Dynamics of a two-level system with Ohmic dissipation in a time-dependent field

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Dynamics of a two-level system with Ohmic dissipation driven by a time-dependent cw field in the noninteracting-blip approximation are studied. It is found that at low temperatures when α, a friction parameter, lies between $\frac{1}{2}$ and 1, the localization predicted by Chakravarty [Phys. Rev. Lett. **49**, 681 (1982)] and Bray and Moore [Phys. Rev. Lett. **49**, 1546 (1982)] disappears, and the rate constant for tunneling of the particle increases with the intensity and frequency of the field. For $0<\alpha<\frac{1}{2}$ quantum coherence is observed, and the transition temperature from the coherent to the decay regime depends on the field intensity. When the intensity parameter approaches the zeros of the zeroth Bessel function coherence disappears. This transition from coherent to incoherent motion oscillates with the intensity of the field, giving rise to a cascade of transitions at different temperatures.

I. INTRODUCTION

Recently there has been considerable interest in the time evolution of an electron in double-well structures in a time-dependent field.[1-7] In the absence of a field, an electron initially localized in one well at time zero will tunnel in an oscillatory manner from one well to the other. Hänggi and co-workers[1] have shown numerically that a laser field with appropriate values for amplitude and frequency can stop the oscillating behavior through a suppression of tunneling. Numerical calculations[3] on model quantum-well systems show that the dynamics of the laser-driven electron can be fairly well reproduced by a two-level Hamiltonian (TLS). A discussion of a two-well potential with a two-level system may be found in the review by Leggett *et al.*[8] The analysis of a TL model provides an integrodifferential kinetic equation for the population in a given well (or a dipole moment in an optical representation). This approach can be used to describe a host of phenomena including electron localization,[4,5] low-frequency generation,[4,5] and even harmonic generation.[5] Additionally, the effect of a static field[6] or the presence of a second laser[7] can also be obtained. The quantum coherent motion of the electron becomes dissipative if an interaction with a bath is included. In the absence of an electric field the system loses coherence when temperature increases. Holstein was the first to consider a transition between coherent and incoherent motions of an electron interacting with phonons in a one-dimensional solid state,[9,10] while Niu[11] studied this problem using the path-integral formalism first introduced by Chakravarty and Leggett.[12] Niu found the dependence of the transition temperature on the parameters of a TLS, as well as a phonon spectrum and electron-phonon coupling.

The interaction with a strong electric field drastically changes the physical picture. A time-dependent evolution of the TLS interacting with (acoustic) phonons in a strong time-dependent electric field has been studied by the present author.[13] If the frequency of the field is much larger than the relaxation phonon energy, the following effect has been found: the particle may be localized in the metastable well. This localization is driven only by the electric field, and is due to quantum interference of the particle motion and electric field. The dissipation does not destroy the effect, and the decay rate becomes negligibly small. Thus localization may be observed even for systems in which the electron interacts strongly with the phonon bath for low temperatures, as shown in Ref. 13.

The physical picture may be different for a specific particle-boson interaction, a so-called Ohmic dissipation.[8,14] This phonon spectrum is used to describe a dissipation in the tunneling of a superconducting phase in Josephson junctions at low temperatures.[15] This specific type of boson gives rise to a qualitatively different effect for the time evolution of the superconducting phase when the coupling exceeds a threshold value. The superconducting phase is trapped in a metastable state of a symmetric double-well system.[16] We study what happens to the system if we irradiate the Josephson junction by a strong microwave field (e.g., a free-electron laser). Will this localization be destroyed?

The transition from coherent to incoherent motion is the other subject of interest. In the absence of a field, Garg has found[8,17] that transition exists and occurs at some temperature T^* for $0<\alpha<\frac{1}{2}$, where α is a dissipation parameter which will be defined below. As shown in Ref. 17, T^* goes to zero with respect to α. We study this transition in a low-temperature limit when $\pi kT \ll \hbar\omega_c$, where ω_c is a characteristic frequency of the bath,[8] and find how T^* depends on the amplitude and frequency of the field. The effect of dissipation on the coherence-incoherence transition has been studied numerically by Dittrich, Oelschlagel, and Hänggi[18] when the tunneling potential has been chosen as a quartic double well (a Duffing oscillator), and a dissipative bath is considered in a stochastic approximation. This approximation is valid in the high-temperature limit, when $\pi kT \gg \hbar\omega_c$.[19] The decay rate was found to increase with temperature. At high temperatures, an application to nonadiabatic chemi-

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cal reactions in a condensed phase has been considered in Ref. 20. In this limit the rate constant depends strongly on the value of the photon frequency, the reaction heat, and the reorganization energy of the solvent. As shown in Ref. 20 the rate constant may be suppressed by three orders of magnitude (localization), or increased by 18 orders of magnitude (delocalization), or may oscillate with respect to the intensity of the field.

To start the analysis, we derive a kinetic equation for the population in a noninteracting-blip approximation first introduced by Chakravarty and Leggett[12] and subsequently discussed by many authors.[14–17,21,22] In Sec. III we derive a formal asymptotic solution for long times for a cw laser field. The analysis of this solution is presented in Sec. IV. Section V contains discussions and conclusions.

The theory presented below may be applied to the tunneling in a superconducting phase in Josephson junctions irradiated by a microwave laser field.

II. GENERAL KINETIC EQUATION IN NONINTERACTING-BLIP APPROXIMATION

In the spirit of earlier works[8,11,14] we consider a two-level subspace of the particle linearly interacting with an oscillator bath. Thus, we have a spin-boson Hamiltonian of the form

$$\mathcal{H}=-\hbar\Delta\sigma_x/2+\hbar\sum_l \omega_l b_l^\dagger b_l +\sigma_z\sum_l g_l(b_l^\dagger+b_l)-\sigma_z V(t) . \quad (1)$$

Here σ_x and σ_z are the Pauli matrices, Δ corresponds to the tunneling matrix element between the two minima (splitting between the levels), b_l and $b_l{}^\dagger$ are the boson (phonon) operators, ω_l is the frequency of the lth boson, and g_l is a particle-boson coupling. The electric field is introduced as a driving force $V(t)$, where

$$V(t)=\mu_0 E(t) .$$

Here μ_0 is a dipole moment, and $E(t)$ is an electric field. As in small polaron theory,[10] we apply the canonical transformation

$$S=\exp[(\sigma_z/2)\sum_l(g_l/\omega_l)(b_l^\dagger-b_l)]\equiv\exp(i\sigma_z\Pi/2) , \quad (2)$$

which eliminates the electron-phonon term in Eq. (1), and renormalizes the tunneling matrix element, so that the Hamiltonian takes the following form:

$$\tilde{\mathcal{H}}\equiv S\mathcal{H}S^{-1}=-(\hbar\Delta\sigma_x/2)[\sigma_+\exp(i\Pi)+\sigma_-\exp(-\Pi)] +\hbar\sum_l\omega_l b_l^\dagger b_l+\sigma_z V(t) , \quad (3)$$

where $\sigma_\pm$ are matrices defined as

$$\sigma_\pm=(\sigma_x\pm i\sigma_y)/2 .$$

The tunneling probability may be defined as[8]

$$P(t)=[1+(\sigma_z(t))_{11}]/2 .$$

Thus we look for the time dependence of matrix element 11 of matrix σ_z. In the Heisenberg picture the equation of motion for the Pauli matrices[21,22] and the boson operators can be found easily, and take the following forms:

$$\dot\sigma_\pm=\pm(i/2)\{\hbar\Delta\sigma_z\exp[\pm i\Pi(t)]-2V(t)\sigma_\pm\} , \quad (4a)$$

$$\dot\sigma_z=i\hbar\Delta\{\sigma_+\exp[-i\Pi(t)]-\exp(i\Pi(t))\sigma_-\} , \quad (4b)$$

$$\dot b_\alpha=(i\hbar\Delta g_\alpha/2)\{\sigma_+\exp[-i\Pi(t)]-\exp(i\Pi(t))\sigma_-\} -i\hbar\omega_\alpha b_\alpha , \quad (4c)$$

$$\dot b_\alpha^\dagger=(i\hbar\Delta g_\alpha/2)\{\exp[-i\Pi(t)]\sigma_+-\sigma_-\exp(i\Pi(t))\} +i\hbar\omega_\alpha b_\alpha^\dagger . \quad (4d)$$

We may eliminate the field dependence by introducing the following substitution:

$$\sigma_+(t)=\exp\left[2i\int_0^t V(\tau)d\tau\right]u_+(t) \equiv\exp[iF(t)]u_+(t) ; \quad (5)$$

thus for σ_z and $\Pi(t)$ we obtain the following equations:

$$\dot\sigma_z=i\hbar\Delta\{\sigma_+\exp[iF(t)-i\Pi(t)]-\text{H.c.}\}-(\Delta^2/2) \times\int_0^t dt_1\sigma_z(t_1)\{\exp[iF(t)-iF(t_1)] \times\exp[-i\Pi(t)]\exp(i\Pi(t_1))+\text{H.c.}\} \quad (6)$$

and

$$\Pi(t)=\Pi^{(0)}(t)-2\sum_l g_l^2\int_0^t dt_l\sin\omega_l(t-t_1)\dot\sigma_z(t_1) , \quad (7)$$

where

$$\Pi^{(0)}(t)=-i\sum_l g_l[\exp(-i\omega_l t)b_l(0)-\text{H.c.}] . \quad (8)$$

In reality it is impossible to solve Eq. (6) analytically owing to the nonlinear properties introduced by $\Pi(t)$, and we therefore suppose that the phonon subsystem is weakly disturbed by the TLS. This is *the first supposition* in the noninteracting-blip approximation.[12] The details and the validity condition have been discussed in Refs. 8 and 14. In the framework of this approach we find the first correction for $\Pi(t)$ in the expansion with respect to Δ using Eq. (4b), i.e., (8) in place of (7) replacing $\exp[i\Pi(t)]$ with $\exp[i\Pi^{(0)}(t)]$. In this way Π is independent of σ_z. Substituting $\Pi(t)$ back into Eq. (6), we obtain the following kinetic equation:

$$\dot\sigma_z=(i\hbar\Delta/2)(\exp(iF(t))\{\exp[-i\Pi^{(0)}(t)+\Pi^{(1)}(t)]\sigma_+(0)+\sigma_+(0)\exp[-i\Pi^{(0)}(t)+\Pi^{(1)}(t)]\}-\text{H.c.}) -(\Delta^2/4)\int_0^t dt_1[\exp\{i(F(t)-F(t_1)]\}\{\exp[i\Pi^{(0)}(t)]\exp[-i\Pi^{(0)}(t_1)]\sigma_z(t_1) +\sigma_z(t_1)\exp[i\Pi^{(0)}(t_1)]\exp[-i\Pi^{(0)}(t)]\}+\text{H.c.}) . \quad (9)$$

$\Pi^{(1)}(t)$ is expected to be a small correction owing to the presence of Δ. Making use of the Feynman identity

$$\exp[s(\hat{A}+\hat{B})]=\exp(s\hat{A})T_c\exp\left[\int_0^s ds_1\exp(-s_1\hat{A})\hat{B}\exp(s_1\hat{A})\right] \tag{10}$$

and the Baker-Hausdorf identity

$$\exp(\hat{A}+\hat{B})=\exp(\hat{A})\exp(\hat{B})\exp([\hat{A},\hat{B}]/2)\;, \tag{11}$$

we expand the exponents with respect to $\Pi^{(1)}(t)$. Here the commutator $[\hat{A},\hat{B}]$ is supposed to be a c number. It is possible to show that

$$\exp[-i\Pi^{(0)}(t)+\Pi^{(1)}(t)]$$
$$=\exp[i\Pi^{(0)}(t)]\exp\left[-i\hbar\Delta\int_0^t dt_1\exp[iF(t_1)-i\Pi^{(0)}(t_1)]\{\exp[-iQ_1(t-t_1)/\pi]-1\}\sigma_+(0)-\text{H.c.}\right]\;, \tag{12}$$

where

$$Q_1(t)/\pi\equiv\sum_l g_l^2\sin(\omega_l t)\;. \tag{13}$$

As we have already mentioned, the transition probability is determined by the (11) element of the $\sigma_z(t)$ matrix. This means that nonvanishing terms appear only in the first-order expansion in Δ of expression (12). Introducing the definition

$$x\equiv(\sigma_z)_{11}\;,$$

we obtain the following kinetic equation for the population x:

$$dx/dt=-\Delta^2\int_0^t dt_1\sin[F(t)-F(t_1)-\Pi^{(0)}(t)+\Pi^{(0)}(t_1)]\sin[Q_1(t-t_1)/\pi]$$
$$-\Delta^2\int_0^t dt_1\cos[F(t)-F(t_1)-\Pi^{(0)}(t)+\Pi^{(0)}(t_1)]\cos[Q_1(t-t_1)/\pi]x(t_1)\;. \tag{14}$$

In this equation $\Pi^{(0)}(t)$ is still an operator over boson fields. To find the population $\langle x\rangle$ we must average over boson fields, assuming an equilibrium distribution. We suppose that the average of the product of the operators $x(t)$ and $\Pi^{(0)}(t)$ in the integrand of the second integral in Eq. (14) may have to be decoupled, and be presented as $\langle x\rangle\langle\Pi^{(0)}\rangle$.[14,21,22] This is *the second supposition* of the noninteracting-blip approximation. Finally, one obtains the following master equation for the average population $\langle x(t)\rangle$:

$$d\langle x\rangle/dt=-\Delta^2\int_0^t dt_1\sin[F(t)-F(t_1)]\exp[-Q_2(t-t_1)/\pi]\sin[Q_1(t-t_1)/\pi]$$
$$-\Delta^2\int_0^t dt_1\cos[F(t)-F(t_1)]\exp[-Q_2(t-t_1)/\pi]\cos[Q_1(t-t_1)/\pi]\langle x(t_1)\rangle\;, \tag{15}$$

with the initial condition

$$x(0)=1\;.$$

To average the exponent, we have used the results of small polaron theory:[10]

$$\langle\exp\{\pm i[\Pi^{(0)}(t)-\Pi^{(0)}(t_1)]\}\rangle=\exp[-Q_2(t-t_1)/\pi]\;, \tag{16}$$

where

$$2Q_2(t)/\pi\equiv\langle[\Pi^{(0)}(t)-\Pi^{(0)}(0)]^2\rangle$$
$$=\sum_l g_l^2[1-\cos(\omega_l t)]\coth(\beta\omega_l/2)\;. \tag{17}$$

It is easy to check that Eq. (15) coincides with the master equation obtained in Refs. 21 and 22 in the absence of the electric field. In the case of a vanishing interaction with phonons, this equation matches the master equation obtained in Ref. 4 ($Q_2=0$, $Q_1=0$). It should be noted that this equation is of a nonconvolution type, and therefore cannot be solved by a usual Laplace transform. It also cannot be derived by the method of Niu,[11] who essentially used the Laplace transform as well.

III. KINETIC EQUATION IN A cw ELECTRIC FIELD

Now we specify the boson field considering the environment with Ohmic dissipation.[8,14] Functions Q_1 and Q_2 [Eqs. (13) and (17)] may be expressed through a spectral density function[8,12,14]

$$J(\omega)\equiv(\pi/2)\sum_l(g_l^2/q_0^2\omega_l)\delta(\omega-\omega_l) \tag{18}$$

as

$$Q_1(t)=\int_0^\infty d\omega(J(\omega)/\omega^2)\sin(\omega t)\;, \tag{19}$$

$$Q_2(t)=\int_0^\infty d\omega(J(\omega)/\omega^2)[1-\cos(\omega t)]\coth(\beta\omega/2)\;, \tag{20}$$

where q_0 is a distance between the wells. A discussion of the connection between a double-well and a two-level one is presented in Ref. 8.

We consider that the spectrum of oscillator frequencies is sufficiently dense, and the distribution of coupling constants sufficiently nonpathological, that $J(\omega)$ may be treated as a continuous and fairly smooth function. We consider the case when the original spectral function $J(\omega)$ has a simple power-law form for $\omega < \omega_c$:[8,12,14]

$$J(\omega)=\eta\omega\exp(-\omega/\omega_c)\ , \tag{21}$$

where η is the friction coefficient and ω_c the cutoff frequency in the exponential cutoff. The quantity η is simply a classically measurable friction coefficient of the extended system. Q_1 and Q_2 are determined from the following equations[8,12]

$$Q_1(t)=\alpha\pi\tan^{-1}(\omega_c t)\ , \tag{22}$$

$$Q_2(t)=(\alpha\pi/2)\ln[1+(\omega_c t)^2] + \alpha\pi\ln[(\pi t/\beta)\mathrm{cosech}(\pi t/\beta)]\ . \tag{23}$$

Here β is defined as $\beta\equiv\hbar/kT$.

$$\alpha\equiv\frac{\eta q_0^2}{2\pi\hbar} \tag{24}$$

is the dimensionless friction, coefficient. In this paper as a particular case we consider only a cw electric (or laser) field

$$V(t)=V_0\cos(\omega_0 t)\ , \tag{25}$$

and

$$F(t)=(2V_0/\omega_0)\sin(\omega_0 t)\equiv a\sin(\omega_0 t)\ ; \tag{26}$$

here V_0 and ω_0 are the amplitude and frequency of the driving force.

To simplify master equation (15), we show how in an asymptotic long-time limit this equation may be transformed into a convolution-type integrodifferential one. To do this we analyze the first correction in the perturbation expansion with respect to $(\Delta/\hbar\omega_0)$ which is considered as a small parameter:

$$\langle x_1\rangle=-(\Delta/\hbar\omega_0)^2\mathrm{Re}\int_0^{\tau}d\tau_1\int_0^{\tau_1}d\tau_2\exp\{ia[\sin(\tau_1)-\sin(\tau_2)]+iQ_1(\tau_1-\tau_2)/\pi-Q_2(\tau_1-\tau_2)/\pi\}\ . \tag{27}$$

Here we have made use of the substitution

$$\tau\rightarrow\omega_0 t\ . \tag{28}$$

Expanding the field-dependent part of the exponent in Eq. (27) into the Fourier series,[23] one obtains

$$\exp[ia\sin(\tau)]=\sum_{m=-\infty}^{\infty}J_m(a)\exp(im\tau)\ , \tag{29}$$

where J_m (a) are mth order Bessel functions.

After the substitution

$$\begin{aligned}\tilde{\tau}_1&\equiv\tau_1\ ,\\ \tilde{\tau}_2&\equiv\tau_1-\tau_2\ ,\end{aligned} \tag{30}$$

with the help of Eqs. (22)–(24) one finds

$$\begin{aligned}\langle x_1\rangle=-(\Delta/\hbar\omega_0)^2\mathrm{Re}\sum_{m_1=-\infty}^{\infty}\sum_{m_2=-\infty}^{\infty}J_{m_1}(a)J_{m_2}(a)\int_0^{\tau}d\tilde{\tau}_1\int_0^{\tilde{\tau}_1}d\tilde{\tau}_2\exp[i(m_1-m_2)\tilde{\tau}_1]\exp[im_2\tilde{\tau}_2+i2\alpha\tan^{-1}(\omega_c\tilde{\tau}_2/\omega_0)]\\ \times\left[\frac{\pi\tilde{\tau}_2/\beta\omega_0}{\sinh(\pi\tilde{\tau}_2/\beta\omega_0)}\frac{1}{1+(\omega_c\tilde{\tau}_2/\omega_0)^2}\right]^{2\alpha}\ ,\end{aligned} \tag{31}$$

In a low-temperature limit when $\gamma\equiv kT\pi/\hbar\omega_c\ll 1$, Eq. (31) may be rewritten as

$$\begin{aligned}\langle x_1\rangle\cong&-(\Delta/\hbar\omega_0)^2(\omega_0/\omega_c)\mathrm{Re}\gamma^{2\alpha-1}\\ &\times\sum_{m_1=-\infty}^{\infty}\sum_{m_2=-\infty}^{\infty}J_{m_1}(a)J_{m_2}(a)\int_0^{\tau}d\tilde{\tau}_1\int_0^{\omega_c\tilde{\tau}_1\gamma/\omega_0}du\exp[i(m_1-m_2)\tilde{\tau}_1+im_2(\omega_0/\omega_c)u/\gamma+i\pi\alpha]\sinh^{-2\alpha}(u)\ .\end{aligned} \tag{32}$$

In Eq. (32), we have introduced the following substitution:

$$\tilde{\tau}_2\equiv(\omega_0/\gamma\omega_c)u\ . \tag{33}$$

In a long-time limit, when

$$\tilde{\tau}_1(\gamma\omega_c/\omega_0)\gg 1\ , \tag{34}$$

the integration over u is convergent and time independent, while the integral over $\tilde{\tau}_1$ gives the term proportional to

$$\{\exp[i(m_1-m_2)\tau]-1]/[i(m_1-mn_2)]\ . \tag{35}$$

In the long-time-scale limit $\tau \gg 1$, the terms with $m_1 = m_2$ dominate, and the other terms may safely be neglected.[13] When $a=0$ is the internal integral the term with $m_1 = m_2 = 0$ is the largest, and the problem reduces to that of one with no dissipation.[4] Thus, collecting all the terms with $m_1 = m_2$ and making use of the following identity:[23]

$$J_0[2a\sin(\omega_0 t/2)] = J_0^2(a) + 2\sum_{m=1}^{\infty} J_m^2(a)\cos(m\omega_0 t) , \tag{36}$$

one obtains

$$\langle x_1 \rangle = -(\Delta/\hbar\omega_0)^2 \mathrm{Re}\int_0^{\tau} d\tau_1 \int_0^{\tau_1} d\tau_2 J_0\{2a\sin[(\tau_1-\tau_2)/2]\}\exp[iQ_1(\tau_1-\tau_2)/\pi - Q_2(\tau_1-\tau_2)/\pi] , \tag{37}$$

where J_0 in Eq. (37) is the zeroth-order Bessel function. In the same way, for $\tau \gg 1$ the estimation of the second correction $\langle x_2 \rangle$ yields

$$\langle x_2 \rangle = (\Delta/\hbar\omega_0)^4 \int_0^{\tau} d\tau_1 \cdots \int_0^{\tau_3} d\tau_4 \mathrm{Re}(J_0\{2a\sin[(\tau_1-\tau_2)/2]\}\exp[iQ_1(\tau_1-\tau_2)/\pi - Q_2(\tau_1-\tau_2)/\pi])$$
$$\times \mathrm{Re}(J_0\{2a\sin[(\tau_3-\tau_4)/2]\}\exp[iQ_1(\tau_3-\tau_4)/\pi - Q_2(\tau_3-\tau_4)/\pi]) . \tag{38}$$

Such an approximation is valid in all orders of the expansion, and therefore for kinetic equation (15) may be rewritten as

$$d\langle x \rangle/d\tau = -(\Delta/\hbar\omega_0)^2 \mathrm{Re}\int_0^{\tau} d\tau_1 J_0\{2a\sin[(\tau-\tau_1)/2]\}\exp[iQ_1(\tau-\tau_1)/\pi - Q_2(\tau-\tau_1)/\pi]\langle x(\tau_1)\rangle . \tag{39}$$

This equation has been derived in the long-time limit. It is of the convolution type. The kernel is dependent on the field parameter a. When $a=0$ Eq. (39) reduces to that of obtained in Refs. 8, 12, 21, and 22.

IV. THE SYSTEM DYNAMICS

Now the kinetic equation (39) is of the convolution type, and it may be solved by using the Laplace transformation defined as

$$x(\lambda) = \int_0^{\infty} dt\, e^{-\lambda t}\langle x(t) \rangle . \tag{40}$$

The formal solution of Eq. (39) is

$$x(\lambda) = \frac{1}{\lambda + \Delta^2 K(\lambda)} , \tag{41}$$

where

$$K(\lambda) = \int_0^{\infty} dt\, e^{-\lambda t} J_0[2a\sin(\omega_0 t/2)] \times \cos[Q_1(t)/\pi]\exp[-Q_2(t)/\pi] . \tag{42}$$

According to Eqs. (22)–(24),

$$\cos[Q_1(t)/\pi]\exp[-Q_2(t)/\pi] = \cos[2\alpha\tan^{-1}(\omega_c t)]\left[\frac{\pi t/\beta}{\sinh(\pi t/\beta)}\frac{1}{1+(\omega_c t)^2}\right]^{2\alpha} . \tag{43}$$

To find integral (42) we expand the zeroth Bessel function back into a Fourier series[23] using Eq. (36). At long time the solution of Eq. (39) is determined by the behavior of $K(\lambda)$ at small λ. As in Refs. 8,12, and 17, integrals (42) and (43) may be performed in a low-temperature limit when

$$\gamma \equiv \pi k T/\hbar\omega_c \ll 1 ,$$

$$K(\lambda) = \gamma^{2\alpha-1}\frac{\sqrt{\pi}\Gamma(1-\alpha)\Gamma(\lambda/\gamma+\alpha)}{2\Gamma(\frac{1}{2}+\alpha)\Gamma(1+\lambda/\gamma-\alpha)}J_0^2(a) + 2^{2\alpha-1}\frac{\pi}{2\Gamma(2\alpha)\sin(\pi\alpha)} \times \sum_{m=1}^{\infty}\left[\lambda + \frac{i\omega_0 m}{2\omega_c}\right]^{2\alpha-1} J_m^2(a) + \text{c.c.} \tag{44}$$

Here Γ is Euler's Γ function. In the second term we can always neglect the λ dependence due to $\lambda \ll \omega_0 m/2\omega_c$. However, there are two possibilities for the first term.

(a) Let us consider $\lambda \ll \gamma$. Then

$$K(0) \cong \gamma^{2\alpha-1}\frac{\sqrt{\pi}\Gamma(\alpha)}{2\Gamma(\frac{1}{2}+\alpha)}J_0^2(a) + \left[\frac{\omega_0}{\omega_c}\right]^{2\alpha-1}\frac{\pi}{\Gamma(2\alpha)}f(a,\alpha) , \tag{45}$$

where

$$f(a,\alpha) \equiv \sum_{m=1}^{\infty} m^{2\alpha-1}J_m^2(a) . \tag{46}$$

This function is presented in Fig. 1. It grows with the field intensity a at $\alpha > \frac{1}{2}$, and never vanishes. The first term in Eq. (45) goes to zero at small temperatures, so that the second one dominates the rate constant. This term is always a nonzero constant at low temperatures, thus a localization phenomenon found by Chakravarty and Bray-Moore[16] no longer occurs whenever the field is applied, (i.e., $a \neq 0$). The rate constant τ^{-1} defined as

$$\langle x(t) \rangle = \exp(-t/\tau)$$

is given by th following equation for $\alpha > \frac{1}{2}$:

$$\tau^{-1} = \frac{\Delta^2}{\hbar^2\omega_c}\left[\frac{\omega_0}{\omega_c}\right]^{2\alpha-1}\frac{\pi}{\Gamma(2\alpha)}f(a,\alpha) . \tag{47}$$

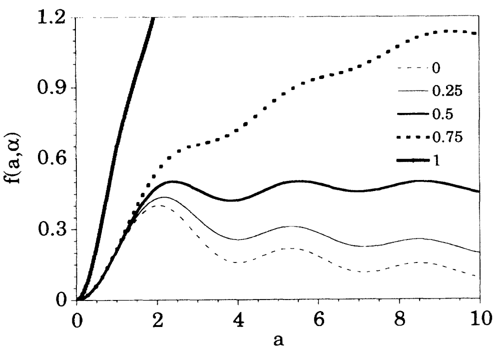

FIG. 1. The a dependence of the function $f(a,\alpha)$ for selected values of α.

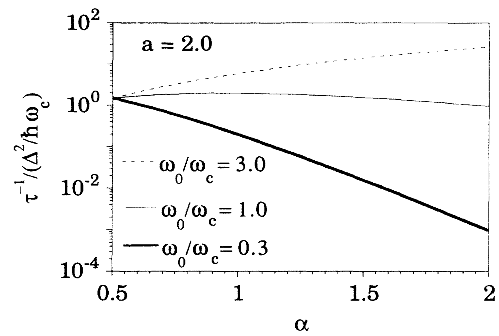

FIG. 2. The dependence of the rate constant on the friction α at different values of the field frequency ω_0/ω_c.

Solution (47) is determined by the condition $\lambda \ll \gamma$, and is valid for temperatures

$$kT \gg \frac{\Delta^2}{\hbar\omega_c}\left[\frac{\omega_0}{\omega_c}\right]^{2\alpha-1}\frac{\pi}{\Gamma(2\alpha)}f(a,\alpha)\ . \tag{48}$$

The other restriction on temperature is due to neglecting the first term in Eq. (44) with respect to the second one, i.e.,

$$\gamma^{2\alpha-1}\frac{\sqrt{\pi}\Gamma(\alpha)}{2\Gamma(\frac{1}{2}+\alpha)}J_0^2(a) \ll \left[\frac{\omega_0}{\omega_c}\right]^{2\alpha-1}\frac{\pi}{\Gamma(2\alpha)}f(a,\alpha)\ . \tag{49}$$

Inequality (49) is always valid when $J_0(a)=0$. The dependence of the rate constant on the friction parameter α is shown in Fig. 2 at different values of the field frequency ω_0. When the ratio $\omega_0/\omega_c=3$, the rate constant goes up with α. At $\alpha=2$ the rate constant increases by one order of magnitude. For small values of the field frequency ($\omega_0/\omega_c=0.3$) the rate constant decreases by three orders of magnitude. We may conclude that lower frequencies are in favor of localization of the particle in the mestastable state.

(b) Let us consider the case of $\lambda \gg \gamma$. The Γ functions in the first term of Eq. (44) may be estimated using the Stirling's asymptotic formula,[20] while the second term is unchanged as in Eqs. (45) and (46). Then for $K(\lambda)$ one obtains

$$K(\lambda)=\Gamma(1-2\alpha)\cos(\pi\alpha)\lambda^{2\alpha-1}J_0^2(a) + \left[\frac{\omega_0}{\omega_c}\right]^{2\alpha-1}\frac{\pi}{\Gamma(2\alpha)}f(a,\alpha)\ . \tag{50}$$

The first term diverges at $\alpha<\frac{1}{2}$ whenever $\lambda \Longrightarrow 0$. Then for the Fourier transform of the population $x(\lambda)$, one obtains

$$x(\lambda)=\frac{\lambda^{1-2\alpha}}{\lambda^{2(1-\alpha)}+\Delta_{\text{eff}}^{2(1-\alpha)}\left[J_0^2(a)+\lambda^{1-2\alpha}\left[\frac{\omega_0}{\omega_c}\right]^{2\alpha-1}\sin(\pi\alpha)f(a,\alpha)\right]}\ , \tag{51}$$

where

$$\Delta_{\text{eff}} \equiv [\Gamma(1-2\alpha)\cos(\pi\alpha)]^{1/2(1-\alpha)}\Delta_r\ , \tag{52}$$

is a renormalized transition matrix element[8,12,17] with

$$\Delta_r \equiv \Delta(\Delta/\omega_c)^{\alpha/1-\alpha}\ . \tag{53}$$

For small λ the second term in the brackets of the denominator in Eq. (51) is small and therefore, as in the case without field,[8,12,17] there are a branch point and two complex poles. The branch point determines the incoherent part of the time-dependent probability, $\langle x_{\text{incoh}}(t)\rangle$ which is the Mittag-Leffler function.[24] The coherent part $\langle x_{\text{coh}}(t)\rangle$ is due to the contribution of a complex-conjugated pair of simple poles

$$\lambda_{0\pm}=\tilde{\Delta}_{\text{eff}}\exp[\pm i\pi/2(1-\alpha)]\ , \tag{54}$$

where

$$\tilde{\Delta}_{\text{eff}} \equiv \Delta_{\text{eff}}|J_0(a)|^{1/1-\alpha} \tag{55}$$

is a renormalization of the tunneling matrix element. The analysis is close to that of Refs. 8, 12, and 17. Therefore, for a time-dependent probability one obtains

$$\langle x(t)\rangle=\langle x_{\text{coh}}(t)\rangle+\langle x_{\text{incoh}}(t)\rangle\ , \tag{56}$$

where

$$\langle x_{\text{coh}}(t)\rangle=\frac{1}{1-\alpha}\cos\left\{\cos\left[\frac{\pi\alpha}{2(1-\alpha)}\right]\tilde{\Delta}_{\text{eff}}t\right\} \times\exp\left\{-\left|\sin\left[\frac{\pi\alpha}{2(1-\alpha)}\right]\right|\tilde{\Delta}_{\text{eff}}t\right\} \tag{57}$$

and

$$\langle x_{\text{incoh}}(t)\rangle = -\frac{\sin(2\pi\alpha)}{\pi} \times \int_0^\infty dz \frac{z^{2\alpha-1} e^{-\tilde{\Delta}_{\text{eff}} z t}}{z^2+2z^{2\alpha}\cos(2\pi\alpha)+z^{4\alpha-2}} . \tag{58}$$

A complete analysis for this case is given in Ref. 8.

In the absence of an electric field, as found by Garg,[8,17] there exists a transition temperature T^* from the coherent to incoherent regimes. In this case two complex poles converge to that of the second-order real one. As in the case of a zero field, we can write two transcendental equations determining this temperature:

$$\lambda^* + \Delta^2 \gamma^{2\alpha-1} \frac{\sqrt{\pi}\,\Gamma(1-\alpha)\Gamma(\lambda^*/\gamma+\alpha)}{2\Gamma(\frac{1}{2}+\alpha)\Gamma(1+\lambda^*/\gamma-\alpha)} J_0^2(a) + \Delta^2 \left[\frac{\omega_0}{\omega_c}\right]^{2\alpha-1} \frac{\pi}{\Gamma(2\alpha)} f(a,\alpha) = 0 , \tag{59}$$

$$1 - \left[\lambda^* + \Delta^2 \left[\frac{\omega_0}{\omega_c}\right]^{2\alpha-1} \frac{\pi}{\Gamma(2\alpha)} f(a,\alpha)\right] \times \Psi(\alpha+\lambda^*/\gamma) - \Psi(1-\alpha+\lambda^*/\gamma)] = 0 . \tag{60}$$

If the following criterion,

$$|J_0(a)|^{(3-4\alpha)/(1-\alpha)} > \left[\frac{\omega_0}{2\omega_c}\right]^{2\alpha-1} \sin(\pi\alpha) f(a,\alpha) \Delta_{\text{eff}}^{-(1-2\alpha)} , \tag{61}$$

is valid [$J_0(a)$ is not too small], we may neglect the third term in Eq. (59). Equations (58) and (60) may now be reduced to those obtained by Garg[17] with the renormalized $\tilde{\Delta}_{\text{eff}}$:

$$2\pi k T^*/\tilde{\Delta}_{\text{eff}}(a) = \left[-\frac{\Gamma(\alpha+\lambda^*/\gamma)}{u^*\Gamma(1-\alpha+\lambda^*/\gamma)}\right]^{1/2(1-\alpha)} , \tag{59a}$$

$$(\lambda^*/\gamma)[\Psi(\alpha+\lambda^*/\gamma) - \Psi(1-\alpha+\lambda^*/\gamma)] = 1 , \tag{60a}$$

where $\Psi(z)$ is the digamma function. The first equation is the condition determining the poles of $x(\lambda)$ in Eq. (41) with $K(\lambda)$ determined by Eq. (44). The derivative of this equation gives condition (60) upon matching of the two poles.[17] The dependence of the transition temperature T^* on the friction parameter α is presented in Fig. 3. The specific feature of the field dependence is the observation of a cascade of phase transitions with respect to the field parameter a. Because $\tilde{\Delta}_{\text{eff}}$ is field dependent according to Eq. (55), we may change the temperature T^* by increasing the field intensity. $J_0(a)$ is an oscillating function, thus T^* oscillates as well too. We also see that at different nonvanishing values of the field amplitude, the temperature T^* is lower than in the zero-field case. This means that in this range of parameters the field destroys coherence.

When the field parameter a is such that $J_0(a)$ is close to zero, the third term in the denominator of Eq. (51) is of importance. We may calculate the rate constant using

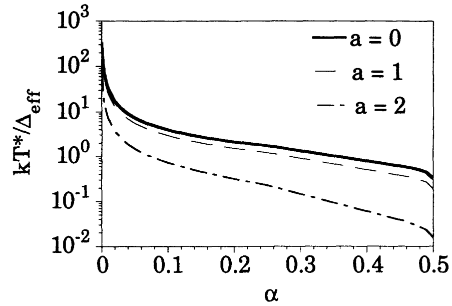

FIG. 3. The dependence of the transition temperature T^* on the friction α at different values of the field intensity a.

the iteration procedure, considering the second term as a perturbation. In this case the estimated rate constant $(T=0)$ is

$$\langle x(t)\rangle = \cos(\Omega t)\exp(-t/\tau) , \tag{62}$$

where the decay of the transition probability τ^{-1} coincides with rate (47). The frequency Ω is determined from the following equation:

$$\Omega = \Delta^{1+2\alpha} \left[\frac{\omega_0}{\omega_c}\right]^{(1-2\alpha)^2} \Gamma(1-2\alpha) \times \left[\frac{\pi}{\Gamma(2\alpha)} f(a,\alpha)\right]^{2\alpha-1} J_0^2(a) . \tag{63}$$

In this case it has been supposed that $\Omega\tau \ll 1$. The long period coherent oscillations disappear whenever $J_0(a)=0$. In such cases there is only an incoherent decay. The decay rate also changes with parameter a in accordance with Fig. 1.

We compare the case of an acoustic-phonon bath, as discussed by the author in Ref. 13, with that of an Ohmic dissipation one. If $J_0(a) \Longrightarrow 0$, the strong quantum decay with long period oscillations takes place when the particle interacts with the Ohmic boson bath. As it was found in Ref. 13 for phonons when $J_0(a) \Longrightarrow 0$, the decay rate decreases as $J_0^2(a)$, while the coherence frequency goes down linearly with $J_0(a)$. Therefore, only the quantum coherence survives near the zeros of $J_0(a)$. For Ohmic dissipation one observes the opposite effect; the quantum coherence is destroyed by the electric field whenever $J_0(a) \Longrightarrow 0$.

V. CONCLUSIONS

In this work we have studied the time-dependent evolution of a TLS interacting with an Ohmic boson bath, and driven by a time-dependent cw field. In the noninteracting-blip approximation, first introduced by Chakravarty and Leggett,[8,12] we derived the integrodifferential master equation (15) for the transition probability $\langle x(t)\rangle$ for an arbitrary electric field. This equation is of a nonconvolution type, thus it cannot be derived by the method of perturbative expansion in a Laplace parameter space.[11] In Sec. III we found that a long-time evolution may be described by a simpler equation (39) with the kernel in convolution. We have found the exact formal solution of this equation by making use of a Laplace transform. The long-time dynamics are determined by the poles of $x(\lambda)$, which are dependent on the behavior of the kernel, $K(\lambda)$, of the integrodifferential kinetic equation at small λ. $K(\lambda)$ is a nonanalytical function of λ, so an analysis depends essentially on the value of the friction parameter α.

In the region when $\frac{1}{2}<\alpha<1$, the localization found by Chakravarty and Bray-Moore,[16] no longer takes place in a low-temperature limit. As shown in Fig. 1, the rate constant is proportional to the function $f(a,\alpha)$ defined by Eq. (46), and grows with the intensity and frequency of the field according to Eq. (47). As shown in Fig. 2, the rate constant increases by one order of magnitude when α changes from 0.5 to 2 if a high-frequency field ($\omega_0>\omega_c$) is applied. For small values of the field frequency ($\omega_0/\omega_c=0.3$), the rate constant decreases by three orders of magnitude. This means that the particle tunneling is suppressed, and the particle is trapped in the metastable well whenever $\omega_0<\omega_c$.

For $0<\alpha<\frac{1}{2}$ we have found an effect of quantum coherence in a low-temperature limit when $\gamma\equiv\pi kT/\hbar\omega_c\ll 1$. Numerical analysis of the coherence-incoherence transition has been performed by Dittrich, Oelschlagel, and Hanggi[18] for a Duffing oscillator as a tunneling potential in the high-temperature limit using a stochastic medium approximation. In our case, to find the transition temperature it is necessary to solve two transcendental equations (59) and (60). The approximate solution has been found for intensities of the field when $J_0(a)\neq 0$, i.e., when inequality (62) is valid. The transition from coherent to incoherent motion obeys the same set of transcendental equations (59a) and (60a) as in the case without field,[17] but with the renormalized transition matrix element $\tilde{\Delta}_{\rm eff}$ in accordance with Eq. (55). T^* depends on the intensity of the field, a, so that the value of T^* is one order of magnitude smaller than in the absence of the field, as shown in Fig. 3. We also found the time-dependent probability $\langle x(t)\rangle$, which consists of two parts determined by Eqs. (56)–(58). The first part describes the coherent motion, while the second is responsible for the incoherent decay. The detailed analysis of this case has been done in Ref. 8. Solutions (62) and (63) have been found in the vicinity of the zeros of the zeroth Bessel function. In this case there is a strong decay of the transition probability with long period oscillations. When $J_0(a)=0$ there is only an exponential decay, with the rate constant determined by Eq. (47). In general, for $0<\alpha<\frac{1}{2}$ the qualitative physical picture of the dynamics is as follows: at $a=0$ there is always coherent motion below some critical temperature T^* (Refs. 8 and 17) (see Fig. 3), when a strong electric field is applied, and T^* is smaller than in the zero-field case. When the intensity a is close to the first zero of the zeroth Bessel function, there are long period oscillations with a strong decay. When $J_0(a)=0$, one observes only a decay of the transition probability according to solutions (62) and (63). In the small friction limit the decay rate vanishes, so the localization discovered by Hänggi and co-workers[1] takes place. This picture qualitatively repeats with the intensity a due to an oscillating behavior of the zeroth Bessel function, giving rise to a cascade of the transitions.

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