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Band tails in diluted magnetic semiconductors

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Tails in the density of electron states in diluted magnetic semiconductors are examined theoretically. The exchange interaction between a charge carrier and magnetic ions makes the energy dependence of the density of states in the tails markedly different from those known for nonmagnetic solid solutions. The cases of uncorrelated and strongly correlated disorder are analyzed. In both cases the density of states in the tail turns out to be nonmonotonic. In the case of uncorrelated disorder the density of states has a gap, while it possesses a deep valley in the case of strongly correlated disorder. The thermal disorder in the spin system of the ions makes the density of states temperature dependent. Some possible experimental manifestations of these features of the tails in diluted magnetic semiconductors are discussed.

I. INTRODUCTION

Band tails in the mobility gap are a common feature of real semiconductors. There are composition fluctuations which lead to band tailing in the case of mixed crystalline semiconductors. Compositional disorder can form a fluctuation potential for electrons and holes. The deeper the state in the mobility gap, the deeper must be the fluctuation potential well causing this state, and the stronger must be the corresponding fluctuation. Since the probability of a fluctuation decreases exponentially with its strength, the density of states in a band tail decays exponentially with the energy E measured from the mobility edge:

$$g(E)=g_0\exp[-\Phi(E)] \ . \tag{1}$$

The first analytical considerations in this field are due to Halperin and Lax,[1] Zittarz and Langer,[2] Alferov *et al.*,[3] and Lifshitz;[4] the reviews are given in Refs. 5–7. The main results in the above works are the following. The exponent $\Phi(E)$ in Eq. (1) is normally a smooth power-type function of energy, whose particular shape depends on the relation between the correlation length of disorder r_c and the characteristic radius of the localized state $r_\Psi \sim \hbar/\sqrt{m|E|}$, where m is the effective mass. In the case of uncorrelated disorder one has $r_c \ll r_\Psi$, and

$$\Phi(E)=\left[\frac{|E|}{E_0}\right]^{1/3}$$

$$\text{with } \varepsilon_0=\frac{(va^{-3})^4}{cB^3}x^2(1-x)^2, \quad B=\frac{\hbar^2}{ma^2} \ , \tag{2}$$

where v is the effective short-range potential of an impurity atom, $x=na^3$ is the relative average concentration of the impurities, a is the lattice constant, and $c \approx 178$ is the numerical coefficient. In the opposite limiting case of strongly correlated disorder $r_c \gg r_\Psi$ the result takes the form

$$\Phi(E)=\frac{E^2}{2v^2b_0} \ ,$$

where (3)

$$b_0=\langle[\delta n(\mathbf{r})]^2\rangle \ ,$$

and $\delta n(\mathbf{r})=n(\mathbf{r})-\langle n\rangle$ is the local concentration fluctuation.

Clearly, it makes sense to consider the exponential dependence in Eq. (1) only if the exponent Φ is considerably larger than unity. The latter implies that fluctuation potential wells do not overlap, being rather isolated from each other. On the other hand, the spectral region of main practical interest is one with a not-too-small density of states corresponding to relatively small extended fluctuations made up of many impurity atoms; such fluctuations obey Gaussian statistics. Under these circumstances Eqs. (2) and (3) describe the density of states localized in isolated wells of the Gaussian random potential. The above restrictions can be expressed in terms of inequalities: $E_0 \ll |E| \ll |v|\langle n\rangle$.

It should be stressed that the above-mentioned approach describes band tails caused by a static disorder, which cannot be influenced by the presence of electrons or holes. The static disorder concept fails for the case of diluted magnetic semiconductors (DMS) which represent a numerous and intensively studied group of mixed crystals. In these materials one kind of lattice atom possesses noncompensated magnetic momenta. Examples include ternary compounds $M_xA^{\mathrm{II}}_{1-x}B^{\mathrm{YI}}$ with substituting magnetic atoms M of the Fe group. The exchange interaction with magnetic ions substantially affects charge carrier properties in DMS.[8] This interaction can result in a sort of magnetic polarization, which in its turn changes the effective potential acting on a charge carrier. Therefore, the shape of a random potential in DMS depends on the presence of charge carriers and thus the disorder can hardly be regarded as a static one. Moreover, since the

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above-mentioned polarization is temperature dependent, one can expect the random potential and corresponding band tails to be temperature dependent. The present work is aimed at considering the band-tailing problem in the case of such flexible disorder.

We consider a mixed crystal with a rather small concentration of magnetic ions $x \ll 1$ at temperatures high enough to neglect magnetic ordering effects. The interaction between an electron (or hole) and a magnetic impurity is described by the Hamiltonian

$$H=-J\sum_i \delta(\mathbf{r}-\mathbf{r}_i)(\mathbf{s}\cdot\mathbf{S}_i)+v\sum_i \delta(\mathbf{r}-\mathbf{r}_i) \ , \qquad (4)$$

where J is the exchange integral. The summation in (4) is undertaken over all impurity sites, $\mathbf{S}_i$ is the impurity spin at site i, and $\mathbf{s}$ is the electron spin ($s=\frac{1}{2}$). Keeping an eye on what follows we note here the character values of the parameters in Eq. (4). The quantity $|Ja^{-3}|$ for the most investigated materials $Cd_{1-x}Mn_xTe$ and $Hg_{1-x}Mn_xTe$ is of the order of 1 eV, being slightly different for electrons and holes. The parameter v, connected with the difference between potentials of the impurity atom and the host one, should be of an atomic scale, that is, $|va^{-3}|>1$ eV. This parameter can also be estimated via the concentration dependence of the energy gap G:

$$va^{-3}=\frac{dG}{dx} \ .$$

By using the data from Ref. 8, one gets values $|va^{-3}|$ which alter from 0.5 eV ($Zn_{1-x}Mn_xTe$) up to ≈ 4 eV ($Hg_{1-x}Mn_xTe$) depending on a material. The magnitude $S=|\mathbf{S}|$ is nominally about several units, e.g., $S=\frac{5}{2}$ for the Mn ions. Bearing in mind the above estimates we shall consider mainly the case $|v|\sim|JSs|$ as a typical one. The case $|v|\ll|JSs|$ is also conceivable, yet it seems to be less probable.

The paper is organized as follows. In Sec. II we present some qualitative arguments which make it possible to guess the energy dependence of the band-tail density of states in DMS. In Sec. III the optimum fluctuation method is developed to obtain the explicit expressions for $g(E)$ for different limiting cases. Section IV contains a discussion of the results and some concluding remarks.

II. QUALITATIVE CONSIDERATION

To begin with let us describe the effect of magnetic polarization in a system consisting of magnetic impurities and a localization electron. For a stationary state of the system the electron-induced polarization of magnetic ion spins produces a potential self-consistently decreasing the electronic state energy. The latter phenomenon known as the magnetopolaronic effect[9] is of great interest to us concerning what follows. When taken into account, this effect changes the standard band-tailing picture drastically, making tails nonmonotonous and temperature dependent.

It is convenient to distinguish between the three different regimes of the magnetopolaronic effect, which can be classified with respect to either temperature or exchange coupling strength.[10] We first consider the temperature driven evolution. It is natural to call as the "saturation" regime that one which takes place at low temperatures, when practically all spins in the localization region are oriented due to their interaction with the electron (this regime can also be called the "fluctuonic" one in connection with the fluctuon concept; see the review[11]). Since the characteristic energy of the interaction is of the order of $|JSs|\langle|\Psi(\mathbf{r})|^2\rangle$, the criterion of the saturation regime has the form

$$\frac{|JSs|\langle|\Psi(\mathbf{r})|^2\rangle}{kT}\gg 1 \ , \qquad (5)$$

where $\Psi(\mathbf{r})$ is the electron wave function with angular brackets implying the averaging over the localization region, T is the temperature, and k is the Boltzmann constant.

With the temperature increase the inequality in Eq. (5) transforms into the opposite one. Then, the magnetic impurities' response to the electron field turns out to be linear in the parameter

$$\frac{|JSs|\langle|\Psi(\mathbf{r})|^2\rangle}{kT}\ll 1 \ . \qquad (6)$$

We note that the effective field acting on the electron is proportional to $|\Psi(\mathbf{r})|^2$, which is typical for the standard acoustic polarons. In view of that we call this regime the "polaron" one. Although thermal fluctuations in the magnetic ion spin subsystem can be considerable, the averaging magnetization still plays the main role in the polaron regime.

Thermal fluctuations in a magnetization exceed its average value at highest temperatures. In this "thermodynamic fluctuation" regime the coupling is mainly due to a proper correlation between the electron spin and the spatial variations in the magnetization. To obtain the criterion of this regime we require the magnetic interaction energy $|JSs|N\langle|\Psi|^2\rangle$ to be much smaller than the characteristic energy $kT\sqrt{N}$ of the thermal fluctuations in a system of $N\sim\langle n\rangle r_\Psi^3$ magnetic ions in the electron localization region of length r_Ψ. The latter energy is estimated by noting that the average magnetization is close to zero, while the typical number of oriented spins in a fluctuation is $\sqrt{N}$, and the energy needed to orient a spin is of order of kT. We then take into account the normalizing condition $\langle|\Psi|^2\rangle r_\Psi^3\sim 1$. Combining these estimates gives

$$\frac{\langle n\rangle J^2(Ss)^2\langle|\Psi|^2\rangle}{(kT)^2}\ll 1 \ . \qquad (7)$$

One can see now that the polaron regime occurs when the characteristic temperature determined in Eq. (7) is considerably larger than the one in Eq. (6), i.e., the window in temperature exists between the saturation and thermodynamic fluctuation regimes. The latter takes place under the conditions $N\gg 1$ and $|J|\langle n\rangle/kT\gg 1$, of which the former means the electron wave function to be spread over a number of magnetic ions, while the latter implies characteristic magnetic energy scale $|J|\langle n\rangle$ considerably exceeding the thermal energy kT. Both conditions are supposed to obey in what follows.

With the above stated in mind, we shall describe qualitatively the energy dependence of band tails in DMS. First of all we note that the criteria (5)–(7) make it possible to define different energy regions in band tails at any given temperature. The classification is based on the standard assumption that the localization length r_Ψ decreases and correspondingly $\langle |\Psi|^2\rangle \sim r_\Psi^{-3}$ increases with the energy E deepening into the mobility gap (normally, $r_\Psi \propto |E|^{-1/2}$). Hence, at any temperature T there exists the characteristic energy $E_1(T)$, dividing the saturation regime region from the polaron one:

$$\frac{|JSs|\langle |\Psi|^2\rangle}{kT} > 1 \quad \text{at } E < E_1(T) ,$$
$$\frac{|JSs|\langle |\Psi|^2\rangle}{kT} < 1 \quad \text{at } E > E_1(T) . \tag{8}$$

In the same way we can introduce the energy $E_2(T)$, dividing the regions of the polaron and thermodynamic fluctuation regimes:

$$\frac{\langle n\rangle (JSs)^2\langle |\Psi|^2\rangle}{(kT)^2} > 1 \quad \text{at } E < E_2(T) ,$$
$$\frac{\langle n\rangle (JSs)^2\langle |\Psi|^2\rangle}{(kT)^2} < 1 \quad \text{at } E > E_2(T) . \tag{9}$$

Thus, different temperature regimes of the magnetic polarization are mapped onto the energy scale.

We now come to grips with the bad-tailing consideration in the above energy regions. In the saturation regime, i.e., at $E < E_1$, when almost all spins are aligned, the strength of electron-ion coupling is to be changed by the maximal amount due to exchange interaction, which, as seen from Eq. (5), can be accounted for by introducing the effective coupling parameter and shifting the origin for the energy scale:

$$v \to v - \frac{|JS|}{2} , \quad E \to E + \frac{|JS|\langle n\rangle}{2} \tag{10}$$

(note that v can be both negative and positive, depending on the impurities). With these changes taken into account the problem reduces to the standard one, described by Eqs. (10 and (2), in which one should replace

$$E \to E + \frac{|JS|\langle n\rangle}{2} , \quad \varepsilon_0 \to \varepsilon_1 \equiv \varepsilon_0 \left[\frac{v - |JS|/2}{v} \right]^4 . \tag{11}$$

Thus, the exchange interaction in DMS causes exponential changes in the band-tail density of states for $E < E_1(T)$.

Consider next the polaron region $E_1(T) < E < E_2(T)$. The magnetic ion response to the electron spin is proportional to $\langle |\Psi|^2\rangle$ in this regime, and so is the corresponding energy gain, which differs from the latter quantity by a multiplier of order of $\langle |\Psi|^2\rangle r_\Psi^3 \sim 1$. The energy gain decreases in absolute value with the energy E approaching the mobility edge, since the electron localization length r_Ψ increases. Therefore, one can expect thermal destruction of those magnetic polarons, whose energies lie above a certain energy exceeding $E_1(T)$. The latter phenomenon should cause a kind of valley in the density of states in the polaron region. As compared with this valley the density of states beneath lines like it has a peak in the proximity of $E_1(T)$. We shall see in what follows that the above-mentioned valley appears to be a true gap in the spectrum, as is depicted in Fig. 1(a).

In the region of thermodynamic fluctuation regime $0 > E > E_2(T)$ the exchange interaction plays only a minor role, causing a relatively small decrease of stationary-state energies $|\Delta E| \ll |E|$. Correspondingly, the band-tail shape undergoes a small change:

$$\Phi(E) \to \Phi(E) \left[1 - \alpha \left| \frac{\Delta E}{E} \right| \right] , \tag{12}$$

where α is a numerical factor and Φ is given by Eq. (2). To estimate ΔE we note that at high temperatures the magnetic interaction induces the average electron spin $\langle s\rangle \sim |J| S \langle |\Psi|^2\rangle / kT$, which results in the energy gain

$$|\Delta E| \sim |JS\langle s\rangle| \langle n\rangle \sim J^2 S^2 \langle |\Psi|^2\rangle \langle n\rangle / kT$$
$$\sim \frac{(SJa^{-3})^2}{BkT} \left[\frac{E}{B} \right]^{1/2} x|E| . \tag{13}$$

Let us explain more in detail the origin of the above-mentioned peculiarity in the $\Phi(E)$ dependence. It is convenient to introduce the polaron shift E_p, which represents the energy gain due to self-consistent mutual alignments of the localized electron and magnetic ion spins, explicitly

$$E_p \sim W \left[\frac{a}{r_\Psi} \right]^3 , \quad W = \frac{(|J| a^{-3} S)^2}{kT} x . \tag{14}$$

The above are standard estimates for the polaron energy due to interaction $\langle |\Psi|^2\rangle JSn(\Delta S/S) r_\Psi^3$ linear in the induced relative spin $\Delta S/S$, which appearance increases the ion subsystem free energy quadratically, as $kTn(\Delta S/S)^2 r_\Psi^3$, so that E_p is the minimal value of the sum. With Eq. (14) taken into account the polaron energy can be written in the form

$$E(r_\Psi) \sim B \left[\frac{a}{r_\Psi} \right]^2 - W \left[\frac{a}{r_\Psi} \right]^3 , \tag{15}$$

where the first term represents the kinetic energy. Note

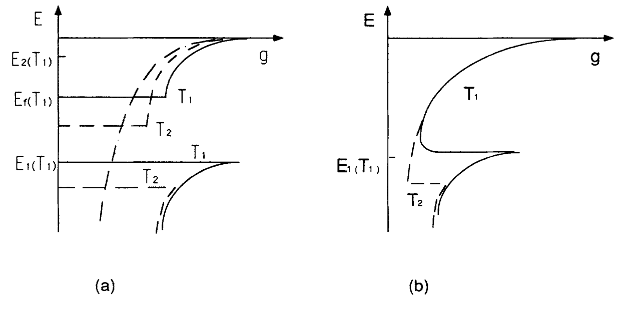

FIG. 1. Density of states in the tail of DMS at temperatures T_1 and $T_2(>T_1)$ in the limiting cases of uncorrelated (a) and strongly correlated (b) disorder. The dot-dashed line shows the density of states in the absence of an exchange interaction.

that there exists the so-called autolocalization barrier at $r_{\Psi b}=3aW/2B$; states of radii smaller than the latter quantity are unstable with respect to spin polarization, which makes them collapse, decreasing in energy. Another way to form a localized state in the system under consideration is to set it up within the proper tight fluctuation suggesting the polaron effect to be relatively small. As is well known[1-6] the typical tight fluctuation exhibits a potential well of a characteristic radius of order of the localization radius r_Ψ in the case of uncorrelated disorder. Consequently,

$$E(r_\Psi)\sim -B\left[\frac{a}{r_\Psi}\right]^2 . \qquad (16)$$

We then consider the energies of Eqs. (15) and (16) as two different electronic terms, which correspond to the external cases of localization: due to polaronic effect only, and without any polaronic effect at all (see Fig. 2). These two intersect at $r_{\Psi i}\sim aW/2B$. Suppose we have the state in an isolated fluctuation well of the radius $r_\Psi > r_{\Psi i}$. To pass to another term at the same energy the electron has to overcome the autolocalization barrier, and thus the initial state is a metastable one. In the opposite case of the initial state of the radius $r_\Psi < r_{\Psi i}$ in the fluctuation well the electron can pass to the polaronic term without the need of overcoming the barrier and then collapse, decreasing in energy to the saturation regime. Therefore, the states of energies below the intersection point are unstable and a kind of a gap in the energy spectrum exists. The upper bound of the gap E_f is determined by the value $r_{\Psi 0}$:

$$-E_f\sim B\left[\frac{a}{r_{\Psi 0}}\right]^2\sim\frac{B^3}{W^2}\sim\frac{(kT)^2B^3}{(Ja^{-3}S)^4x^2} . \qquad (17)$$

The gap extends down to the energy which corresponds to completely saturated polarization. The latter energy is $-\frac{1}{2}|J|S$ at zero temperature. As the temperature increases it goes even lower. Indeed, at zero temperature states in the nearest proximity of the lower bound are formed by very large spatial regions ($r_\Psi \gg a$) of completely saturated polarization, provided by very weak interaction ($\propto r_\Psi^{-3}$). Such states can be easily destroyed by thermal fluctuations. Based on this picture an estimate of the width δE of the energy layer within which bound states are destroyed takes the form $kT\sim J|Ssr_\Psi^{-3}\sim|J|Ssa^{-3}(\delta E/B)^{3/2}$, so that

$$\delta E\sim B\left[\frac{kT}{Ja^{-3}Ss}\right]^{2/3} , \qquad (18)$$

and the lower bound of the gap lies at the energy $-(|J|S\langle n\rangle/2+\delta E)$ (measured from the mobility edge).

The above arguments have been restricted to the uncorrelated disorder case, as is reflected in Eq. (16). The opposite case (of strongly correlated disorder) criterion $r_\Psi \ll r_c$ implies fluctuation potential wells to be classical in the sense that the electron kinetic energy in the well is negligibly small as compared with the well depth. Neglecting the kinetic energy and considering the potential well bottom as completely flat, one arrives at the standard problem of electronic states in a uniform medium in the presence of the magnetic polaron effect.[9,11] Two different groups of states are known for the latter case. The first one corresponds to a rather small degree of ion spin alignment, so that the magnetic polaron effect plays only a minor role, and states remain extended in a uniform medium. This kind of state in a random medium must have a density close to that given by Eq. (3). The states of another group involve strong, almost saturated spin polarization, being essentially autolocalized even in a uniform medium. Smooth fluctuations in a random medium are expected to cause the density of such states of the form of Eq. (3), but with parameters renormalized in accordance with Eq. (10). Because of the threshold character of the autolocalization in three-dimensional systems there are no types of states intermediate between the above-mentioned two types. Consequently, the characteristic energy E_1' must exist, dividing the spectrum, with the band tail given by the original Eq. (3) above E_1' and by the renormalized [in accordance with Eq. (10)] Eq. (3) below E_1'. The latter energy has the same physical meaning as the energy E_1 introduced above in the course of uncorrelated disorder consideration. This energy is expected to coincide with $-(\langle n\rangle/2)|J|S$ at zero temperature, being slightly deeper at finite temperatures due to thermal destruction of weakly bound states, qualitatively the same as that for the case of uncorrelated disorder above. In view of this analogy one can repeat estimations leading to Eq. (18). The only difference is that in the fluctuation potential well, which is parabolic near its minimum, the localization length is[6] $r_\Psi\sim[r_c(\delta E/B)^{-1/2}]^{1/2}$, rather than $r_\Psi\sim(\delta E/B)^{-1/2}$ used in Eq. (18). As a result we have

$$\delta E\sim B\left[\frac{kT}{Ja^{-3}Ss}\right]^{4/3}\left[\frac{r_c}{a}\right]^2 . \qquad (19)$$

We presume therefore that the density of states in the case of strongly correlated disorder has a hump at the energy $-(|J|S\langle n\rangle/2+\delta E)$, as well as in the previous case of uncorrelated disorder [Fig. 1(b)]. However, there is no

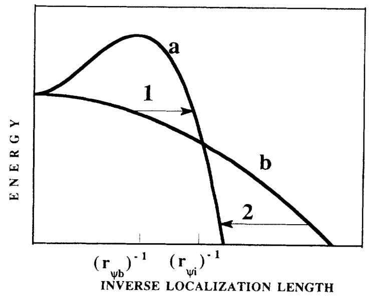

FIG. 2. Electronic terms vs the inverse localization radius $(r_\Psi)^{-1}$. The terms correspond to the extremal cases of localization (a) due only to polaron effects and (b) without any polaronic effects. The arrows indicate the interterm transitions: (1) involves overcoming the autolocalization barrier, while (2) occurs without the need of overcoming the barrier.

evidence of the gap in the spectrum, and the above qualitative consideration reveals the only characteristic energy E'_1 in this case, as opposed to the previous one.

It is worth noting that band tails, corresponding to the cases of strongly correlated and uncorrelated disorder, can overlap. In other words, both short and long radii (as compared with the correlation radius r_c) states can coexist at the same energy being provided by the appropriate fluctuations. As a result the entire density of states composed of both uncorrelated and correlated case contributions may have a rather complicated shape with either one or two humps, either possessing a gap or without it. The visual images can be obtained by putting together Figs. 1(a) and 1(b).

To estimate the density of states at its peaks one can proceed directly from Eqs. (2) and (3) for the cases of uncorrelated and strongly correlated disorder, respectively. Substituting δE given by Eq. (18) into Eq. (2) and δE given by Eq. (19) into Eq. (3), one gets

$$g_p \sim g_0 \exp\left[-\left[\frac{T}{T_0}\right]^{1/3}\right],$$
$$T_0 = \frac{\beta\varepsilon_0^2|J|}{v^2 k\langle n\rangle}\left[\frac{V-|J|S/2}{v}\right]^6 \tag{20}$$

for the case of uncorrelated disorder, and

$$g_p \sim g_0 \exp\left[-\left[\frac{T}{T'_0}\right]^{8/3}\right],$$
$$T'_0 = \beta'\left[\frac{a}{rc}\right]^{3/2}\left[\frac{b_0 v^2}{B^2}\right]^{3/8}\frac{|J|a^{-3}}{k} \tag{21}$$

for the case of strongly correlated disorder. Here β and β' are numerical coefficients, which cannot be determined in the above course of semiquantitative consideration; these coefficients will be obtained in Sec. III below [see Eq. (62)]. Peaks in the density of states at energies $E \approx E_1$ in the case of uncorrelated disorder or at $E \approx E'_1$ in the case of strongly correlated disorder are the most important feature of the above drawn picture. These peaks appear due to the exchange-interaction-induced shift of the origin for the energy scale given in Eq. (10). Their temperature decay, described by Eqs. (20) and (21), reflects the effect of thermal destruction of the spin polarization.

We have been tacitly assuming magnetic field to be absent above. When taken into account this field produces a magnetization which in its turn influences tail states. The field-induced magnetization shifts the mobility edge and bare (polaronless) tail states providing nonzero value $\langle \mathbf{S}\rangle$ in Eq. (4). In the strong-field regime $\mu H \gg kT$, where μ is the Bohr magneton, the average spin $\langle \mathbf{S}\rangle$ almost approaches its saturated value $S\mathbf{H}/H$, while its dispersion becomes relatively small: $\langle(\Delta\mathbf{S})^2\rangle \sim kTS/\mu H$. Correspondingly, the magnetic polaron effect becomes suppressed in this regime, and Eqs. (1)–(3) remain true with the renormalization given in Eq. (10). Hence, the above described complicated picture (Fig. 1) of band tails in DMS transforms to the renormalized standard scheme when subjected to a strong magnetic field.

One more remark regarding the underlaying standard band-tail scheme is in order. It should be clear that only spherically symmetric tail states have been discussed above, since they are known to be only important in the case of nonmagnetic semiconductors.[1–7] Recently a concept of resonant (quasilocalized) fluctuation states has been advanced,[12] suggesting looplike states of nonzero angular momenta superimposed on the continuous spectrum background. It has been shown that these states form a tail, which is more or less symmetric to the standard band tail with respect to the mobility edge in a disordered semiconductor. Nonzero angular momenta of such states can hardly be important in DMS, since their magnetic interaction with ions spins is negligibly small as compared with the exchange interaction. Taking into account only the latter one, it is possible to classify different magnetic polaron regimes in the same way as has been done for the spherically symmetric case. As a result, two new characteristic energies appear, which are symmetric to E_1 and E_2 with respect to the mobility edge. The states of (relatively) small radii above $|E_1|$, which are most sensitive to the magnetic polaron effect, decrease in the energy on approximately $(\langle n\rangle/2)|J|S$ ($\approx|E_1|$), while less sensitive states of large radii below $|E_1|$, and especially below $|E_2|$ ($<|E_1|$) undergo relatively small shifts. It follows then that states which were resonant in the absence of the polaron effect do not contribute much to the density of localized tail states in DMS.

We shall end this section with a remark concerning the actual temperature range in which band-tail localization effects can be important. Namely, the thermal energy kT must be noticeably smaller than the maximum energy scale in the tail. Hence, we put in what follows:

$$kT \ll \max\{\varepsilon_0, \varepsilon_1, |J|S\langle n\rangle/2\}\ . \tag{22}$$

In fact, we have already obtained all of the main results of the present work in the course of the above qualitative consideration. The next section is aimed at confirming these results by means of a more rigorous mathematical treatment.

III. THE OPTIMUM FLUCTUATION METHOD

We shall find an explicit form of the function $g(E)$ in the region of exponentially small density of states:

$$g(E) = g_0(E)\exp[-\Phi_0(E)] \ll g_0\ .$$

Since the space overlap of states is negligibly small in this region, we can restrict ourselves to consideration of individual localized states, using the so-called optimum fluctuation method (see the reviews[5–7]). It is assumed in the framework of the method that a state of any given energy is dominated by a corresponding particular shape of the local concentration fluctuation and large deviations from this shape are very unlikely to occur. The reasoning behind this assumption is that fluctuations much stronger than the optimal one are unlikely because a probability of fluctuation decreases exponentially with its strength increase; on the other hand, fluctuations much narrower than the optimal one become unlikely because it can hardly produce a localized state. The optimum fluctuation method enables us to determine optimal fluctuation shape, and by that to calculate $\Phi_0(E)$. The specific character of the present problem is that we study the states of a given free energy F rather than of energy E, which formulation reflects the necessity of accounting for thermodynamic fluctuations in the spin system. Thus, we look for optimal fluctuations which are responsible for states of a given free energy F, regarding the density of states as a function of F. Nevertheless, the final result is expected to be expressible as a function of energy E, as long as the latter is connected with F.

We consider the Gaussian part of the spectrum, in which the density of states is still of appreciable magnitude. This region is dominated by small deviations of the local concentration from its average value. The potential-energy functional

$$u[n(\mathbf{r})]=v\int|\Psi|^2(\mathbf{r})n(\mathbf{r})d^3r$$

of a carrier localized within such fluctuation is a random quantity, characterized by its dispersion

$$\sigma=v^2\int d^3r\,d^3r'\,b(\mathbf{r}-\mathbf{r}')|\Psi(\mathbf{r})|^2|\Psi(\mathbf{r}')|^2\ , \tag{23}$$

where $b(\mathbf{r}-\mathbf{r}')\equiv\langle\delta n(\mathbf{r})\delta n(\mathbf{r}')\rangle$ is the dispersion of the local concentration. The functional of probability of the deviation $u_1=u-\langle u\rangle$ has the form

$$P(u_1)=\exp\left[-\frac{u_1^2}{2\sigma}\right]\ . \tag{24}$$

In order to find the most effective (optimal) fluctuation one has to optimize the probability in Eq. (24) under two additional conditions, one of which is the normalization condition for the wave function, while another one expresses a given value of the free energy,

$$F=\frac{\hbar^2}{2m}\int|\nabla\Psi(\mathbf{r})|^2d^3r+U(\mathbf{r})\ , \tag{25}$$

where the effective potential energy has the form

$$\begin{aligned}U(\mathbf{r})&=v\int|\Psi(\mathbf{r})|^2\delta n(\mathbf{r})d^3r-T\ln Z\\&=u_1-T\ln Z\end{aligned} \tag{26}$$

and Z is the partition function of the spin system,

$$Z=Sp_sSp_{\mathbf{S}_i}\exp\left[\frac{J}{kT}\sum_i(\mathbf{S}_i\mathbf{s})|\Psi(\mathbf{r}_i)|^2\right]\ . \tag{27}$$

Optimization of the probability in Eq. (24) under these additional conditions is tantamount to finding the absolute extremum of the functional

$$\Phi=\frac{u_1^2}{2\sigma}+\lambda\left[F-\varepsilon\int|\Psi(\mathbf{r})|^2d^3r\right]\ , \tag{28}$$

where λ and ε are determined Lagrange multipliers; we shall see in what follows [Eq. (34)] that ε coincides with the electron self-energy. The recipe is to find u_1 satisfying Eq. (28) and then substitute it in the exponent in Eq. (24), which gives the density-of-states exponent Φ_0 in Eq. (1). From now on we restrict ourselves to the case of zero external magnetic field.

Two comments regarding the recipe above are in order. The first one concerns an explicit form of the correlation function $b(\mathbf{r})$ in Eq. (23). Two limiting cases, uncorrelated and strongly correlated, can be pointed out as independent of particular details of a disorder. For the uncorrelated disorder we have the inequality $r_\Psi\gg r_c$, which enables us to set

$$b(\mathbf{r})=\langle n\rangle\delta(\mathbf{r}-\mathbf{r}')\ . \tag{29}$$

For the strongly correlated disorder the opposite inequality makes it possible to restrict ourselves to the leading terms in the expansion

$$b(\mathbf{r})=b_0+b_1\frac{r^2}{r_c^2}+\cdots\ , \tag{30}$$

where the coefficients b_0 and $b_1\sim b_0$ are of order of$\langle n\rangle r_c^{-3}$. The second comment concerns the partition function in Eq. (27), which can be presented in more convenient form. Using standard transformations from Ref. 14, one can rewrite the partition function as

$$Z=2\prod_i f(\xi_i)+\left[1+\sum_i\xi_iB_s(\xi_i)\right]\ , \tag{31}$$

where

$$\xi_i=\frac{J}{2kT}|\Psi(\mathbf{r}_i)+|^2,\quad f(\xi)=\frac{\sinh[(S+\frac{1}{2})\xi]}{\sinh(\xi/2)}\ , \tag{32}$$

$$B_s(\xi)=\frac{f'(\xi)}{f(\xi)}=(S+\tfrac{1}{2})\coth[(S+\tfrac{1}{2})\xi]-\tfrac{1}{2}\coth(\xi/2)\ . \tag{33}$$

Here $B_s(\xi)$ is the usual Brillouin function, and besides that one can replace the summation by the integration in the expression $-T\ln Z$ for spin contribution to the free energy.

Consider first the case of uncorrelated disorder. On substituting Eqs. (23) and (31) into Eq. (28), one can find variational derivatives of the latter equation with respect to both $\Psi(\mathbf{r})$ and δn. On equating them to zero [in order to optimize the functional in Eq. (28)], one arrives at the following equations:

$$\begin{gathered}-\frac{\hbar^2}{2m}\Delta\Psi\\-\left[\tfrac{1}{2}JnB_s(\xi)-vn+\frac{Jn}{2}\frac{\xi B_s'(\xi)+B_s(\xi)}{1+I}\right]\Psi=\varepsilon\Psi\ ,\\\delta n+\lambda\langle n\rangle\{v|\Psi|^2-T\ln[f(\xi)]\}-\lambda T\langle n\rangle\frac{\xi B_s(\xi)}{1+I}=0\ ,\end{gathered} \tag{34}$$

where we have introduced the quantity

$$I=\langle n\rangle\int\xi(\mathbf{r})B_s[\xi(\mathbf{r})]d^3r\ . \tag{35}$$

The system in Eq. (34) can be reduced to the Schrödinger-type equation

$$-\frac{\hbar^2}{2m}\Delta\Psi+U\Psi=E\Psi,\quad E=\varepsilon-v\langle n\rangle\ , \tag{36}$$

with the energy E is reckoned from the shifted band edge and an effective potential depending on the wave function

$$U=\left\{v\langle n\rangle-\tfrac{1}{2}J\langle n\rangle B_s(\xi)\left[1+\frac{1+\xi B_s'(\xi)/B_s(\xi)}{1+I}\right]\right\}\left\{1-\lambda\left[v|\Psi|^2-T\ln[f(\xi)]-\tfrac{1}{2}J\frac{B_s(\xi)}{1+I}|\Psi|^2\right]\right\}-v\langle n\rangle\ . \quad (37)$$

With Eqs. (29) and (34) taken into account the density-of-states exponent in Eq. (24) takes the form

$$\Phi_0(E)=\frac{\lambda^2\langle n\rangle}{2}\int d^3r\left[v|\Psi|^2-kT\ln[f(\xi)]-\frac{JB_s(\xi)/2}{1+I}|\Psi|^2\right]^2 \quad (38)$$

with $\Psi(\mathbf{r})$ and λ satisfying Eq. (36). Since we are not able to find an analytical solution of Eq. (36) with the potential given in Eq. (37) in the whole range of parameters, we restrict ourselves to consideration of the limiting cases, corresponding to the different parts of the energy spectrum classified in Sec. II above.

As is seen from Eq. (6), the saturation regime occurs at $\xi\gg 1$. In the latter case the approximations are valid,

$$\begin{gathered}B_s(\xi)\approx S\,\mathrm{sgn}J,\quad \ln[f(\xi)]\approx|\Psi|^2JS/2kT\ ,\\ I\approx|JS|\langle n\rangle/2kT\ ,\end{gathered} \quad (39)$$

upon which Eq. (36) takes the form

$$-\frac{\hbar^2}{2m}\Delta\Psi-\lambda\langle n\rangle(v-|J|S/2)^2|\Psi|^2\Psi=(E+|J|S\langle n\rangle/2)\Psi\ . \quad (40)$$

A nonlinear Schrödinger equation of the type in Eq. (40) has been studied in detail in Refs. 5–7 and 13 in connection with the standard optimum fluctuation method. The results of importance here are summarized by the expressions

$$\begin{gathered}E+\frac{|J|S\langle n\rangle}{2}=-\frac{c}{4}\frac{\hbar^6}{m^3}\left[\langle n\rangle\lambda\left[v-\frac{|J|S}{2}\right]^2\right]^{-2},\\ \int|\Psi|^4d^3r=c\frac{\hbar^6}{m^3}\left[\langle n\rangle\lambda\left[v-\frac{|J|S}{2}\right]^2\right]^{-3}.\end{gathered} \quad (41)$$

Using these results, we obtain Eqs. (1) and (2) with the parameters in Eq. (10). Where necessary one can also express the density of states in terms of the free energy, which is the case at hand is

$$F=E-kT(\ln 2)(1+|J|S\langle n\rangle/2kT)\ . \quad (42)$$

We turn now to the polaron regime, in which we again have $I\gg 1$, but $\xi\ll 1$. In this case the following approximations hold:

$$\begin{gathered}B_s\approx\frac{S(S+1)|\Psi|^2}{6\langle n\rangle T},\quad \ln[f(\xi)]\approx\frac{S(S+1)}{6}\left[\frac{J|\Psi|^2}{2kT}\right]^2,\\ I\approx\frac{S(S+1)}{3}\left[\frac{J}{2kT}\right]^2\langle n\rangle\int|\Psi|^4d^3r\ ,\end{gathered} \quad (43)$$

and Eq. (36) reads

$$-\frac{\hbar^2}{2m}\Delta\Psi-\langle n\rangle\left[\lambda v^2+\frac{S(S+1)J^2}{12kT}\right]|\Psi|^2=E\Psi\ . \quad (44)$$

Equation (44) differs from Eq. (40) in that λ appears in the only first component of the effective potential. However, expressions of the type in Eq. (41), suitably modified, are again applicable. On using them, we find

$$\begin{gathered}\Phi_0(E)=\left[\frac{|E|}{\varepsilon_0}\right]^{1/2}\left[1-\left[\frac{|E|}{\varepsilon_p}\right]^{1/2}\right]^2,\\ \varepsilon_p=\frac{T^2}{\varepsilon_0}\left[\frac{6v^2}{S(S+1)J^2}\right]^2.\end{gathered} \quad (45)$$

As expressed in terms of the energy, the free energy becomes

$$F=E\left[1-2\left[\frac{|E|}{\varepsilon_p}\right]^{1/2}\right]-kT\ln\left[\frac{8\varepsilon_p}{kT}\left[\frac{|E|}{\varepsilon_p}\right]^{3/2}\right]. \quad (46)$$

The remarkable feature is that for the values under consideration, $|E|\gg T$, the free energy has a minimum as a function of the energy E at

$$|E_f|\approx\frac{\varepsilon_p}{9}\left[1+\frac{27}{2}\frac{kT}{\varepsilon_p}\right], \quad (47)$$

which means that there are no states of energies $E<E_f$ in the polaron regime, and, thus, a sort of gap in the energy spectrum appears. The physical origin of this gap has been discussed in Sec. II above. There is nothing new with the above mathematical description of the gap, which has been given more in detail in Ref. 13 for the acoustic polaron band tailing.

Our last topic is the thermodynamic-fluctuation regime. In this case approximations in Eq. (43) remain valid, but we have $I\ll 1$. Equation (36) accordingly takes the form (45) with the second term in the effective potential greater by a factor of 3. It thus becomes possible to formally make use of Eq. (45) with $\varepsilon_p\rightarrow\varepsilon_p/9$. In this regime the second term in Eq. (45) is small, so we have

$$\Phi_0(E)\approx\left[\frac{|E|}{\varepsilon_0}\right]^{1/2}\left[1-6\left[\frac{|E|}{\varepsilon_p}\right]^{1/2}\right], \quad (48)$$

and correspondingly the free energy becomes

$$F=E[1-10(|E|/\varepsilon_p)^{1/2}]-T\ln 2\ . \quad (49)$$

To verify that the inequality $(|E|/\varepsilon_p)^{1/2}\ll 1$ obeys, we need to express its left-hand side in terms of the parameter $I\ll 1$. Using the expressions in Eq. (41) for this purpose, we find

$$\left[\frac{|E|}{\varepsilon_p}\right]^{1/2}=\frac{kT}{|E|}I \ll 1 \ . \tag{50}$$

Note that Eq. (48) agrees with the qualitative results in Eqs. (12) and (13).

We turn now to the correlated disorder case. We proceed again from Eqs. (24)–(28) and use the correlation function in Eq. (30). On varying Φ in Eq. (28) with respect to $\Psi(\mathbf{r})$ and u_1, we arrive at the set of equations similar to those in Eq. (34):

$$\begin{aligned}\varepsilon\Psi=&-\frac{\hbar^2}{2m}\Delta\Psi+\left\{v-\tfrac{1}{2}JB_s(\xi)-\tfrac{1}{2}J\frac{B_s(\xi)+\xi B_s'(\xi)}{1+I}\right\}\langle n\rangle\Psi\\ &-\frac{u_1^2}{\lambda\sigma^2}\left\{v-\tfrac{1}{2}JB_s(\xi)-\tfrac{1}{2}J\frac{B_s(\xi)+\xi B_s'(\xi)}{1+I}\right\}\left\{\int d^3r'\,b(\mathbf{r}-\mathbf{r}')\left[v|\Psi(\mathbf{r}')|^2-kT\ln[f(\xi(\mathbf{r}'))]\right.\right.\\ &\left.\left.-\frac{1}{2}\frac{JB_s[\xi(\mathbf{r}')]|\Psi(\mathbf{r}')|^2}{1+I}\right]\right\}\Psi\ ,\\ u_1+\lambda=&\,0\ .\end{aligned}\tag{51}$$

Substituting correlation function (30) into Eq. (52) leads to the equation

$$-\frac{\hbar^2}{2m}\Delta\Psi-U_0\Psi+U_1\frac{r^2}{r_c^2}\Psi=E\Psi \tag{53}$$

with the parameters

$$\begin{aligned}U_0=&\left\{v\langle n\rangle-\tfrac{1}{2}J\langle n\rangle B_s(\xi)\left[1+\frac{1+\xi B_s'(\xi)/B_s(\xi)}{1+I}\right]\right\}\left\{1-\frac{\lambda}{\langle n\rangle}\int d^3r'\left[b_0+\frac{r'^2}{r_c^2}b_1\right]\frac{\delta u(\mathbf{r}')}{\delta n}\right\}\ ,\\ U_1=&\left\{v-\tfrac{1}{2}JB_s(\xi)\left[1+\frac{1+\xi B_s'(\xi)/B_s(\xi)}{1+I}\right]\right\}\lambda b_1\int d^3r'\frac{\delta u(\mathbf{r}')}{\delta n}\ .\end{aligned}\tag{54}$$

At the same time the density-of-states exponent becomes

$$\Phi_0(E)=\frac{\lambda^2}{2}\sigma\ . \tag{55}$$

Again, the procedure cannot be implemented analytically for arbitrary values of the parameters in Eqs. (53)–(55), and we have to consider separately the various part of the spectrum according to the classification given in Sec. II.

For the saturation regime, at $|E| \gg |E_1'|$, the approximations of the form in Eq. (39) enable us to rewrite Eq. (53) as

$$-\frac{\hbar^2}{2m}\Delta\Psi+\lambda(v-\tfrac{1}{2}|J|S)^2b_1\frac{r^2}{r_c^2}\Psi=\left[E+\frac{\langle n\rangle|J|S}{2}-\lambda\left[v-\frac{|J|S}{2}\right]^2\left[b_0+b_1\frac{r_\Psi^2}{r_c^2}\right]\right]\Psi\ . \tag{56}$$

The latter is the equation for the three-dimensional harmonic oscillator of the frequency $\omega=|v-|J|S/2|(\lambda b_1/3mr_c^2)^{1/2}$, whose solution is

$$E+\frac{\langle n\rangle|J|S}{2}-\lambda\left[v-\frac{|J|S}{2}\right]^2\left[b_0+b_1\frac{r_\Psi^2}{r_c^2}\right]=\left[\frac{3\hbar^2U_1}{2mr_c^2}\right]^{1/2}=\left[4\lambda b_0E_c\left[v-\frac{|J|S}{2}\right]^2\right]^{1/2} \tag{57}$$

with a characteristic energy

$$E_c=\frac{3b_1}{8b_0}\frac{\hbar^2}{mr_c^2}\ . \tag{58}$$

On expressing $\lambda(E)$ from Eq. (57) and substituting it and $\sigma=b_0(v-|J|S/2)^2$ into Eq. (55), one arrives at Eq. (3) with the renormalized parameters given in Eq. (10).

For the polaron regime, i.e., at $E_1'<E<E_2'$, the approximation in Eq. (43) are valid, so that Eq. (53) reduces to

$$-\frac{\hbar^2}{2m}\Delta\Psi-\frac{\langle n\rangle S(S+1)}{12kT}J^2|\Psi|^2\Psi+\lambda v^2b_1\frac{r^2}{r_c^2}\Psi=\left[E-\lambda v^2\left[b_0+b_1\frac{r_\Psi^2}{r_c^2}\right]\right]\Psi\ . \tag{59}$$

We note then, that the second and the third terms on the left-hand side of Eq. (59) should be considered as negligibly small in accordance with the above interpretation of the strongly correlated disorder case, which implies the states in a fluctuation potential well to be classical, and, correspondingly, the bottom of the well to be completely flat. Hence, we equate the right-hand side of Eq. (59) to zero. On taking into account the conditions $r_\Psi \ll r_c$ and $b_1 \sim b_0$, we finally get

$$E-\lambda v^2 b_0=0 . \tag{60}$$

Substituting this result into Eq. (55) leads to the standard expression (3) for the density-of-states exponent in the polaron region $E_2' \gg E \gg E_1'$.

Lastly, in the thermodynamic-fluctuation regime Eq. (53) also takes the form (59), but with the first component of the effective potential three times larger. Accordingly, the above result remains valid.

We can now write expressions for the characteristic energies E_1 and E_2 (and, respectively, E_1' and E_2'), which separate the regions in which the three above stated regimes occur. These energies are determined from the conditions $\xi=1$ and $I=1$. In writing the first condition we express $|\Psi|^2$ in terms of I. For this purpose we note that the potential well corresponding to the optimal fluctuation is approximately square,[6] allowing the approximation

$$|\Psi|^2=\int|\Psi|^4 d^3r .$$

For I we use Eq. (50). As a result we have

$$\begin{aligned}
|E_1|&=B\left[\frac{kT}{|J|a^{-3}}\right]^{2/3}\left[\frac{c}{16}\right]^{1/3}+\frac{|J|S\langle n\rangle}{2} ,\\
|E_2|&=B\left[\frac{kT}{|J|a^{3}}\right]^{4/3}\left[\frac{6\sqrt{c}}{S(S+1)x}\right]^{2/3} ,\\
|E_1'|&=B\left[\frac{2kT}{|J|a^{-3}}\right]^{4/3}\left[\frac{r_c}{a}\right]^{2}\frac{2b_0}{3b_1}+\frac{|J|S\langle n\rangle}{2} ,\\
|E_2'|&=B\left[\frac{kT}{|J|a^{3}}\right]^{8/3}\left[\frac{12}{S(S+1)x}\right]^{4/3}\left[\frac{r_c}{a}\right]^{2}\frac{2b_0}{3b_1} .
\end{aligned} \tag{61}$$

The second terms in the expressions for E_1 and E_1' correspond to the shift of the origin for the energy scale determined in Eq. (10), while the first terms can be interpreted as describing the thermal destroying of large radii's states, in accordance with the estimates in Eqs. (18) and (19). The relations $|E_1| \gg |E_2|$ and $|E_1'| \gg |E_2'|$ follow from the inequality $|J|S\langle n\rangle/2 \gg kT$, which expresses the condition that the thermal energy is small in comparison with the maximum energy of the exchange coupling. Only in this case can the effects of the magnetic interaction be seen against the background of the thermal disorder.

The same inequality $|J|S\langle n\rangle/2 \gg kT$ guarantees satisfaction of the condition $E_f > E_1$, which means that there exists a gap in the density of states at $E_1 < E < E_f$. It should be kept in mind however, that the latter conclusion is applicable only to the case of Gaussian fluctuations. When we take non-Gaussian fluctuations into account, we can say no more than that there exists an exponentially deep valley in the density of states between E_f and E_1.

We are now in a position to estimate the density of states in its peaks at energies $E \approx E_1$ in the case of uncorrelated disorder or at $E \approx E_1'$ in the case of strongly correlated disorder. The estimates for these two cases yield, respectively,

$$g_p \sim g_0 \exp\left[-\left[\frac{|E_1|-|J|S\langle n\rangle/2}{\varepsilon_1}\right]^{1/2}\right]$$

or

$$g_p \sim g_0 \exp\left[-\frac{(|E_1'|-|J|S\langle n\rangle/2)^2}{2v^2 b_0}\right] .$$

On substituting E_1 and E_1' from Eq. (61) in, we come to Eqs. (20) and (21), but with specified numerical coefficients:

$$\begin{aligned}
T_0&=\frac{4\varepsilon_0^2|J|}{v^2 k\langle n\rangle}\left[\frac{v-|J|S/2}{v}\right]^{6} ,\\
T_0'&=2^{7/8}\left[\frac{E_c^2 b_0 v^2}{B^4}\right]^{3/8}\frac{|J|a^{-3}}{k} .
\end{aligned} \tag{62}$$

Therefore all the conclusions based on the qualitative consideration in Sec. II above are in agreement with those obtained in the framework of the rigorous optimum fluctuation method.

IV. DISCUSSION OF RESULTS

The main result of this study is a prediction of a nonmonotonic energy dependence of the tails of the density of states in DMS. The established deviations from the standard monotonic behavior are described by several energy parameters, which we will not estimate. Assuming typical values of parameters $-va^3 \sim 3/4|J|Sa^{-3} \sim 1.5$ eV, $\varepsilon_0 \sim 1-10$ meV, $x \sim 0.1$, Eqs. (9), (40), (36), and (41) give

$$\begin{aligned}
&\varepsilon_1 \approx 8\varepsilon_0, \quad |E_1| \sim |E_1'| \sim \frac{|J|S\langle n\rangle}{2} \sim 0.1 \text{ eV} ,\\
&|E_2| \sim \varepsilon_0\left[\frac{kT}{\varepsilon_0}\right]^{4/3}, \quad |E_f| \sim \frac{\varepsilon_0}{4}\left[\frac{kT}{\varepsilon_0}\right]^{2} ,\\
&T_0 \sim 3-300 \text{ K}, \quad T_0' \sim \left[\frac{a}{r_c}\right]^{3/4} \times 100 \text{ K} .
\end{aligned} \tag{63}$$

The relative positions of the characteristic energies in the spectrum are shown in Fig. 1. Estimates (63) show that there exists a wide temperature range $\varepsilon_0 \ll T \ll |E_1|$ in which the structure of the spectrum in Fig. 1—i.e., $|E_1| \gg T$, $|E_f| \gg T$, $|E_2| \gg T$—might be observable.

It should be stressed that the nonmonotonic behavior of the density-of-states tails in DMS stems from the interaction of localized charge carriers with the spins of magnetic ions and, thus, could show up in processes involving initially occupied tail states only. Luminescence might be one such process. According to our results, the emission should be dominated by carriers localized in the peaks in the density of states at $E \sim E_1$. This conclusion means, that the emission spectrum of a DMS should have a peak at energies approximately $|E_1|$ less than the band gap. In addition, the luminescence intensity is expected to manifest a sort of temperature quenching, falling off with the temperature in accordance with Eqs. (20) or (21). Experimentally, peaks in the photoluminescence of DMS have been observed in several places[15,16] at energies slightly below the width of the band gap. These peaks have been interpreted in those studies as resulting from the existence of excitons or impurity levels. We would pin our hopes for an experimental test of our interpretation of the photoluminescence peaks on an observation of the specific temperature dependence of the photoluminescence temperature quenching given by Eqs. (20) and (21). Besides, in accordance with Eq. (61), the energy difference between the width of the band gap and the peak in the luminescence should be proportional to the exchange integral J, which can be independently extracted from experimental data.[8]

States in the peaks in the band tails might be seen as trapping levels in the photoconductivity. A distinguishing feature of such levels would be the temperature dependence of their concentration, which is similar to one of those given in Eqs. (20) and (21). The latter dependence is expected to increase charge carrier lifetimes and the corresponding photoconductivity exponentially with a temperature at $T > T_0$ or $T > T_0'$.

There is another prediction, which concerns the spectrum of the magnetization which arises during exposure of DMS to polarized light. Specifically, the magnetization should reach its maximum at photon energies which generate charge carriers directly in the peaks in the band tails. The production of charge carriers of higher energies would be accompanied by their depolarization over the time taken for their trapping in the tail states. On the other hand, the number of carriers produced by photons of lower energies would be exponentially small. Experiments on optical magnetization[17] have indeed revealed a peak at energies a few tens of millielectron volts less than the width of the band gap. The latter energy could be interpreted as $|E_1| \sim |J|S/2$ in accordance with the above consideration. Besides, there is some evidence in the data in Ref. 17 in favor of the thermal quenching of the observed optical magnetization, which could be speculatively related to Eqs. (20) or (21) of the present work.

To avoid any misunderstanding, it should be stressed that the nonmonotonic behavior of the density of states described above can hardly be seen in the optical-absorption tails due to interband transitions. Such transitions can be treated in the Franck-Condon approximation, since the characteristic times for spin flips are comparatively long. Therefore, the absorption tails are determined by fluctuations created in the absence of localized charge carriers. Since fluctuations in the concentration and the spins are independent in this case, one can assert that the energy dependence of the tails of the interband absorption would be described by expressions like (1) and (2), in which we would have to replace v^2 by $v^2+(JS/2)^2$. On the other hand, a peak in the tail of the density of states might be manifested in the absorption if it were occupied by charge carriers, as it apparently is in degenerate DMS.

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