Elzain, J. Phys. C **11**, 3425 (1978).
[29] A. Isihara and L. Smrčka, J. Phys. C **19**, 6777 (1986).
[30] U. Ekenberg, Phys. Rev. B **40**, 7714 (1989).
[31] W. Müller, H. T. Grahn, R. J. Haug, and K. Ploog, Phys. Rev. B **46**, 9800 (1992).