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Eikonal approximation in the theory of two-dimensional fermions with long-range current-current interactions

D. V. Khveshchenko
Theoretische Physik, Eidgenössische Technische Hochschule Zürich-Hönggerberg, CH-8093 Zürich, Switzerland and Landau Institute for Theoretical Physics, 2 Kosygina Street, 117940 Moscow, Russia

P. C. E. Stamp
Physics Department, University of British Columbia, 6224 Agricultural Road, Vancouver, Canada V6T 1Z1
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We study the behavior of response functions of two-dimensional (2D) fermions interacting via a long-range transverse gauge field in the eikonal approximation. We observe that an exponentially vanishing wave-function renormalization prevents divergences in the density-density correlation function and the pairing susceptibility. The wave-function renormalization also shows up in a suppression of the de Haas–van Alphen effect. Elaborating on an observation recently made by Ioffe, Lidsky, and Altschuler, we also infer an effective bosonic description which makes it possible to reproduce our eikonal results in terms of free 2D bosons.

I. INTRODUCTION

Recently, much attention has been paid to the general problem of the existence of a metal-like non-Fermi-liquid ground state for interacting fermions in $D \geq 2$. Stimulated by Anderson's "Luttinger liquid" hypothesis about the ground state of the (2D) two-dimensional Hubbard model,[1] the previous studies mainly concentrated on short-ranged interactions. In this framework Anderson's hypothesis was disputed by a number of papers which claim the absence of anomalies in the conventional perturbation expansion and the validity of the Landau-Fermi-liquid picture for quasiparticle excitations.[2,3] However these conclusions cannot be simply extended onto the case of long-ranged interactions which are, in fact, of primary physical importance.

A 2D example provided by the Coulomb interaction $V(q) \sim \frac{1}{q^2}$ was recently shown to demonstrate some features of a breakdown of the regular perturbation expansion due to the pecularity of the Debye screening in 2D.[4]

Another relevant example is provided by the retarded current-current interaction of charged fermions mediated by a transverse gauge field. It was noticed in Ref. 5 and then elaborated in Refs. 6 and 7 that a relativistic transverse electromagnetic interaction in 3D metals leads to anomalies in perturbation theory.

In 2D a similar problem of the interaction via the transverse gauge field arises in the context of the modern gauge theory of strongly correlated electrons in doped Mott insulators[8] which is supposed to be an adequate description of the normal properties of high-T_c compounds. It has been also argued that the $\nu = 1/2$ fractional quantum Hall effect can be effectively described by the same kind of theory.[9]

The gauge model of nonrelativistic spinless fermions with chemical potential μ is given by the Hamiltonian written in the gauge $A_0 = 0$:

$$H = \Psi^\dagger \left(\frac{1}{2m}(-i\nabla - g\mathbf{A})^2 - \mu \right) \Psi + \frac{1}{2}(\nabla \times \mathbf{A})^2 + \frac{1}{2c^2}\left(\frac{\partial \mathbf{A}}{\partial t}\right)^2 . \quad (1.1)$$

Intending to concentrate on the effects of the transverse gauge field $A_\perp(k) = \frac{\mathbf{k}\times\mathbf{A}}{k}$, we shall discard the longitudinal component $A_\parallel(k) = \frac{\mathbf{k}\cdot\mathbf{A}}{k}$ responsible for the density-density Coulomb interaction, which was shown in Ref. 4 to lead to weaker singularities.

On the bare level the gauge field propagator

$$D_{ij}^{(0)}(\omega, \mathbf{q}) = \langle A_i(\omega, \mathbf{q}) A_j(-\omega, -\mathbf{q}) \rangle = \frac{\delta_{ij} - \frac{q_i q_j}{q^2}}{-\omega^2 c^{-2} + q^2}$$

has a pole corresponding to a propagating mode $\omega = cq$. However in a metallic state with gapless charge excitations the gauge field spectrum becomes strongly renormalized due to particle-hole excitations. The gauge field dispersion corresponds to an overdamped mode $\omega \sim iq^3$:

$$D_{ij}(\omega, q) = \frac{\delta_{ij} - \frac{q_i q_j}{q^2}}{i\gamma\frac{\omega}{q} + \chi q^2}. \quad (1.2)$$

Formally this form of the propagator can be understood as a result of the fermion polarization (Fig. 1) which is governed by the Landau damping term, the coefficient γ being some function of the coupling constant. The random phase approximation (RPA) result is $\gamma_{\text{RPA}} \sim g^2$ while $\chi_{\text{RPA}} = 1 + O(g^2)$. Moreover at small ω and q the form of the gauge field propagator is quite insensitive to details of the charge excitation spectrum unless it develops a gap.

In the lowest order the fermion self-energy is given by the diagram of Fig. 2. In 2D this expression was calcu-

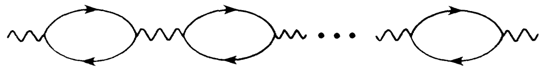

FIG. 1. RPA fermion polarization corrections to the gauge propagator.

FIG. 2. Fermion self-energy corrections.

lated in the form[8,9]

$$\Sigma(\epsilon) \approx -g^2 \frac{p_F}{m\chi^{2/3}\gamma^{1/3}}(i\epsilon)^{2/3} \quad (1.3)$$

if $\epsilon \gg \frac{\xi_p^3}{\mu^2}$, where $\xi_p = \frac{p^2}{2m} - \mu$ is the bare quasiparticle spectrum, otherwise the self-energy behaves as $\mathrm{Re}\Sigma(\epsilon,p) \sim -g^2\frac{\epsilon}{\xi_p}$, $\mathrm{Im}\Sigma(\epsilon,p) \sim -g^2(\epsilon^2/\xi_p^4)\mathrm{sgn}\epsilon$.

Despite the obviously singular character of the correction (1.3) it was argued in Ref. 9 that the system still could be considered in the framework of some modified Landau-Fermi-liquid theory. Namely, it was conjectured that the fermion Green function still has a pole corresponding to the renormalized quasiparticle dispersion $\epsilon(p) \sim (p-p_F)^{3/2}$, although both real and imaginary parts of the spectrum were supposed to be of the same order.

Indeed, it can be easily shown that the functional form of $\Sigma(\epsilon)$ given by (1.3) does not change if one takes into account only completely uncrossed (ladder) diagrams.[10,11] However, one can neglect the vertex corrections to (1.3) only at $\epsilon \gg \frac{g^6}{\gamma}$, otherwise one cannot fulfill the Ward identity

$$\Lambda(\epsilon,p) = 1 - \frac{\partial\Sigma(\epsilon,p)}{\partial\epsilon}, \quad (1.4)$$

where $\Lambda(\epsilon,p)$ stands for the irreducible three-point vertex at zero transferred momentum $\Lambda(\epsilon,\mathbf{p};\frac{\omega}{\mathbf{k}}\to 0,\mathbf{k}\to 0)$. Equation (1.4) implies that close to the Fermi surface there exists a whole series of infrared divergent terms, the expression (1.3) being the first term of this series. Obviously, a complete summation of higher order self-energy and vertex corrections requires essentially nonperturbative methods. In our previous paper[12] we first applied the so-called eikonal approximation to study the behavoir of the one-particle Green function. In the present paper we undertake a consistent summation of higher order corrections to two-point response functions by means of the eikonal approximation.

The use of this approximation in the case of the conventional relativistic 3D electrodynamics provides an elegant and efficient way to find the well-known nonpole infrared asymptotics of the one-particle Green function as well as the double-logarithmic asymptotics of the three-point vertex function.[13]

In our case of nonrelativistic 2D fermions with singular interactions a general possibility in applying the eikonal approximation follows from the fact that for quasiparticles near the Fermi surface the small angle scattering becomes dominant. More exactly, the validity of the eikonal approximation is restricted by those amplitudes which receive their main contributions from excitations in the vicinity of the Fermi surface.

In addition, in the case of the transverse gauge interaction the overdamped dispersion (1.2) provides essentially different scales for energy and momentum transfer

$$\omega \ll v_F q_\parallel \sim \frac{1}{m}\left(\frac{\gamma\omega}{\chi}\right)^{2/3} \ll v_F q_\perp \sim v_F\left(\frac{\gamma\omega}{\chi}\right)^{1/3}, \quad (1.5)$$

where $q_{\parallel,\perp}$ are the longitudinal and transverse components of $\mathbf{q}$ with respect to the particle's momentum $\mathbf{p}$.

This circumstance opens a very interesting avenue of research initiated by recent discussions of a possibility in formulating a consistent bosonization procedure in $D > 1$.[14–17] It was argued in Ref. 18 that it might be possible to describe a long-wavelength behavior of 2D fermions interacting via the transverse gauge field in terms of free bosons. This conjecture was made in the framework of the 2D generalization of the bosonization procedure originally proposed by Haldane.[15]

In a recent paper by Ioffe, Lidsky, and Altshuler[19] it was explicitly shown how an effective bosonic quasi-1D description arises in the course of a straightforward diagrammatic calculation of the one-particle Green function. In the present paper we elaborate this observation and demonstrate that the results which can be found by means of the effective bosonic description are in agreement with those obtained in the eikonal approximation.

II. ONE-PARTICLE PROPERTIES

A formal eikonal expansion of the one-particle Green function starts from a derivation of the Green function $G(\mathbf{r},\mathbf{r}';t,t';\mathbf{A})$ in a given external field $\mathbf{A}(\mathbf{r},t)$

$$\left(i\frac{\partial}{\partial t} - \frac{1}{2m}(-i\boldsymbol{\nabla} - g\mathbf{A})^2 + \mu\right)G(\mathbf{r},\mathbf{r}';t,t';\mathbf{A})$$

$$= \delta(\mathbf{r}-\mathbf{r}')\delta(t-t'). \quad (2.1)$$

For details of this procedure we refer to our previous paper.[12] To find the one-particle Green function one then has to average $G(\mathbf{r},\mathbf{r}';t,t';\mathbf{A})$ over Gaussian gauge fluctuations which are governed by the propagator $D_{ij}(\omega,\mathbf{q}) = \langle A_i(\omega,\mathbf{q})A_j(-\omega,-\mathbf{q})\rangle$ given in (1.2).

The resulting expression can be written in the integral form (n_p is the Fermi distribution function):

$$G(\epsilon,\mathbf{p}) = i\int_0^\infty d\alpha e^{i\alpha(\epsilon-\xi_p+i\delta_p)} \exp\left(ig^2\int\frac{d\mathbf{q}d\omega}{(2\pi)^3}D_{ij}(\omega,\mathbf{q})\frac{p_ip_j}{m^2}\frac{1-e^{i\alpha(\xi_\mathbf{p}-\xi_{\mathbf{p}-\mathbf{q}}+\omega)}}{(\xi_\mathbf{p}-\xi_{\mathbf{p}-\mathbf{q}}+\omega)^2}\right). \quad (2.2)$$

Expanding (2.2) in g^2 one generates all crossed graphs including the so-called "maximally crossed" ones which are supposed to give the dominant contributions in the case of singular (long-range) interactions.[20]

When calculating the exponential "Debye-Waller factor" in (2.2) it is important to notice that the integrations over longitudinal and transversal components of $\mathbf{q}$ can be done separately because of the condition (1.5).

The behavior of the Green function in the vicinity of the Fermi energy can be simply found at $p = p_F$.[12] Estimating the integral over α one can see that at $\epsilon \gg \tilde{g}^6$ the lowest order results of perturbation theory are easily reproduced

$$G(\epsilon, p_F) = \frac{1}{\epsilon}\left(1 - \frac{\tilde{g}^2 i^{2/3}}{\epsilon^{1/3}} + \cdots\right). \tag{2.3}$$

On the contrary, at $\epsilon \ll \tilde{g}^6$ we find an essentially nonperturbative asymptotic behavior

$$G(\epsilon, p_F) \sim \frac{\tilde{g}^{3/2}}{\epsilon^{5/4}} \exp\left(-\frac{\tilde{g}^3}{\epsilon^{1/2}}\right). \tag{2.4}$$

Note that the asymptotics (2.4) is basically a result of the saddle point approximation in the integral over α in (2.2), hence it cannot be expanded in a power series in $\tilde{g}^2$.

It is worthwhile mentioning that the asymptotic behavior (2.4) implies a more radical breakdown of the Fermi-liquid theory for the model (1.1) than any kind of Luttinger liquid behavior proposed, for instance, in the Anderson's scenario of the "Tomographic Luttinger Liquid."[1]

In particular, the wave-function renormalization found from (2.4) vanishes exponentially on the Fermi surface: $Z_{p_F}(\epsilon) \sim \frac{\tilde{g}^{3/2}}{\epsilon^{1/4}} \exp(-\frac{\tilde{g}^3}{\epsilon^{1/2}})$. As a consequence, the distribution function $n(p)$ remains analytic in the vicinity of the Fermi surface and has a finite slope

$$n(p) = \frac{1}{2} - \frac{v_F}{\tilde{g}^6}(p - p_F). \tag{2.5}$$

The unusual low-energy behavior implied by (2.4) does affect physical observables which are sensitive to one-particle properties near the Fermi surface. In particular, it leads to a drastic suppression of oscillations of the orbital magnetization [de Haas–van Alphen effect (dHvA)] in a weak external magnetic field, when $\omega_c \ll \tilde{g}^6$ (and $\omega_c = \frac{H}{m}$ is the cyclotron frequency). The oscillatory part of the magnetic moment is conventionally calculated from the thermodynamic potential, but it can be also found in terms of the one-particle Green function

$$\tilde{M}(H) = \frac{\partial}{\partial H}\int_{-\infty}^{\mu} d\mu \, \mathrm{Tr} \int_{-\infty}^{\mu} d\epsilon \, \mathrm{Im} G(\epsilon; \mathbf{r}, \mathbf{r}'). \tag{2.6}$$

When $\omega_c \ll \tilde{g}^6$, the Green function can be taken in the semiclassical form (we use the Landau gauge for the background field):

$$G(\epsilon; \mathbf{r}, \mathbf{r}') \approx \bar{G}(\epsilon; \mathbf{r} - \mathbf{r}') e^{\frac{iH}{2}(x'-x)(y'+y)}. \tag{2.7}$$

Then to calculate the trace in (2.6) one can use the basis of states provided by Landau levels and to apply the Poisson summation formula.[21] The result is given by the expression

$$\tilde{M}(H) \sim p_F^2 \sum_{r=1}^{\infty} \frac{I_r}{r} \sin\left[2\pi r\left(\frac{F}{H} - \phi\right)\right], \tag{2.8}$$

where the phase factor ϕ depends on the fermion spectrum far from the Fermi surface, and $F = p_f^2/2$ is the "dHvA frequency" of the orbit in $\mathbf{k}$ space. For the case of free fermions the amplitude I_r of the rth harmonic is equal to 1, giving the characteristic sawtooth structure (for 2D fermions) of $\tilde{M}(H)$. If g is finite, then for $\omega_c \ll \tilde{g}^6$, one has

$$I_r = \frac{2\pi i r}{H}\int_{-\infty}^{0} d\epsilon \exp\left(\frac{2\pi i r}{H}[\epsilon - \Sigma_{p_F}(\epsilon)]\right) \tag{2.9}$$

which is evaluated to give

$$\tilde{M}(H) \sim p_F^2 \frac{\tilde{g}^3}{H^{1/2}} \sum_{r=1}^{\infty} \frac{1}{r^{1/2}} \exp\left(-\frac{\pi}{6}\frac{\tilde{g}^6}{\omega_c} r\right) \times \sin\left[2\pi r\left(\frac{F}{H} - \phi\right)\right]. \tag{2.10}$$

Under the condition $\omega_c \ll \tilde{g}^6$, only the first ($r = 1$) harmonic is important because of the exponential suppression of amplitudes of higher harmonics. The exponential form of the amplitudes I_r in (2.10) looks similar to either the case of impurity scattering (with the identification of scattering time as $\tau \sim \tilde{g}^{-6}$) or the case of finite temperatures $T \sim \tilde{g}^6$. However the entire result (2.10) is not identical because of the magnetic field dependent prefactor, and the extra factor of $r^{1/2}$.

The result (2.10) is physically transparent, because according to (2.5) the jump of $n(p)$ on the Fermi surface is smoothed out over an interval of order $\tilde{g}^6$. It is quite important to notice that the formula (2.17) manifests an effect of the transverse gauge field on the gauge invariant quantitity $\tilde{M}(H)$. It takes place because the infrared corrections studied in this section affect the modulus of the one-particle Green function rather than its phase.

III. TWO-POINT CORRELATION FUNCTIONS

One can also use the eikonal technique to calculate the entire partition function and various multipoint Green functions summing up the most infrared divergent contributions. In particular, this method is capable of studying correlations in particle-particle as well as particle-hole channels. An important consistency check is a fulfillment of the Ward identity (1.4).[20]

A useful and efficient way to derive the appropriate expression for the four-point Green function in an external field $\mathbf{A}(x)$ is via Schwinger's "bilinear shift operator:"[22]

$$G_2[x_1,x_2;x_3,x_4;\mathbf{A}(x)] = \exp\left(\frac{i}{2}g^2\int_{x_1}^{x_3}dx\int_{x_2}^{x_4}dx'\frac{\delta}{\delta A_i(x)}D_{ij}(x-x')\frac{\delta}{\delta A_j(x')}\right)$$
$$\times\{G[x_1,x_3;\mathbf{A}(x)]G[x_2,x_4;\mathbf{A}(x)]-(x_3\leftrightarrow x_4)\} \tag{3.1}$$

where $G[x,x';\mathbf{A}(x)]$ is a solution of (2.1). In the context of the eikonal expansion this equation plays the same role as the Bethe-Salpeter equation in Fermi-liquid theory. The averaging of (3.1) over the gauge fluctuations governed by (1.2) then generates insertions of the propagator $D_{ij}(x-x')$ joining two fermion lines in all possible ways, including vertex corrections, which preserves the relevant Ward identity (1.4).

Fourier transform of the expression (3.1) gives a two-particle Green function

$$G_2(p,p';p+k,p'-k) = \int d^3x e^{ikx}\int_0^\infty d\alpha_1\int_0^\infty d\alpha_2 e^{i\alpha_1(\epsilon-\xi_p+i\delta)-i\alpha_2(\epsilon'-\xi_{p'}-i\delta)}$$
$$\times\exp\left[i\frac{g^2}{m^2}\int\frac{d\mathbf{q}d\omega}{(2\pi)^3}D_{ij}(\omega,\mathbf{q})\left(p_ip_j\frac{1-e^{2i\alpha_1(\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{q}}+\omega)}}{(\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{q}}+\omega)^2}\right.\right.$$
$$+p_i'p_j'\frac{1-e^{2i\alpha_2(\xi_{\mathbf{p}'}-\xi_{\mathbf{p}'-\mathbf{q}}-\omega)}}{(\xi_{\mathbf{p}'}-\xi_{\mathbf{p}'-\mathbf{q}}-\omega)^2}$$
$$\left.\left.-2e^{iqx}p_ip_j'\frac{1-e^{i\alpha_1(\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{q}}+\omega)}}{\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{q}}+\omega}\frac{1-e^{i\alpha_2(\xi_{\mathbf{p}'}-\xi_{\mathbf{p}'-\mathbf{q}}-\omega)}}{\xi_{\mathbf{p}'}-\xi_{\mathbf{p}'-\mathbf{q}}-\omega}\right)\right]. \tag{3.2}$$

Contracting two pairs of external lines in a way corresponding to the particle-hole channel we obtain the following eikonal formula for the density-density correlation function:

$$K(\Omega,\mathbf{Q}) = \int d^2pn_p\int_0^\infty d\alpha e^{i\alpha(\Omega+\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{Q}}+i\delta)}$$
$$\times\exp\left[i\frac{g^2}{m^2}\int\frac{d^2\mathbf{q}d\omega}{(2\pi)^3}D_\perp(\omega,\mathbf{q})\left((p_\perp+Q_\perp)(1-n_{\mathbf{p}+\mathbf{Q}+\mathbf{q}})\frac{1-e^{i\alpha(\xi_{\mathbf{p}+\mathbf{Q}}-\xi_{\mathbf{p}+\mathbf{Q}+\mathbf{q}}+\omega)}}{\xi_{\mathbf{p}+\mathbf{Q}}-\xi_{\mathbf{p}+\mathbf{Q}+\mathbf{q}}+\omega}\right.\right.$$
$$\left.\left.-p_\perp n_{\mathbf{p}+\mathbf{q}}\frac{1-e^{i\alpha(\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{q}}+\omega)}}{\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{q}}+\omega}\right)^2\right]+(\Omega\to-\Omega). \tag{3.3}$$

Calculating the integral in the exponent at small $\mathbf{Q}$ and Ω we find that, in agreement with the Ward identities, the self-energy and vertex corrections almost compensate each other and the result reads (hereafter we include irrelevant numerical factors in the definition of $\tilde{g}$)

$$K(\Omega,\mathbf{Q}) = \int d^2pn_p\int_0^\infty d\alpha e^{i\alpha(\Omega+\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{Q}}+i\delta)}$$
$$\times\exp\left(-\tilde{g}^2\frac{Q^2}{p}\alpha^{1/3}\right)+(\Omega\to-\Omega). \tag{3.4}$$

One can see that the exponential singularities of the one-particle Green functions (2.4) are integrated out and have no effect at small $\mathbf{Q}$. As a result, the compressibility $K(0,\mathbf{Q}\to 0)$ as well as a Fourier transform of the equal time correlator $\int d\Omega K(\Omega,\mathbf{Q}\to 0)$ remain finite. At small g these almost coincide with the results obtained for free fermions. In addition, we observe that the diffusion pole of the scattering amplitude found in Ref. 19 [such that $\Gamma(\epsilon,p;\omega,q)\sim\frac{(\omega+\epsilon)^{2/3}+\epsilon^{2/3}}{q_\parallel+\omega^{1/3}q_\perp}$] as a result of a summation of "fan-shaped" diagrams, does not affect the density-density correlator (3.3).

It was first pointed out in Ref. 23 that the lowest order vertex correction to $\Lambda(\Omega,Q)$ is logarithmically divergent at $Q\approx 2p_F$. At transferred momentum Q close to $2p_F$ the momenta of scattered particles $-\mathbf{p}$ and $\mathbf{p}+\mathbf{Q}$ have almost the same direction and then the current-current interaction provides an attraction which may lead to a physical singularity of (3.3) at $Q\to 2p_F$. Summing up the infrared divergent terms one obtains a 2D counterpart of the powerlike Kohn singularity at $\frac{\tilde{g}^6}{\mu}\ll\frac{|Q-2p_F|}{p_F}\ll 1$ when the behavior of $K(0,\mathbf{Q})$ is mainly governed by the vertex corrections (Fig. 3)

$$K(0,\mathbf{Q}) = \int d^2pn_p\int_0^\infty d\alpha e^{i\alpha(\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{Q}}+i\delta)}$$
$$\times\exp\left(\frac{g^2}{\gamma}\ln(\mu\alpha)-\tilde{g}^2p\alpha^{1/3}\right)$$
$$\sim \text{const}+|Q-2p_F|^{\frac{1-\eta}{2}}, \tag{3.5}$$

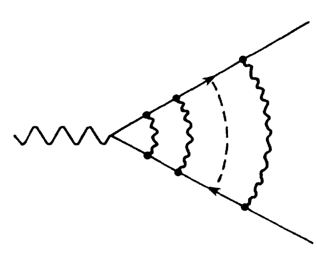

FIG. 3. Ladder-type vertex corrections.

where $\eta \sim \frac{g^2}{\gamma}$ and Q approaches $2p_F$ from above.

In the opposite regime $\frac{|Q-2p_F|}{p_F} < \frac{\tilde{g}^6}{\mu}$ the last term in the exponent associated with the wave-function renormalization dominates and renders the integral in (3.5) finite: $K(0, 2p_F) \sim \text{const} + \tilde{g}^{3(1-\eta)}$.

Some remnant of the weak "$2p_F$ singularity" also appears at $\frac{|Q-2p_F|}{p_F} \gg \frac{\tilde{g}^6}{\mu}$ as a nonanalytic contribution to the Fourier transform of the equal time correlator

$$\int d\mathbf{x} e^{i\mathbf{Q}\mathbf{x}} \langle \Psi^\dagger \Psi(\mathbf{x}) \Psi^\dagger \Psi(0) \rangle = \int d\Omega K(\Omega, \mathbf{Q}) \sim \left(\max \left\{ \frac{|Q - 2p_F|}{p_F}, \frac{\tilde{g}^6}{\mu} \right\} \right)^{2-\eta}. \tag{3.6}$$

The results (3.5) and (3.6) are consistent with the singular behavior of the renormalized vertex $\Lambda(\Omega, Q) \sim (\max\{\frac{|2p_F-Q|}{p_F}, (\frac{\Omega}{\mu})^{1/3}\})^{-\eta}$ found in Ref. 19. The density response function (3.6) manifests a kind of a crossover at critical value $\eta_c(g) = 2$. This value of the critical coupling constant is in agreement with the estimate obtained by the authors of Ref. 19 who argued that at some $\eta_c \sim 1$ a phase transition occurs. However it was also mentioned in Ref. 19 that the statement about the phase transition is based on diagrammatic calculations which assume $\eta \ll 1$ and, strictly speaking, do not account for all crossed diagrams becoming important at $\eta \sim 1$.

On the other hand, the eikonal approximation *is* supposed to provide a proper account of crossed diagrams. However, we only observe a crossover behavior of the density response function (3.6) at $Q \approx 2p_F$ and not a real singularity. The origin of this behavior is the exponential wave-function renormalization (2.4) which cuts off the powerlike divergency of $\Lambda(\Omega, Q \approx 2p_F)$.

In view of this we are inclined to the conclusion that a real phase transition at finite couplings does not seem likely. However, one cannot rule out the possibility of some kind of topological phase transition which does not manifest itself in correlations of any local field operators.[24]

It might be also interesting to study pairing correlations in the particle-particle channel. In contrast to the aforementioned Kohn singularity, which appears to be quite sensitive to the effective phase volume of scattered fermions, the singularity in the Cooper channel depends primarily on the sign of interaction. Indeed the Cooper instability persists even in the two-particle problem ($p_F \to 0$).

The corresponding (nongauge invariant) correlation function is given by the formula

$$\begin{aligned}\Delta(\Omega, \mathbf{Q}) = \int d^2p(1 - n_{\mathbf{p}} - n_{-\mathbf{p}+\mathbf{Q}}) \int_0^\infty d\alpha e^{i\alpha(\Omega - \xi_{\mathbf{p}} - \xi_{\mathbf{p}-\mathbf{Q}} + i\delta)} \\ \times \exp\left[i\frac{g^2}{m^2} \int \frac{d\mathbf{q}d\omega}{(2\pi)^3} D_\perp(\omega, \mathbf{q}) \left((-p_\perp + Q_\perp)(1 - n_{-\mathbf{p}+\mathbf{Q}+\mathbf{q}}) \right.\right. \\ \left.\left. \times \frac{1 - e^{i\alpha(\xi_{\mathbf{p}-\mathbf{Q}} - \xi_{-\mathbf{p}+\mathbf{Q}+\mathbf{q}} + \omega)}}{\xi_{\mathbf{p}-\mathbf{Q}} - \xi_{-\mathbf{p}+\mathbf{Q}+\mathbf{q}} + \omega} + p_\perp(1 - n_{\mathbf{p}-\mathbf{q}}) \frac{1 - e^{i\alpha(\xi_{\mathbf{p}} - \xi_{\mathbf{p}-\mathbf{q}} - \omega)}}{\xi_{\mathbf{p}} - \xi_{\mathbf{p}-\mathbf{q}} - \omega} \right)^2 \right].\end{aligned} \tag{3.7}$$

By calculating (3.7) at small Q we find no essential difference with the case of free fermions

$$\Delta(\Omega, \mathbf{Q}) = \int d^2p(1 - n_{\mathbf{p}} - n_{-\mathbf{p}+\mathbf{Q}}) \int_0^\infty d\alpha e^{i\alpha(\Omega - \xi_{\mathbf{p}} - \xi_{\mathbf{p}+\mathbf{Q}} + i\delta)} \exp\left(-\tilde{g}^2 \frac{Q^2}{p} \alpha^{1/3} \right) \sim \ln \frac{\mu}{\max\{\Omega, \frac{Q^2}{2m}\}}. \tag{3.8}$$

Although the expression (3.8) is logarithmically divergent it does not lead to any instability because the current-current interaction is repulsive at small Q. Indeed one could only expect to find some singularity of the effective vertex $\Gamma(p, p', k)$ if momenta of scattered particles are parallel and the current-current interaction becomes attractive, that is when $Q = |\mathbf{p} + \mathbf{p}'|$ approaches $2p_F$. One also has to have $\Omega \sim \mu$ otherwise as Q becomes comparable with p_F the Cooper loop ceases to be logarithmically divergent. In this regime (3.7) yields the result which is similar to (3.5): at $|\Omega - \frac{Q^2}{2m}| \gg \tilde{g}^6$ the pairing correlation function has a nonanalytic powerlike intermediate asymptotics while at $|\Omega - \frac{Q^2}{2m}| < \tilde{g}^6$ it remains finite even for large $\eta(\tilde{g})$:

$$\begin{aligned}\Delta(\Omega, \mathbf{Q}) = \int d^2p(1 - n_{\mathbf{p}} - n_{-\mathbf{p}+\mathbf{Q}}) \int_0^\infty d\alpha e^{i\alpha(\Omega - \xi_{\mathbf{p}} - \xi_{\mathbf{p}+\mathbf{Q}} + i\delta)} \\ \times \exp\left(\frac{g^2}{\gamma} \ln(\mu\alpha) - \tilde{g}^2 p\alpha^{1/3} \right) \sim \frac{1}{\eta} \left(\max\left\{ \left| \Omega - \frac{Q^2}{2m} \right|, \tilde{g}^6 \right\} \right)^{-\eta}.\end{aligned} \tag{3.9}$$

The anomalous exponent $\eta \sim \frac{g^2}{\gamma}$ coincides with the one in (3.5) although this equality holds only for spinless fermions.

Notice that the effect of the exponentially vanishing Z factor on (3.9) is exactly the same as in the case of the gauge invariant density-density correlation function. This should not be surprising because it derives from the modulus of the one-particle Green function rather than its phase.

A similar conclusion can be drawn about the equal time pairing susceptibility

$$\int d\mathbf{x} e^{i\mathbf{Q}\mathbf{x}} \langle \Psi^\dagger \Psi^\dagger(\mathbf{x}) \Psi\Psi(0)\rangle = \int d\Omega \Delta(\Omega, \mathbf{Q}) \sim \left(\max\left\{ \frac{|2p_F - Q|}{p_F}, \frac{\tilde{g}^6}{\mu} \right\} \right)^{2-\eta}. \quad (3.10)$$

Thus we do not find an instability in the Cooper channel either.

Nevertheless in the course of our eikonal calculations we discovered that the exponential factors appearing in (3.3) and (3.7) are related to each other and can be understood as correlation functions of some vertex operators:exp($i\Phi$): built from a free boson field Φ. It is this property of the eikonal calculus which enables one to apply an effective bosonic description to study the problem (1.1).

IV. EFFECTIVE BOSONIC DESCRIPTION

In the Hamiltonian approach an attempt to construct a bosonic theory which captures mostly relevant small scattering angle processes and then reproduces long-wavelength properties of the model (1.1) was made in Ref. 17. Later on the authors of Ref. 19 explicitly showed how a quasi-1D (bosonizable) effective Lagrangian description occurs in the context of a straightforward diagrammatic calculation of the one-particle Green function. The Green function obtained in this manner is equivalent to our formula (2.4).

More precisely, in Ref. 19 the parameter γ was used to control the diagrammatic expansion instead of our coupling constant g which was, in turn, put equal to unity. This alternative choice of the expansion parameter corresponds to the case of the so-called "zero current" theory which follows from (1.1) in the strong coupling limit $g \to \infty$. At $\gamma \gg 1$ it turns out that within the weak coupling ($g \ll 1$) eikonal calculations the two expansions can be linked together by the relation $\gamma \sim g^{-6}$.

It was noticed in Ref. 19 that the asymptotic behavior (2.4) can be found by means of the effective quasi-1D bosonic theory at $\gamma < 1$.

The above consideration shows that at $\gamma < 1$ the asymptotics (2.4) simply extends over the entire energy range $\epsilon \sim \mu$. In the opposite case $\gamma \sim g^{-6} \gg 1$, the necessary condition $\epsilon \ll \tilde{g}^6$ for the nonperturbative asymptotics (2.4) to hold means a small scattering angle

$$\theta \sim q_\perp / p \sim \left(\frac{\gamma\epsilon}{\chi} \right)^{1/3} \Big/ p_F \ll 1. \quad (4.1)$$

It is the condition (4.1) which provides an effective quasi-1D dynamics along the direction of the particle's momentum $\mathbf{p}$. In other words, the eikonal condition (4.1) means that the space of momenta $\mathbf{p}$ can be effectively split into 1D "rays" in accordance with the heuristic picture of the "Luttinger Tomographic Projection."[1] However (and just as for Anderson's model[1,20]), there is still a "residual coupling" between these rays, as we shall see.

The 1D bosonic theory of Ref. 19 was invented primarily for the calculation of $G(\epsilon, \mathbf{p})$. However it is highly tempting to formulate a refined bosonic description which would be capable to reproduce the two-point response functions as well.

In the 3D case of free fermions this problem was first addressed by Luther[13] who showed that one can recover two-point correlation functions

$$\langle \Psi^\dagger(x) \cdots \Psi(x) \Psi^\dagger(0) \cdots \Psi(0) \rangle$$

substituting fermion operators by the bosonic ones

$$\Psi(\mathbf{x}) \to \int \frac{d\mathbf{n}}{2\pi} e^{ip_F \mathbf{n}\cdot\mathbf{x}} \exp i\Phi_{\mathbf{n}}(\mathbf{x}). \quad (4.2)$$

The bosonic field $\Phi_{\mathbf{n}}(\mathbf{x})$ should be understood as a chiral (right moving) field associated with the direction pointed by the unit vector $\mathbf{n}$ normal to the spherical Fermi surface which parametrizes a continuous manifold of Fermi points. This representation can be further generalized to the case of arbitrary shape of the Fermi surface and can also be used to treat interacting fermions.[15]

To get a complete correspondence between fermionic and bosonic operators as well as the corresponding Hilbert spaces one has to solve a subtle (and ambiguous) problem of operator ordering in $D > 1$.[14] Fortunately, to deal with two-point response functions one only needs to establish a relationship between currents.

Using a bosonic field $\Phi_{\mathbf{n}}(\mathbf{x})$ one can construct a corresponding (chiral) current operator $J_{\mathbf{n}}(\mathbf{x}) = (\mathbf{n}\boldsymbol{\nabla})\Phi_{\mathbf{n}}(\mathbf{x})$ obeying commutation relations[15]

$$[J_{\mathbf{n}}(\mathbf{x}), J_{\mathbf{n}'}(\mathbf{x}')] = \frac{1}{2\pi} \delta^2(\mathbf{n} - \mathbf{n}')(\mathbf{n}\boldsymbol{\nabla})\delta(\mathbf{x} - \mathbf{x}'). \quad (4.3)$$

To proceed with a conventional quantization $\Phi_{\mathbf{n}}(\mathbf{x})$ can be expanded in the form

$$\Phi_{\mathbf{n}}(\mathbf{x}) = \sum_{\mathbf{nq}>0} e^{i\mathbf{q}\cdot\mathbf{x}} \frac{J_{\mathbf{n}}(\mathbf{q})}{i\sqrt{\mathbf{nq}\Lambda}}, \quad (4.4)$$

where $\Lambda \ll p_F$ stands for an ultraviolet cutoff for a transverse component of $\mathbf{q}$.

On the basis of the commutation relations (4.3) one can identify $J_{\mathbf{n}}(\mathbf{x})$ with a bilinear product of fermion operators: $J_{\mathbf{n}}(\mathbf{q}) = \sum_{|\mathbf{p}-p_F\mathbf{n}|<\Lambda} \Psi^\dagger_{\mathbf{p}+\mathbf{q}} \Psi_{\mathbf{p}}$. The transverse component of the space current coupled with $A_\perp(\mathbf{k})$ is given by the dual gradient: $\Psi^\dagger(\mathbf{x})(\frac{\mathbf{n}\boldsymbol{\nabla}}{i})\Psi(\mathbf{x}) = \frac{\partial \Phi_{\mathbf{n}}(\mathbf{x})}{\partial t}$.

In contrast to the case of the one-particle Green function it turns out that to reproduce the response functions of the preceding section the intended bosonic theory has to incorporate kinetic couplings between bosons $\Phi_{\mathbf{n}}$ with different $\mathbf{n}$. This is necessary to account for vertex corrections on an equal footing with self-energy ones.

The effective bosonic action can be written in the form (v_F is put equal to unity)

$$S = \frac{1}{2}\int d\omega d^2q\left(\int \frac{d\mathbf{n}}{2\pi}\Phi_{\mathbf{n}}(-\omega,-\mathbf{q})(\mathbf{qn})(\omega-\mathbf{nq})\Phi_{\mathbf{n}}(\omega,\mathbf{q}) + \tilde{g}^2\omega^{5/3}\int\frac{d\mathbf{n}}{2\pi}\int\frac{d\mathbf{n}'}{2\pi}\Phi_{\mathbf{n}}(-\omega,-\mathbf{q})(\mathbf{nn}')\Phi_{\mathbf{n}'}(\omega,\mathbf{q})\right). \quad (4.5)$$

To avoid a double counting of loop corrections which were already accounted for in (1.2) one can introduce N bosonic replicas $\Phi^a_{\mathbf{n}}, a = 1, 2, ..., N$ and then put N equal to zero.[19] In other words, one has to exclude any renormalizations of a kinetic part of the bosonic Lagrangian. In the limit $N \to 0$ the boson propagator following from (4.5) becomes equivalent to the expression

$$\langle\Phi^a_{\mathbf{n}}(\omega,\mathbf{q})\Phi^b_{\mathbf{n}'}(-\omega,-\mathbf{q})\rangle = \frac{\delta_{ab}\delta_{\mathbf{n},\mathbf{n}'}}{\Lambda(\mathbf{nq})(\omega-\mathbf{nq})} + \frac{\tilde{g}^2}{\Lambda\omega^{1/3}}\frac{(\mathbf{nn}')}{[\omega-\mathbf{nq}-(\gamma\omega)^{2/3}][\omega-\mathbf{n}'\mathbf{q}-(\gamma\omega)^{2/3}]}. \quad (4.6)$$

More strictly, the formulas (4.5) and (4.6) enable one to reproduce those one- and two-particle eikonal amplitudes which are dominated by configurations of momenta obeying one of the conditions $|\mathbf{n}' \pm \mathbf{n}| \ll 1$. In this case all integrations over the transverse components of transferred momenta $q_\perp = \mathbf{q}\times\mathbf{n}$ simply cancel the factors Λ in the denominators.

On calculating the one-particle Green function with the use of the Lagrangian (4.5) one ends up with a quasi-1D theory with right and left moving bosons (two opposite Fermi points) associated with directions of $\mathbf{p}$ and $-\mathbf{p}$. The form of the Lagrangian (4.5) makes it possible to clarify the physical origin of the exponential asymptotics (2.4). The well-known powerlike behavior of 1D one-particle Green functions, as well as correlation functions, takes place for a quite special family of Lagrangians bilinear in current operators, the most popular example being the Sugawara construction which is a local bilinear form $L =: J_LJ_L - J_RJ_R :$. An example of a nonlocal coupling with the strength $V(x) \sim \frac{1}{x^2}$ is provided by the Calogero model. In all these cases the 1 + 1-dimensional theory possesses conformal invariance. As soon as this symmetry is lost the decay of 1D correlators ceases to be powerlike [see, for instance, the example of the Coulomb interaction $V(x) \sim \frac{1}{x}$ recently considered in Ref. 25].

Thus in the effective model (4.5) one simply encounters the more general situation where the Lagrangian bilinear form is nonlocal in time. It is obvious that to get a powerlike behavior one has to have not only an effectively one-dimensional dynamics but also a conformally invariant one.

As another example, we sketch the derivation of the asymptotics (3.6) using the bosonic theory (4.5):

$$\int d\Omega K(\Omega,\mathbf{Q}) = \int d^2\mathbf{x}e^{i\mathbf{Qx}}\langle\Psi^\dagger\Psi(\mathbf{x})\Psi^\dagger\Psi(\mathbf{0})\rangle = \int d\mathbf{x}\int\frac{d\mathbf{n}}{2\pi}\int\frac{d\mathbf{n}'}{2\pi}e^{i[\mathbf{Q}+p_F(\mathbf{n}'-\mathbf{n})]\cdot\mathbf{x}}$$
$$\times\exp\left(-\frac{1}{2}\int d\omega\int d^2q(\langle\Phi_{\mathbf{n}}\Phi_{\mathbf{n}}\rangle + \langle\Phi_{\mathbf{n}'}\Phi_{\mathbf{n}'}\rangle - 2\langle\Phi_{\mathbf{n}}\Phi_{\mathbf{n}'}\rangle)(1-\cos\mathbf{qx})\right). \quad (4.7)$$

The exponent in (4.7) can be found in the following approximate form:

$$\exp\left[-\int_0^\infty dq_\parallel\int_0^\infty d\omega[2-\cos(q_\parallel\mathbf{nx})-\cos(q_\parallel\mathbf{n}'\mathbf{x})]\left(\frac{1}{(\omega+q_\parallel)^2}+\frac{\tilde{g}^2}{\omega^{1/3}(\omega+q_\parallel)^2}\right.\right.$$
$$\left.\left.+\frac{\tilde{g}^2}{\omega^{1/3}[\omega-q_\parallel-(\gamma\omega)^{2/3}][\omega+q_\parallel-(\gamma\omega)^{2/3}]}\right)\right]$$
$$\sim\frac{(\max\{\mathbf{n}\cdot\mathbf{x},\mathbf{n}'\cdot\mathbf{x}\})^\eta}{(\mathbf{n}\cdot\mathbf{x}+i\delta)(\mathbf{n}'\cdot\mathbf{x}-i\delta)}\exp(-\tilde{g}^2x^{1/3}). \quad (4.8)$$

At $g = 0$ and $Q \approx 2p_F$ (4.7) and (4.8) yield

$$\int d\Omega K_0(\Omega,\mathbf{Q}) = \int d\mathbf{x}\int\frac{d\mathbf{n}}{(\mathbf{nx}+i\delta)}\int\frac{d\mathbf{n}'}{(\mathbf{n}'\mathbf{x}-i\delta)}$$
$$\times e^{i[\mathbf{Q}+p_F(\mathbf{n}'-\mathbf{n})]\cdot\mathbf{x}}\sim(Q-2p_F)^2. \quad (4.9)$$

The consistency of the applied approximations is achieved due to the fact that in the relevant region of integration $\mathbf{n}' \approx -\mathbf{n}$. Performing the calculation of (4.7) at finite values of g and using (4.9) we recover the result (3.6).

As was shown in Ref. 15, all the information about long-wavelength properties (in particular, the equilibrium thermodynamics) is encoded in eigenvalues of the generalized Landau equation for collective bosonic modes

$$(\omega-\mathbf{nq})\Phi_{\mathbf{n}}(\omega,\mathbf{q}) = \tilde{g}^2\omega^{2/3}\int\frac{d\mathbf{n}'}{2\pi}(\mathbf{n}\cdot\mathbf{n}')\Phi_{\mathbf{n}'}(\omega,\mathbf{q}). \quad (4.10)$$

Formally one can find two independent solutions of Eq. (4.10)

$$\Phi_n^{\parallel,\perp}(\omega,\mathbf{q}) = \frac{q_{\parallel,\perp}}{\omega - \mathbf{nq} + i\delta} f(\omega). \tag{4.11}$$

The spectra of the corresponding collective modes are given by the equations

$$1 = \tilde{g}^2 \omega^{2/3} \int \frac{1 \pm \cos 2\phi'}{\omega - q\cos\phi' + i\delta} \frac{d\phi'}{2\pi}. \tag{4.12}$$

The dispersion of the mode associated with $\Phi_n^{\parallel}$ is close to linear $[\omega = q - O(\tilde{g}^4 q^{1/3})]$ at $\omega \gg \tilde{g}^6$. In the opposite limit $\omega < \tilde{g}^6$ it acquires a form $\omega \sim (\frac{q}{\tilde{g}})^{6/5}$. The solution corresponding to the second mode $\Phi_n^{\perp}$ can only be found at $\omega < \tilde{g}^6$. Its spectrum demonstrates even a stronger nonlinearity: $\omega \sim (\frac{q}{\tilde{g}^2})^{3/2}$. Notice that in contrast to fermionic (one-particle) excitations which are completely incoherent the bosonic branches of the spectrum correspond to real quasiparticles.

Using the effective free boson description we can calculate the specific heat $C_V(T)$ as a function of temperature

$$C_V(T) = \frac{\partial}{\partial T} \sum_{\lambda=\parallel,\perp} \int \frac{d\mathbf{n}}{2\pi\Lambda} \int d^2\mathbf{q} \frac{\omega_\lambda(\mathbf{n}\cdot\mathbf{q})}{\exp(\frac{\omega_\lambda(\mathbf{n}\cdot\mathbf{q})}{T}) - 1}, \tag{4.13}$$

where $\omega_\lambda(q)$ denote different solutions of (4.12). Estimating the integral in (4.13) we obtain that at $T \gg \tilde{g}^6$ an ordinary Fermi-liquid result holds $[C_V(T) \sim T]$ which is solely due to the contribution of the mode $\Phi_n^{\parallel}$. On the contrary, at low temperatures $T < \tilde{g}^6$ the specific heat becomes nonlinear and it is mainly determined by the $\Phi_n^{\perp}$ contribution: $C_V(T) \sim \tilde{g}^2 T^{2/3}$.

Remarkably this estimate coincides with the result of the RPA calculations[8,9] (we are reminded that the RPA result is nonanalytic in g since $\gamma_{\text{RPA}} \sim g^2$). In addition, we confirm the hypothesis made in Ref. 18 that the RPA result remains valid in an effective bosonic theory as well. The reason is that RPA contributions basically represent the effect of states with an arbitrary number of low-energy particle-hole pairs and the bosonization scheme takes an account of just this subspace of the entire Hilbert space.[15,17]

It is worthwhile mentioning that the bosonic representation can be also used to investigate the transport properties of the model (1.1) which are supposed to be quite unusual.

V. CONCLUSIONS

The above results provide further support for the statements about a breakdown of Fermi-liquid theory in the 2D model (1.1) made in Refs. 12 and 19. Moreover, the low-energy behavior found within the eikonal approximation appears to be quite different from the "orthogonality catastrophe"[1] [which involves exponentiation of logarithmic divergences to give $Z(\epsilon) \sim \epsilon^\eta$]. The initial deviation from the Fermi-liquid theory is demonstrated by the lowest infrared divergent diagram (Fig. 2), but at $\epsilon \sim \tilde{g}^6$ this power-law behavior turns into the exponential asymptotics of Eq. (2.4).

One might think that an exponential behavior of the one-particle Green function (2.4) in the vicinity of the Fermi surface is an artifact caused by a gauge noninvariance of the object. However we show that the trace of the exponentially decaying Z factor does appear in both gauge invariant and noninvariant response functions which typically receive their singular contributions from momenta close to the Fermi surface. In particular, due to this fact we do not find real divergencies of susceptibilities in both particle-particle and particle-hole channels which would demonstrate a tendency toward pairing or a formation of charge density wave. Moreover, the behavior of the one-particle Green function is reflected in experiments which are sensitive to the behavior of those fermions near the Fermi energy; thus it leads to a dramatic suppression of the oscillations of orbital magnetization in a weak external magnetic field, in the dHvA effect.

It may happen, of course, that an intrinsic instability of the model (1.1) cannot be detected as a divergent susceptibility of some local order parameter and then the corresponding phase transition is not of second order. As a plausible example one could consider the intriguing possibility of a spontaneous generation of uniform magnetic flux commensurate with the particle's density first discussed by Wiegmann.[24]

We have also shown that to recover the results of the eikonal approximation, capturing the most relevant features of the long-wavelength dynamics of the model (1.1), one has to use the effective bosonic Lagrangian which is not purely one dimensional.

The very existence of the approximate free boson representation means a possibility of a partial diagonalization of the problem involved in the scaling limit.

In other words, using this representation one is able to restore the (non-Fermi liquidlike) properties of the low-energy particle-hole subspace of the entire Hilbert space. This is supposed to be an intrinsic feature of any bosonization scheme.

As an example of the application of this technique we have found the spectrum of the bosonic collective mode governing particle-hole dynamics, and its contribution to specific heat.

We intend to further address these and other related issues (for instance, a generalization on the case of fermions with spin) elsewhere.

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