# Nonlinear absorption and refractive index of a Brillouin-scattered mode in magnetoactive centrosymmetric semiconductor plasmas

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An analytical investigation has been made for the nonlinear refractive index and absorption coefficient of the Brillouin-scattered Stoke's mode resulting from the nonlinear interaction of an intense pumping light beam with acoustic perturbations internally generated due to the electrostrictive property of the doped semiconductor plasma. The origin of this nonlinear interaction lies in the third-order optical susceptibility arising from the nonlinear current density and electrostriction of the medium. The magnitude of third-order optical susceptibility determined from our analysis is found to agree with the theoretically and experimentally quoted values. The total refractive index and absorption coefficient are determined through the effective susceptibility derived with the help of the coupled-mode theory of plasmas. The magnetic field and the obliquity of the pump are found to augment the refractive index. The absorption coefficient decreases with increasing scattering angle and achieves a minimum value for the backscattered mode. The analysis reveals that the large refractive index and small absorption coefficient can easily be obtained in a magnetized centrosymmetric semiconducting crystal, and establishes their potentials as candidate materials for fabrication of cubic nonlinear devices.

## I. INTRODUCTION

In the present paper, an analytical investigation is presented for the nonlinear refractive index and absorption coefficient of the Brillouin-scattered Stoke's mode in doped centrosymmetric semiconductor plasmas. This scattered mode results from the nonlinear interaction of an intense pumping light beam with acoustic perturbations internally generated due to the electrostrictive property of the medium. The present analysis is based on the consideration that the origin of nonlinear optical properties lies in the third-order nonlinear optical susceptibility of the medium.

The experimental and theoretical investigations of the measurement of nonlinear optical susceptibilities in different media have attracted the attention of many workers in view of their potential uses in modern optoelectronic devices. In centrosymmetric materials, the dominant nonlinear optical processes may be characterized by the third-order nonlinear optical susceptibility  $x^{(3)}$ . It is this nonlinear crystal property that has been extensively exploited for the construction of modern optoelectronic devices.<sup>1-3</sup> Due to the existence of sophisticated fabrication technology and the experimental observations of large optical nonlinearities in the vicinity of band-gap resonant transitions, semiconductors have been the natural choice as active media for the study of nonlinear optical phenomena,  $4^{-6}$  in preference to materials such as gases and liquids.

Recently, the availability of a large number of stable laser sources in the infrared regime has opened a new area of research, viz., nonlinear optical effects (refractive and absorptive) in the semiconductors near the fundamental absorption edges.<sup>7</sup> The nonlinear part of refractive index and absorption coefficient is related to the nonlinear optical susceptibility of the crystal. This makes the investigation of nonlinear susceptibility very important and several approaches to understand the mechanisms involved have already appeared in the literature.<sup>8-10</sup> Since the susceptibility is rapidly converging with respect to pump intensity  $I_{\rm in}$ , only the third-order component  $\chi^{(3)}$  is normally considered; the higher-order components such as  $\chi^{(5)}$  and  $\chi^{(7)}$  are neglected and the nonlinear refraction in its first order (proportional to  $I_{\rm in}$ ) and the effective absorption coefficient can be determined if the effective  $\chi^{(3)}$ of the medium is known.

Optical properties of a material can be modified by an externally applied electric or magnetic field. The refractive index and absorption coefficient as functions of the applied electric and magnetic fields are responsible for many electro-optical and magneto-optical effects. Thus recent attention has focused on the theoretical and experimental investigations of stimulated emission and resonant amplification of far-infrared radiation in the pres-ence of the electric and magnetic fields.<sup>11-14</sup> In the presence of a static magnetic field, the medium supports many new modes and offers many new channels for scattering. Gadkari and Ghosh<sup>15</sup> have studied the stimulated Brillouin scattering (SBS) of a plane polarized pump wave in magnetized crystals. In their paper they investigated the parametric decay of a high-amplitude pump wave into a low-frequency acoustic wave and a scattered helicon wave. Their case corresponds to the propagation of a pump wave exactly parallel to the external magnetic field. Such an exact parallel propagation may not be experimentally feasible. Moreover the electric field of the pump considered by them is parallel to the propagation vector. This again is not the case of realistic situation.<sup>16</sup> For a finite semiconductor plasma  $E_0$  must have components that are both parallel and perpendicular to propagation direction. Thus the most realistic case is to consider a hybrid mode propagating obliquely to the external

magnetic field. However, as far as we know, no such attempt has been made to determine the third-order optical susceptibility arising due to the induced current density and electrostriction and subsequently the nonlinear refractive index and effective absorption coefficient.

In the present report, we have investigated analytically the nonlinear refractive index and absorption coefficient arising due to the SBS of an electromagnetic hybrid wave propagating obliquely in a magnetoactive semiconductor plasma. The semiconductor crystals are assumed to be centrosymmetric (viz., cubic) so that the sound waves can be treated as purely longitudinal or transverse. Using the coupled-mode theory of plasmas, the effective third-order (Brillouin) susceptibility of the crystal is derived. Using this complex Brillouin susceptibility, the nonlinear refractive index and effective absorption coefficient of the Brillouin-scattered Stoke's mode are determined. This theory is based on the principle of parametric coupling of three finite waves and the time-varying field which gives rise to electrostrictive polarization. Finally we have applied the analysis to the cubic semiconductor (n-InSb) data to appreciate the numerical values of refractive index and absorption coefficient.

# **II. THEORETICAL FORMULATIONS**

### A. Induced current density

This section deals with the theoretical formulation of the total induced current density  $\mathbf{J}$  for the signal and Stoke's component of the scattered electromagnetic wave in magnetized doped semiconductor plasmas. We have considered the well-known hydrodynamic model of a homogeneous one-component (electron) plasma subjected to an electromagnetic pump field under thermal equilibrium. In order to study the total induced current density  $\mathbf{J}$ , we consider the propagation of an electromagnetic pump wave

$$\mathbf{E}_{0} = [E_{0x}\mathbf{\hat{x}} + E_{0y}\mathbf{\hat{y}}] \exp[i(k_{0}x - \omega_{0}t)]$$
(1)

in a homogeneous semiconductor plasma having an external magnetostatic field  $\mathbf{B}_s$  in a direction making an arbitrary angle  $\theta$  with the x axis in the x-z plane. As the crystal is assumed to be centrosymmetric, the effect of any pseudopotential can safely be neglected for analytical simplicity. We employ the coupled-mode scheme to obtain the total current density.

The basic equations considered for the analysis are

$$\frac{\partial^2 u}{\partial t^2} - \frac{C}{\rho} \frac{\partial^2}{\partial x^2} u + 2\Gamma_a \frac{\partial u}{\partial t} = \frac{\gamma}{2\rho} \frac{\partial}{\partial x} \{ (E_{\text{eff}})_x E_{1x}^* \} , \quad (2)$$

$$\frac{\partial \mathbf{v}_0}{\partial t} + v \mathbf{v}_0 = \frac{e}{m} \mathbf{E}_{\text{eff}} , \qquad (3)$$

$$\frac{\partial \mathbf{v}_1}{\partial t} + v \mathbf{v}_1 + v_{0x} \frac{\partial \mathbf{v}_1}{\partial x} = \frac{e}{m} (\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_s) , \qquad (4)$$

$$\frac{\partial n_1}{\partial t} + n_e \frac{\partial v_1}{\partial x} + n_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial n_1}{\partial x} = 0 , \qquad (5)$$

$$\mathbf{P}_{es} = -\gamma \mathbf{E}_{\text{eff}} \frac{\partial u^*}{\partial x} , \qquad (6)$$

$$\frac{\partial E_{1x}}{\partial x} = \frac{n_1 e}{\epsilon} + \frac{\gamma}{\epsilon} (E_{\text{eff}})_x \frac{\partial u^*}{\partial x} , \qquad (7)$$

where

$$\mathbf{D} = \boldsymbol{\epsilon} \mathbf{E}_{\text{eff}}, \ \mathbf{E}_{\text{eff}} = \mathbf{E}_0 + \mathbf{v}_0 \times \mathbf{B}_s$$

Equation (2) represents the motion of the lattice in the crystal. Hence  $\mathbf{u}(x,t) = u \exp\{i(k_a x - \omega_a t)\}$  denotes the relative displacement of oscillators from the mean position of the lattice. C is the linear elastic modulus of the crystal and  $\rho$  is the mass density.  $(\omega_a, \mathbf{k}_a)$  represents the frequency and the wave vector of the generated acoustic phonon mode under consideration;  $\Gamma_{\alpha}$  is the phenomenological damping parameter.  $\mathbf{F}(x,t)$  $[=(\gamma/2)(\partial/\partial x)\{(E_{\text{eff}})_x E_{1x}^*\}]$  is the driving force experienced by the lattice due to the electromagnetic pump wave. Because of this electromagnetic field the ions within the lattice move into nonsymmetrical position, usually producing a contraction in the direction of the field and an expansion across it. The electrostatic force thus produced is the origin of the electrostriction in the medium with  $\gamma$  as the electrostriction coefficient of the crystal.

Equations (3) and (4) represent the rate equations for the pump and the signal beam under the influence of a magnetostatic field, respectively.  $\mathbf{B}_s$ ,  $\mathbf{B}_1$  and  $\mathbf{v}_0$ ,  $\mathbf{v}_1$  are, respectively, the equilibrium and perturbed magnetic fields and oscillatory fluid velocities of the electrons of effective mass m and charge e. v is the electron collision frequency. In Eq. (3),  $E_{eff}$  represents the effective electric field which includes the Lorentz force  $(\mathbf{v}_0 \times \mathbf{B}_s)$  due to the external magnetic field. The electron continuity equation is given by Eq. (5) in which  $n_e$  and  $n_1$  are the equilibrium and perturbed carrier densities, respectively. In a Brillouin active centrosymmetric crystal, an acoustic mode is generated due to electrostrictive strain leading to the energy exchange between electromagnetic and acoustic fields and gives rise to electrostrictive polarization  $\mathbf{P}_{es}$ [Eq. (6)]. At very high frequencies of the field which are quite large compared to the frequencies of the motion of the electrons in the medium, the polarization of the medium is considered when neglecting the interactions of the electrons with one another and with the nuclei of the atoms. Thus the electric induction in the presence of an external magnetostatic field is given by  $D = \epsilon E_{eff}$ .<sup>11</sup> The space-charge field  $E_1$  is determined by the Poisson relation [Eq. (7)], where  $\epsilon$  is the dielectric function of the semiconductor expressed equivalently to  $\epsilon_0 \epsilon_1$ ;  $\epsilon_0$  and  $\epsilon_1$ are the absolute permittivity and relative dielectric constant of the crystal, respectively. In the above relations, we have neglected the effect due to  $\mathbf{v}_0 \times \mathbf{B}_1$  by assuming that the propagating acoustic mode is producing a longitudinal electric field.

A carrier density perturbation is produced within the Brillouin active medium due to the electrostrictive force. In a doped semiconductor, these density perturbations can be obtained by the method adopted by Guha, Sen, and Ghosh.<sup>12</sup> Differentiating Eq. (5) with respect to time, and using Eqs. (3) and (4), one gets

$$\frac{\partial^2 n_1}{\partial t^2} + v \frac{\partial n_1}{\partial t} + \overline{\omega}_p^2 n_1 - \frac{n_e \gamma^2 k_a^2 u^* v^z}{\epsilon (\omega_{cz}^2 + v^2)} \overline{E}$$
$$= -i(k_0 + k_a) n_1 \overline{E} , \quad (8)$$

where  $\overline{E} = [(e/m)E_{0x} - \omega_{cz}v_{0y}]$  and  $\overline{\omega}_P^2 = \omega_p^2(\omega_{cx}^2 + v^2)/(\omega_c^z + v^2)$ . Here  $\omega_{cx,z}(= -eB_{sx,z}/m)$  are the components of cyclotron frequency along the x and z axes and  $\omega_P [= (n_e e^2/m\epsilon)^{1/2}]$  is the electron plasma frequency of the medium. In deriving Eq. (8) we have neglected the Doppler shift under the assumption that  $\omega_0 \gg v \gg \mathbf{k}_0 \cdot \mathbf{v}_0$ .

The perturbed electron concentration  $n_1$  will have two components, namely, a fast one  $(n_{1f})$  and a slow one  $(n_{1s})$ , such that  $n_1 = n_{1f} + n_{1s}$ . The slow component is associated with the low-frequency acoustic wave  $(\omega_{\alpha})$ whereas the fast components oscillate at the highfrequency electromagnetic wave frequencies  $(\omega_0 \pm \omega_a)$ . Here we have considered only the resonant sideband frequencies  $(\omega_0 \pm \omega_{\alpha})$  on assuming a long interaction path so that the higher-order scattering terms which are off resonant are neglected<sup>13</sup> and the only waves coupled by the sound waves are the incident wave  $(\omega_0)$  and the scattered waves at  $\omega_0 \pm \omega_{\alpha}$ . As we are interested in the Stoke's component of the scattered electromagnetic wave, the phasematching conditions which are to be satisfied in the present case are as follows:  $\omega_s = \omega_0 - \omega_\alpha$  and  $\mathbf{k}_s = \mathbf{k}_0 - \mathbf{k}_\alpha$ in which  $(\omega_s, \mathbf{k}_s)$  represents the Stoke's mode.

From Eq. (8) we obtain the following coupled equations under the rotating-wave approximation:

$$\frac{\partial^2 n_{1s}}{\partial t^2} + v \frac{\partial n_{1s}}{\partial t} + \overline{\omega}_P^2 n_{1s} = -i(k_0 - k_a)\overline{E}n_{1f}^* , \qquad (9a)$$

$$\frac{\partial^2 n_{1f}}{\partial t^2} + v \frac{\partial n_{1f}}{\partial t} + \overline{\omega}_P^2 n_{1f} - \frac{n_e \gamma k_a^2 v^2 \overline{E} u^*}{\epsilon (\omega_{cz}^2 + v^2)} = -i(k_0 - k_a) \overline{E} n_{1s}^* .$$
(9b)

The above equations exhibit that the nonlinear polarization induced by the intense pump wave leads to the coupling of high-frequency electromagnetic waves and the material excitations wave generated within the medium. The density perturbation associated with the acoustic phonon mode at frequency  $\omega_{\alpha}$  beats with the pump at frequency  $\omega_0$  and produces fast components of the density leading to an exchange of energy among the electromagnetic fields separated in frequency by integral multiples of  $\omega_{\alpha}$ . Since we have neglected the higher harmonics, the Stoke's mode of the scattered component at  $\omega_s = \omega_0 - \omega_{\alpha}$  can be obtained from Eqs. (2) and (9) as

$$n_{1s} = \frac{n_e \gamma^2 k_a^2 \nu^2 (E_{\text{eff}})_x E_{1x}}{2\epsilon \rho (\omega_a^2 - k_a^2 v_a^2 + 2i \Gamma_a \omega_a) (\omega_{cz}^2 + \nu^2)} \\ \times \left[ 1 - \frac{(\delta_i^2 + i\omega_s \nu) (\delta_z^2 - i\omega_a \nu)}{(k_0 - k_a)^2 |\overline{E}|^2} \right]^{-1}, \quad (10)$$

in which  $v_{\alpha} [=(c/\rho)^{1/2}]$  is the acoustic velocity in the medium. Also in Eq. (10) we have defined  $\delta_1^2 = \overline{\omega}_P^2 - \omega_s^2$  and  $\delta_z^2 = \overline{\omega}_P^2 - \omega_a^2$ .

It is evident from Eq. (10) that  $n_{1s}$  depends upon the input pump intensity  $(I_{in})$ , where  $I_{in} = \frac{1}{2} \eta \epsilon_0 c |E_0|^2$  with  $\eta$  and c being the background refractive index of the crystal and the velocity of light, respectively. The density perturbations thus produced affect the propagation characteristics of the generated waves.

The resonant Stoke's component of the induced current density due to finite nonlinear polarization of the medium has been deduced by neglecting the transition dipole moment, which can be represented as<sup>10</sup>

$$= \frac{\epsilon \omega_P^2 \bar{E}_1}{(\omega_0^2 - \omega_{cz}^2)} + \frac{i e \bar{\omega}_P^2 \gamma^2 k_a^2 |(E_{\text{eff}})_x|^2 E_{1x}^*}{2m \rho \omega_0 (\omega_a^z - k_a^2 v_a^2 - 2i \Gamma_a \omega_a)} \left[ 1 - \frac{(\delta_1^2 - i \omega_s v)(\delta_2^2 + i \omega_a v)}{(k_0 - k_a)^2 |\bar{E}|^2} \right]^{-1} = \mathbf{J}_l(\omega_s) + \mathbf{J}_{nl}(\omega_s) , \qquad (11)$$

where  $\overline{E}_1 = i\omega_0 E_{1x} + \omega_{cz} v_{1y}$ . The first part of  $\mathbf{J}(\omega_s)$  [i.e.,  $\mathbf{J}_l(\omega_s)$ ] represents the linear component of the induced current density and is strongly influenced by magnitude and orientation of the external magnetic field via  $\omega_{cz}$ , while the latter part represents the nonlinear current density  $\mathbf{J}_{nl}(\omega_s)$ .

#### B. Effective optical susceptibility

Now treating the induced polarization  $\mathbf{P}_{cd}(\omega_s)$  as the time integral of induced current density  $\mathbf{J}(\omega_s)$ , we may write

$$\mathbf{P}_{cd}(\omega_s) = \int \mathbf{J}(\omega_s) dt \quad . \tag{12}$$

In order to study the effective optical susceptibility  $\kappa_{eff}$ , we can express the induced polarization in a crystal pos-

sessing the property of inversion symmetry in the form of an expansion series

$$\mathbf{P}_{cd}(\omega_s) = \epsilon_0 \varkappa \mathbf{E}_1(\omega_s)$$
$$= \epsilon_0 [\varkappa^{(1)} + \varkappa^{(3)} |\mathbf{E}_{\mathbf{eff}}|^2 + \cdots ] \mathbf{E}_1(\omega_s) , \qquad (13)$$

where  $\kappa^{(1)}$  and  $\kappa^{(3)}$  are the first- and third-order susceptibilities due to induced current density, respectively. Thus one can find out the intensity-dependent optical susceptibility of the crystal by using Eqs. (12) and (13). Because of inversion symmetry,  $\kappa^{(2)}, \kappa^{(4)}$  etc. are all zero; moreover we are not interested in these components as they are responsible for various passive optical properties such as parametric amplification, second-harmonic generation, etc.  $\kappa^{(1)}, \kappa^{(3)}, \kappa^{(5)}$ , etc. account for the linear as well as nonlinear refraction and absorption processes in semiconductors. We have neglected the contribution of  $x^{(5)}$ ,  $x^{(7)}$ , etc. to the active nonlinear optical processes resulting from  $x^{(3)}$ .

Using Eqs. (11) and (12), we will obtain an expression for  $\mathbf{P}_{cd}(\omega_e)$ , now equating the expressions corresponding to the same power of  $\mathbf{E}_1(\omega_s)$  in this derived equation and Eq. (13), we find

$$\kappa_{cd}^{(1)} = \frac{i\epsilon_1 \overline{\omega}_P^2}{\omega_s(\omega_0^2 - \omega_{cz}^2)}$$
(14)

and

-0.04

-1.08

-2.12

-3.16

- 4.20

a (10<sup>9</sup> mks units)

$$\kappa_{cd}^{(3)} = \frac{-\gamma^2 \overline{\omega}_P^2 k_a^2}{2\epsilon_0 \rho \omega_a \omega_s (\omega_a^2 - k_a^2 v_a^2 - 2i\Gamma_a \omega_a)} . \tag{15}$$

From Eqs. (14) and (15), one notices that both  $\kappa_{cd}^{(1)}$  and  $\kappa_{cd}^{(3)}$  are complex. The real part of  $\kappa_{cd}^{(1)}$  is responsible for the linear refraction of laser beam while the imaginary part takes account of the linear absorption process within the crystal. The optical nolinearities are explained by finite real and imaginary parts of  $\kappa_{cd}^{(3)}$  considering that the higher-order nonlinear susceptibilities contribute negligibly.

The electrostrictive strain interacts with the pump wave in the Brillouin active medium giving rise to an electrostrictive polarization  $\mathbf{P}_{es}(\omega_s)$ , analogous to the polarization due to molecular vibrations in the stimulated Raman scattering phenomenon. Thus, besides the nonlinear-induced polarization due to the perturbed current density, the system should also possess an electrostrictive polarization. This electrostrictive polarization  $\mathbf{P}_{es}(\omega_s)$  is obtained from Eq. (6) as

$$\mathbf{P}_{es}(\omega_s) = \frac{-\gamma^2 k_a (k_0 - k_a) |(E_{\text{eff}})_x|^2 E_{1x}}{2\rho(\omega_a^2 - k_a^2 v_a^2 - 2i\Gamma_a \omega_a)}$$
$$= \epsilon_0 \varkappa_{es}^{(3)} |(E_{\text{eff}})_x|^2 E_{1x} . \tag{16}$$

Thus in a doped centrosymmetric crystal the effective third-order optical susceptibility  $x^{(3)}$  is given by

$$\chi_{\rm eff}^{(3)} = \chi_{cd}^{(3)} + \chi_{es}^{(3)} \,. \tag{17}$$

Using Eqs. (15)-(17) one obtains

$$\kappa_{\text{eff}}^{(3)} = \frac{-\gamma^2 k_a^2}{2\rho\epsilon_0(\omega_a^2 - k_a^2 v_a^2 - 2i\Gamma_a \omega_a)} \left\{ \frac{k_0 - k_a}{k_a} + \frac{\overline{\omega}_p^z}{\omega_0 \omega_s} \right\},$$
(18)

in which  $k_a = [k_0^2 + k_s^2 - 2k_0k_s\cos\phi]^{1/2}$ , where  $\phi$  is the scattering angle, i.e., the angle between  $\mathbf{k}_s$  and  $\mathbf{k}_0$ .

#### C. Total refractive index and absorption coefficient

The total refractive index of the system is given by<sup>14</sup>

$$\eta = \eta_l + \eta_{nl} |E_{\text{eff}}|^2 , \qquad (19)$$

in which  $\eta_l$  and  $\eta_{nl}$  are, respectively, the linear and nonlinear parts of the refractive index of the material. From the real parts of  $\kappa^{(1)}$  [  $\operatorname{Re} \kappa^{(1)}$ ] and  $\kappa^3_{eff}$ [ $\operatorname{Re} \kappa^{(3)}_{eff}$ ], one can study the phenomenon of the total refractive index in centrosymmetric crystals because  $\eta_l$  and  $\eta_{nl}$  are given by

$$\eta_l = \sqrt{\{1 + \operatorname{Re}^{(1)}\}} \tag{20}$$

and

12

ゴ4 10

E<sub>eff</sub> (10<sup>7</sup> vm<sup>-1</sup>) ----

FIG. 1. Variation of the refractive index ( $\eta$ ) and the absorption coefficient (*a*) with the effective electric field ( $E_{\text{eff}}$ ) when  $\omega_c = 10^{12} \text{ sec}^{-1}$ ,  $\theta = 45^\circ$  and  $\phi = 60^\circ$ .



$$\eta_{\rm nl} = \operatorname{Re}_{\varkappa_{\rm eff}^{(3)}}.$$
 (21)

Now to study the total absorption coefficient a one may use the following standard relation:

$$a = a_l + a_{nl} |E_{\text{eff}}|^2$$
, (22)

in which  $a_l$  and  $a_{nl}$  are the linear and nonlinear absorption coefficients. From the imaginary parts of  $\chi^{(1)}[\text{Im}\chi^{(1)}]$  and  $\chi^{(3)}_{\text{eff}}[\text{Im}\chi^{(3)}_{\text{eff}}]$ , in the following relations, one can study the phenomenon of total absorption in the centrosymmetric crystals. These relations are

$$a_l = \sqrt{\mathrm{Im} \varkappa^{(1)}} \tag{23}$$

and

$$a_{nl} = \frac{-k_s}{2\epsilon_1} [\operatorname{Im} \varkappa_{\text{eff}}^{(3)}] . \qquad (24)$$

Now Eqs. (19) and (22) can be safely employed to study the nonlinear parameters (refractive and absorptive) of a Brillouin active magnetized semiconductor plasma.

# **III. RESULTS AND DISCUSSION**

Now we shall try to get the numerical appreciation of the analytical results obtained. To do so an *n*-InSb crystal is assumed to be irradiated by a 10.6- $\mu$ m CO<sub>2</sub> laser and the physical parameters used are  $m = 0.014m_0$ , with  $m_0$  being the free-electron mass,  $\epsilon_1 = 15.58$ ,  $\gamma = 5 \times 10^{-10}$  F m<sup>-1</sup>,  $\rho = 5.8 \times 10^9$  kg m<sup>-3</sup>,  $\nu = 3.5 \times 10^{11}$  sec<sup>-1</sup>,  $\Gamma_a = 2 \times 10^{10}$  sec<sup>-1</sup>, and  $n_e = 2.44 \times 10^{24}$  m<sup>-3</sup>. The results are plotted in Figs. 1–4.

The nonlinear optical susceptibility [Eq. (18)] resulting from the stimulated scattering process is found to increase with a rise in the carrier concentration of the medium via plasma frequency  $\omega_P$ . The cubic Brillouin suscepetibility with a carrier concentration of the order of  $10^{24}$  m<sup>-3</sup> is found to be  $\sim 8 \times 10^{-11}$  esu. In this representative calculation, we have used the conversion formula  $\chi^{(3)}$  (SI)/ $\chi^{(3)}$  (esu)=1.4×10<sup>-8</sup> (see Ref. 2, p. 316). However, at lower doping level the magnitude of  $\chi^{(3)}_{\text{eff}}$  is lowered by about five orders and is potentially not very useful in the fabrication of cubic nonlinear devices. The third-order susceptibility due to electrostriction  $\chi^{(3)}_{\text{es}}$  is around  $5 \times 10^{-11}$  esu. The magnitude of the third-order susceptibility agrees well with the experimentally observed<sup>15</sup> and theoretically quoted values.<sup>17,18</sup>

Figure 1 depicts the variation of the refractive index  $\eta$ and the absorption coefficient a with the effective electric field  $E_{\text{eff}}$ . It is seen from this graph that with an increase in the effective electric field  $\eta$  increases while a decreases. This result is quite obvious because with the increase in the transmitted power, absorption should decrease. The variation of the refractive index  $\eta$  and the absorption coefficient a with the magnetic field via the cyclotron frequency is depicted in Fig. 2. Here again the refractive index increases while the absorption coefficient decreases with the increment in cyclotron frequency. It may be inferred that a very slow rate of change is observed with both parameters in the regime  $2 \times 10^{12}$  sec<sup>-1</sup>  $\leq \omega_c \leq 5 \times 10^{13}$  sec<sup>-1</sup>. Below and above this regime rates of change are quite fast. This type of variation can be understood from the fact that the nonlinear parameters for the Brillouin mode depend upon two factors: first the coupling of the electron motion with the acoustic mode responsible for induced current density and second the nonlinear force due to the electrostriction responsible for

FIG. 2. Variation of the refractive index ( $\eta$ ) and the absorption coefficient (*a*) with cyclotron frequency ( $\omega_c$ ) when  $\theta = 45^{\circ}$ ,  $E_{\text{eff}} = 10^8$  V m<sup>-1</sup>, and  $\phi = 60^{\circ}$ .







FIG. 3. Variation of the refractive index  $(\eta)$  and the absorption coefficient (a) with the inclination of magnetic field  $(\theta)$ when  $E_{\text{eff}} = 10^8 \text{ Vm}^{-1}$ ,  $\omega_c$  $= 5 \times 10^{12} \text{ sec}^{-1}$ , and  $\phi = 60^\circ$ .

driving the absorption characteristics. The contribution of the coupling between electron motion and the acoustic wave to  $\chi_{\text{eff}}^{(3)}$  can be seen through the second term in curly brackets in Eq. (18). For  $\omega_c < 3 \times 10^{12} \text{ sec}^{-1}$ , this coupling parameter increases rapidly while polarization due to electrostriction remains almost constant; thus  $\eta$  increases while *a* decreases sharply due to the induced current density. In the strong magnetic-field regime when  $\omega_c > 5 \times 10^{13} \text{ sec}^{-1}$ , polarization due to electrostriction dominates the process which again resulted in a sharp increment in  $\eta$  and decrement in *a*. In the middle regime, both effects combine to give a slow rate of



FIG. 4. Variation of the refractive index ( $\eta$ ) and the absorption coefficient (a) with the scattering angle ( $\phi$ ) when  $E_{\text{eff}}$ = 10<sup>8</sup> V m<sup>-1</sup>,  $\omega_c$  = 5×10<sup>12</sup> sec<sup>-1</sup>, and  $\theta$ =45°.

change. Figure 3 depicts the variation of the refractive index and the absorption coefficient with the inclination of the magnetic field with respect to the direction of propagation. It is found that the refractive index  $\eta$  increases whereas a decreases down to  $\theta < 30^{\circ}$  and for  $\theta > 30^{\circ}$  both become independent of it. This is quite natural as the cross magnetic field should have a greater effect on the growth (negative absorption) and propagation characteristics (refractive index) as compared to the parallel magnetic field. This is why  $\eta$  increases and a decreases as  $\theta \rightarrow 90^\circ$ . It may be inferred from Fig. 3 that when the magnetic field is directed with an angle  $\theta \ge 30^{\circ}$ to the incident photon propagation axis, the field configuration becomes more profitable to obtain larger values of the refractive index and smaller values of the absorption coefficient. The dependence of  $\eta$  and a on scattering angle  $\phi$  are shown in Fig. 4. Both decrease with the increase in the scattering angle. This can be understood from the fact that increasing values of the scattering angle tend to increase the coupling term

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 $|k_a(E_{\text{eff}})_x|$  and hence the absorption decreases. Thus one may get minimum absorption for the backscattered mode  $(\phi = 180^\circ)$ .

The above discussion reveals that the large refractive index and small absorption coefficient can be easily achieved in magnetized centrosymmetric semiconducting crystals in the configuration for the magnetic field oblique to the propagation direction of the incident photon. This establishes its potential as a candidate material for the fabrication of phase conjugate mirrors, etc. when the crystal is heavily doped.

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