Degeneracy effects on the relaxation and recombination of adsorbed doubly polarized atomic hydrogen

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We determine the influence of the Kosterlitz-Thouless transition on the stability of doubly spin-polarized atomic hydrogen adsorbed on a superfluid helium film and find that the lifetime of the gas is enhanced dramatically in the superfluid phase. Moreover, the relevant rate constants have a discontinuous behavior at the critical point. The observation of these effects is an unambiguous way to establish the achievement of the phase transition experimentally.

I. INTRODUCTION

Since the observation by Kosterlitz and Thouless[1] that in two-dimensional systems a phase transition can be brought about by the unbinding of topological excitations, it has been possible to apply their ideas not only to such seemingly unrelated phenomena as melting[2] and surface roughening[3] but also to the critical behavior of granular superconductors, Josephson-junction arrays,[4] neutral anyonic gases[5] and superfluid ^{4}He films. In particular, in the latter case various experiments[6,7] have clearly confirmed the predictions of the Kosterlitz-Thouless theory and, most notably, the universal jump in the superfluid density at the transition temperature which was stressed by Nelson and Kosterlitz.[8]

Being in the same universality class as liquid helium, a gas of doubly spin-polarized atomic hydrogen ($H\downarrow\Downarrow$) adsorbed on a superfluid helium film is believed to have a Kosterlitz-Thouless transition as well. Although this notion is not new,[9] there has not been any experimental effort until recently to actually observe the transition. The main reason for this is that at the high densities required, the atomic gas decays rapidly due to inelastic three-body processes leading to the formation of hydrogen molecules. However, Svistunov, Hijmans, Shylapnikov, and Walraven[10] have suggested that the adsorbed gas has effectively a much longer lifetime if it is in contact with a large buffer volume. Moreover, they note that in such a configuration the realization of the Kosterlitz-Thouless transition can most easily be established by observing the discontinuity in the adsorbtion isotherm or its influence on the decay of the density in the buffer volume.

Motivated by these promising ideas, we developed in a previous paper[11] a microscopic theory for doubly spin-polarized atomic hydrogen, which is valid at essentially all temperatures for the densities of interest ($n \simeq 1\times 10^{13}$ cm^{-2}) because it includes the contribution of all two-body or ladder diagrams. It, therefore, greatly improves upon the one-loop theory of Popov,[12] which turns out to be unable to give an accurate description of the gas at the experimentally relevant temperatures ($T \simeq 100$ mK).[11,13] Furthermore, we calculated various important equilibrium properties of an atomic hydrogen gas adsorbed on superfluid helium, such as the superfluid density, the critical temperature, and the adsorption isotherm.

However, we did not consider the effect of the Kosterlitz-Thouless transition on the decay of the gas. This topic was first discussed by Kagan, Svistunov, and Shylapnikov,[14] who found (but see Sec. III) that the three-body recombination-rate constant near zero temperature is about a factor of 6 smaller than slightly above the critical temperature. They also conjectured that the rate constant shows a discontinuous drop at the critical temperature but their discussion makes use of the Popov theory and can, therefore, only be trusted at temperatures very far below the critical one. This is not the case for our theory of the dilute Bose gas, which can be used to settle this interesting issue of the discontinuity and even estimate the magnitude of the effect if it exists. How this works out is explained below.

The remaining part of the paper is organized as follows. In Sec. II we discuss the two- and three-body processes that lead to the decay of doubly spin-polarized atomic hydrogen and derive the density and temperature dependence of the relevant rate constants. In Sec. II A we consider the normal phase of the gas just above the critical temperature, whereas in Sec. II B we turn to the required modifications if the gas is superfluid and we have to deal with the presence of a quasicondensate. Our final results are presented in Sec. III and the conclusions of this work are summarized in Sec. IV.

II. DECAY PROCESSES IN ADSORBED $H\downarrow\Downarrow$

A gas of doubly spin-polarized hydrogen atoms adsorbed on superfluid helium with a binding energy of about one kelvin would be absolute stable and an almost ideal realization of a two-dimensional weakly interacting Bose gas without the dipole interaction between the magnetic moments of the electrons and protons constituting the hydrogen atoms. The magnetic dipole interaction, however, leads to depolarization and the formation of hydrogen molecules, which diminish the density of atoms in the doubly polarized state $|\downarrow\Downarrow\rangle$. The rate equation for the gas is thus

$$\frac{dn}{dt} = -Gn^2 - Ln^3 , \tag{1}$$

where we have introduced the two-body relaxation- and three-body recombination-rate constants G and L, respectively.

For the strong magnetic fields that are used to increase the stability, the relaxation is dominated by collisions that flip one proton spin, whereas in recombination events one and two electron spin-flip processes are roughly equally important. Besides the magnetic-field strength, the rate constants depend also on the temperature and, in the degenerate regime, on the density of the gas. As we shall see shortly, the determination of these dependencies is greatly simplified by the fact that at subkelvin temperatures, the thermal de Broglie wavelength $\Lambda=(2\pi\hbar^2/mk_BT)^{1/2}$ of the hydrogen atoms is much larger than the range of the interaction and only s-wave scattering is of importance. Hence, the momentum dependence of various collisional quantities can be neglected and we can use effective-range theory to express our results solely in terms of the scattering length.

Furthermore, in the three-body process a large amount of energy is released, which makes it highly improbable for the particles involved to remain adsorbed on the surface. As a result, the rate constant is nearly independent of the angle ϑ between the magnetic-field direction and the surface normal.[15] For two-body relaxation this is not true and we have the strongly anisotropic result,[16]

$$G(\vartheta)=G_0\sin^2(2\vartheta)+G_2\sin^2(\vartheta)[1+\cos^2(\vartheta)] , \quad (2)$$

with the term proportional to $G_{\Delta m}$ corresponding to a transition that changes the angular momentum by $\pm\Delta m$. Thus, for a magnetic field perpendicular to the surface, the relaxation rate vanishes. Unfortunately, this ideal situation cannot be realized everywhere on the superfluid helium film for the experimental configuration under construction[17] and relaxation processes are allowed.

Despite this fact, it is well known that three-body dipolar recombination is the dominant decay mechanism at the high densities needed for the establishment of the Kosterlitz-Thouless transition at moderate temperatures.[9] Nevertheless, we discuss here also the decay of the gas due to two-body collisions, because in the superfluid phase the recombination rate is reduced considerably and it is *a priori* not clear that relaxation does not contribute significantly to the decay of the system.

A. Normal phase

Treating the weak magnetic dipole interaction V^d as a perturbation and neglecting the influence of the surrounding gas on the collision, the relaxation-rate constant is given by

$$G(T)=C_G\int d\hat{p}_f\langle p_f|\langle\Psi^{(-)}_{\mathbf{p}_f}|V^d\mathcal{S}|\Psi^{(+)}_{\mathbf{p}_i}\rangle|^2\rangle_{\text{th}} . \quad (3)$$

Here C_G is a proportionality constant, $\mathcal{S}$ is an unnormalized symmetrization operator, and $\langle\cdots\rangle_{\text{th}}$ denotes a thermal average over the initial relative momentum $\mathbf{p}_i$ of the colliding particles. The integral is only over the direction of the final relative momentum $\mathbf{p}_f$, since the magnitude is determined by energy conservation. Denoting the energy released in the relaxation process by Δ_{rel}, this simply implies that $\mathbf{p}_f^2=\mathbf{p}_i^2+m\Delta_{\text{rel}}$. The initial and final states of the transition matrix element are continuum eigenstates of the zeroth-order Hamiltonian, including the central (singlet-triplet) interaction between the hydrogen atoms, and obey ingoing and outgoing asymptotic boundary conditions, respectively. Notice also that we have suppressed the spin indices to simplify the notation.

At sufficiently low temperatures ($k_BT\ll\Delta_{\text{rel}}$), the temperature dependence of the relaxation rate is determined by the low-energy behavior of the initial state, which factorizes as[18]

$$\Psi^{(+)}_{\mathbf{p}}(\mathbf{r})\underset{p\downarrow 0}{\sim}T^{2B}(0,0;2\epsilon_{\mathbf{p}})\phi(\mathbf{r}) , \quad (4)$$

introducing the kinetic energy $\epsilon_{\mathbf{p}}=\mathbf{p}^2/2m$ and the (off-shell) two-body T matrix $T^{2B}(0, 0; E)$ at energy E. In two dimensions, the T matrix develops a logarithmic singularity and we must use

$$T^{2B}(0,0;\epsilon_{\mathbf{p}})=\frac{4\pi\hbar^2/m}{\pi i-\ln(\mathbf{p}^2a^2/8\hbar^2)-2\gamma} , \quad (5)$$

with $\gamma\simeq 0.5772$ Euler's constant and $a\simeq 2.40\ a_0$ the two-dimensional scattering length for a collision of two doubly polarized atoms.

Substituting these results in Eq. (3) and performing the thermal average, we expect the relaxation-rate constant to have the asymptotic behavior,

$$G(T)\underset{T\downarrow 0}{\sim}C_{\text{rel}}\frac{(2\pi)^2}{\ln^2(a/\Lambda)} , \quad (6)$$

and, in particular, to vanish at zero temperature. However, this last feature is an artifact caused by the neglect of the influence of the surrounding gas, which is only justified if the degeneracy parameter $n\Lambda^2$ is small. In the degenerate regime where $n\Lambda^2=\mathcal{O}(1)$, the average kinetic energy is comparable to the average interaction energy and we must add the self-energy $\hbar\Sigma$ to $\epsilon_{\mathbf{p}}$ to obtain the dispersion of the atoms. Consequently Eq. (4) is modified into

$$\Psi^{(+)}_{\mathbf{p}}(\mathbf{r})\underset{p\downarrow 0}{\sim}T^{2B}(0,\ 0;2\epsilon_{\mathbf{p}}-2\hbar\Sigma)\phi(\mathbf{r}) \quad (7)$$

and we find

$$G(T)\underset{T\downarrow 0}{\sim}C_{\text{rel}}\left|\frac{m}{\hbar^2}T^{2B}(0,0;-2\hbar\Sigma)\right|^2 , \quad (8)$$

which does not vanish at zero temperature because the self-energy obeys[11]

$$\hbar\Sigma\underset{T\downarrow 0}{\sim}2nT^{2B}(0,\ 0;\ -\hbar\Sigma) \quad (9)$$

and is nonzero in this limit.

A similar discussion is possible for the recombination process. In the first instance, the decay rate is written as

$$L(T)=C_L\sum_{vlm}\int d\hat{q}_f\langle q_f|\langle\Psi^{(-)}_{vlm,\mathbf{q}_f}|V^d\mathcal{S}|\Psi^{(+)}_{\mathbf{p}_i,\mathbf{q}_i}\rangle|^2\rangle_{\text{th}} , \quad (10)$$

where $\mathbf{p}_i$ and $\mathbf{q}_i$ are the Jacobi momenta[19] of the three incoming atoms and $\mathbf{q}_f$ is the relative momentum of the outgoing atom with respect to the center of mass of the hydrogen molecule, having the rovibrational quantum numbers (v,l,m). In addition, V^d now represents the sum of the dipole interactions between the three pairs.

If $k_B T \ll \Delta_{\rm rec}$, the temperature dependence is again solely determined by the low-energy behavior of the initial state. However, an effective-range theory for a three-body collision in two dimensions is not available at present. Nevertheless, we can make progress by realizing that in three dimensions, a Jastrow-type wave function is an excellent approximation for the initial state. Applying the same idea to two dimensions we are led to

$$\Psi^{(+)}_{\mathbf{p},\mathbf{q}}(\{\mathbf{r}_j\}) \underset{p,q\downarrow 0}{\sim} \prod_j T^{2B}(0,0;2\epsilon_{\mathbf{p}_j})\phi(\mathbf{r}_j) , \tag{11}$$

with $\mathbf{r}_j$ the distance between the particles of pair j, and $\mathbf{p}_j$ their relative momentum.[18] As a result we find that

$$L(T) \underset{T\downarrow 0}{\sim} C_{\rm rec}\frac{(2\pi)^6}{\ln^6(a/\Lambda)} , \tag{12}$$

which is incorrect in the degenerate regime where we must take into account that the particles do not collide in a vacuum. The correct behavior is then

$$L(T) \underset{T\downarrow 0}{\sim} C_{\rm rec}\left|\frac{m}{\hbar^2}T^{2B}(0,0;-2\hbar\Sigma)\right|^6 . \tag{13}$$

At this point it is important to realize that the dominant corrections to Eqs. (8), (9), and (13) can be neglected if the average kinetic energy is small compared to the self-energy $\hbar\Sigma$. For atomic hydrogen this implies roughly that $n\Lambda^2 > 2$, which is sufficient for our purposes because at the critical temperature we have $n\Lambda_c^2 \simeq 6$ and the above asymptotic expressions can, therefore, be used in the temperature interval $T_c \le T < 3T_c$. Clearly, below the critical temperature modifications are needed to also take the effect of the quasicondensate into account. This is our next objective.

B. Superfluid phase

The decay rates derived above are in their region of validity independent of temperature and depend only on the density of the gas. Due to this feature we can incorporate the influence of the quasicondensate by means of the correlator method devised by Kagan, Svistunov, and Shylapnikov.[14] Here we present their approach in a somewhat different language, which is more convenient for a discussion of the wave-function renormalization that is required to cancel certain ultraviolet divergences and that was performed incorrectly in Ref. 14. Furthermore, we use the simpler two-body process as an example.

We describe the relaxation of the gas by means of the contact interaction,

$$V^{\rm rel} = \frac{1}{2}\int d\mathbf{x}\,\mathcal{V}\psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x})\psi(\mathbf{x}) , \tag{14}$$

and the usual field operators $\psi^\dagger(\mathbf{x})$ and $\psi(\mathbf{x})$, creating or annihilating a particle at position $\mathbf{x}$. Here $\mathcal{V}$ is an effective potential amplitude, which must be adjusted at the end of the calculation to reproduce the results of Sec. II A above the critical temperature. Bearing this in mind, the decay rate out of an initial state $|i\rangle$ with energy E_i is found from Fermi's Golden Rule and equal to

$$\Gamma_i = \frac{2\pi}{\hbar}\int d\mathbf{x}\int d\mathbf{x}'\sum_f \left\langle i\left|\frac{\mathcal{V}}{2}\psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x})\psi(\mathbf{x})\right|f\right\rangle\left\langle f\left|\frac{\mathcal{V}}{2}\psi^\dagger(\mathbf{x}')\psi^\dagger(\mathbf{x}')\psi(\mathbf{x}')\psi(\mathbf{x}')\right|i\right\rangle\delta(E_f-E_i) . \tag{15}$$

In particular, we can obtain the intrinsic two-body rate without degeneracy effects by using $|i\rangle = |\mathbf{p},\mathbf{p}'\rangle$ and taking the limit $p,p'\downarrow 0$ for $\mathbf{p}\neq\mathbf{p}'$. The final state $|f\rangle$ then equals $|\mathbf{p}'',-\mathbf{p}''\rangle$ and we have

$$\begin{aligned}\Gamma_0 = \frac{2\pi}{\hbar}\int d\mathbf{x}\int d\mathbf{x}'\sum_{\mathbf{p}''}\frac{\mathcal{V}^2}{V^2}&\langle 0|\psi(\mathbf{x})\psi(\mathbf{x})|\mathbf{p}'',-\mathbf{p}''\rangle \\ &\times\langle \mathbf{p}'',-\mathbf{p}''|\psi^\dagger(\mathbf{x}')\psi^\dagger(\mathbf{x}')|0\rangle \\ &\times\delta(2\epsilon_{\mathbf{p}''}-\Delta_{\rm rel}) ,\end{aligned} \tag{16}$$

denoting the vacuum by $|0\rangle$ and the volume of the system by V. The energy $\Delta_{\rm rel}$ released in the relaxation process is, of course, due to the spin degrees of freedom, which we have again suppressed for clarity reasons.

Returning to the relaxation rate in a gas we must take for $|i\rangle$ and $|f\rangle$ N-body states. However, if the energy released is large compared to the thermal energy $k_B T$ we can approximate $E_f - E_i$ by $\epsilon_{\mathbf{p}'}+\epsilon_{\mathbf{p}''}-\Delta_{\rm rel}$ and $|f\rangle$ by $|f'\rangle\otimes|\mathbf{p}',\mathbf{p}''\rangle$, where the state $|f'\rangle$ contains only particles with small momenta compared to $\mathbf{p}'$ and $\mathbf{p}''$ because the relevant initial states have the same property. As a result we find first of all that

$$\Gamma_i \simeq \Gamma_0\frac{V^2}{4}\langle i|\psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x})\psi(\mathbf{x})|i\rangle , \tag{17}$$

and after a thermal average over the initial states, that

$$\begin{aligned}\Gamma(T) &\simeq \Gamma_0\frac{1}{2n^2}\langle\psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x})\psi(\mathbf{x})\rangle_{\rm th}\frac{N^2}{2} \\ &\equiv \Gamma_0 K^{(2)}(T)\frac{N^2}{2} .\end{aligned} \tag{18}$$

Above the Kosterlitz-Thouless transition temperature the correlator $K^{(2)}(T)$ is one and we have the obvious result that the decay rate in the gas is equal to the intrinsic two-body rate Γ_0 times the number of pairs. Below the transition temperature we have to deal with a quasicondensate, which implies that $\langle \psi(\mathbf{x}) \rangle_{\rm th}$ is nonzero and equal to the square root of the quasicondensate density n_0. We then use $\psi(\mathbf{x})=\sqrt{n_0}+\psi'(\mathbf{x})$ to calculate the correlator. This gives

$$K^{(2)}(T)=\frac{1}{2n^2}\left\{n_0^2\left[1+\frac{n_a'}{n_0}\right]^2+4n_0n'+2(n')^2\right\}, \qquad (19)$$

in terms of the normal and anomalous averages $n'=\langle \psi'^{\dagger}(\mathbf{x})\psi'(\mathbf{x})\rangle_{\rm th}=n-n_0$ and $n_a'=\langle \psi'(\mathbf{x})\psi'(\mathbf{x})\rangle_{\rm th}$, respectively.

As mentioned previously, we use the T-matrix approximation to evaluate these expectation values. Within this framework, the quasicondensate density is obtained from $n_0=-\mu'/T^{MB}(0,0,0;0)$ and the many-body T matrix:

$$T^{\rm MB}(0,0,0;0)=T^{2B}(0,0;2\mu')\left[1+T^{2B}(0,0;2\mu')\times\frac{1}{V}{\sum_{\mathbf{p}}}'\frac{N(\hbar\omega_{\mathbf{p}})}{\hbar\omega_{\mathbf{p}}}\right]^{-1}, \qquad (20)$$

which incorporates the influence of the surrounding gas on the collision of two hydrogen atoms. Here $N(\hbar\omega_{\mathbf{p}})$ is the Bose distribution for quasiparticles with the Bogoliubov dispersion relation $\hbar\omega_{\mathbf{p}}=(\epsilon_{\mathbf{p}}^2-2\mu'\epsilon_{\mathbf{p}})^{1/2}$, μ' is determined by the equation of state,

$$n=\frac{-\mu'}{T^{2B}(0,0;2\mu')}\left[1+2T^{2B}(0,0;2\mu')\frac{1}{V}{\sum_{\mathbf{p}}}'\frac{N(\hbar\omega_{\mathbf{p}})}{\hbar\omega_{\mathbf{p}}}\right]+\frac{1}{V}{\sum_{\mathbf{p}}}'\left[\frac{\epsilon_{\mathbf{p}}}{\hbar\omega_{\mathbf{p}}}N(\hbar\omega_{\mathbf{p}})+\frac{\epsilon_{\mathbf{p}}-\mu'-\hbar\omega_{\mathbf{p}}}{2\hbar\omega_{\mathbf{p}}}\right], \qquad (21)$$

and the prime denotes that the summation is only over such momenta that $\epsilon_{\mathbf{p}}>-c_0\mu'$ since the quasicondensate density n_0 accounts for the occupation of momentum states below this cutoff.[11] Moreover, the anomalous expectation value is found to be

$$\frac{n_a'}{n_0}=-\frac{1}{V}{\sum_{\mathbf{p}}}'\frac{2N(\hbar\omega_{\mathbf{p}})+1}{2\hbar\omega_{\mathbf{p}}}T^{MB}(0,0,0;0) \qquad (22)$$

and diverges in the ultraviolet, which makes Eq. (19) meaningless as it stands.

However, it is clear from the structure of the correlator what the physics of this divergence is. From the use of a contact interaction we expect the decay rate to be proportional to the probability for two atoms to be at the same position and in the T-matrix approximation this probability is

$$|\Psi_0^{(+)}(0)|^2=\left[1-\frac{1}{V}{\sum_{\mathbf{p}}}'\frac{2N(\hbar\omega_{\mathbf{p}})+1}{2\hbar\omega_{\mathbf{p}}}T^{\rm MB}(0,0,0;0)\right]^2=\left[1+\frac{n_a'}{n_0}\right]^2. \qquad (23)$$

Therefore, the divergence of the term in the correlator that is proportional to n_0^2, and thus corresponds to the collision of two atoms in the quasicondensate, is exactly cancelled by a wave-function renormalization. Consequently the two-body T matrix in Γ_0 [cf. Eq. (8)] is renormalized to the many-body T matrix and we obtain

$$G(T)=C_{\rm rel}\left|\frac{m}{\hbar^2}T^{\rm MB}(0,0,0;0)\right|^2K_R^{(2)}(T), \qquad (24)$$

where the renormalized correlator is

$$K_R^{(2)}(T)=\frac{1}{2n^2}\left\{n_0^2+4n_0n'+2(n')^2\right\}=\frac{1}{2n^2}\left\{2n^2-n_0^2\right\}, \qquad (25)$$

and finite.

In exact analogy with the above treatment of the two-body case, we find for the recombination-rate constant, the result

$$L(T)=C_{\rm rec}\left|\frac{m}{\hbar^2}T^{\rm MB}(0,0,0;0)\right|^6K_R^{(3)}(T), \qquad (26)$$

with

$$K_R^{(3)}(T)=\frac{1}{6n^3}\left\{6n^3-9n_0^2n+4n_0^3\right\} \qquad (27)$$

the correlator $\langle \psi^{\dagger}(\mathbf{x})\psi^{\dagger}(\mathbf{x})\psi^{\dagger}(\mathbf{x})\psi(\mathbf{x})\psi(\mathbf{x})\psi(\mathbf{x})\rangle_{\rm th}/6n^3$ after the cancellation of the divergences due to the anomalous expectation value n_a'.

Notice that, although Eqs. (24) and (26) are derived here in a somewhat formal manner, they are easy to interpret. Starting from the expressions for the decay rates in the normal phase, the presence of the quasicondensate brings about two modifications. First, we have to take into account that if two or three particles in the quasicondensate collide, the correct normalization of the initial wave function requires a decrease of the rate by a factor of 2! or 3!, respectively.[20] This is taken care of by the renormalized correlator. Second, the scattering of two atoms is influenced by the surrounding gas, which renormalizes the two-body T matrix to the many-body T matrix. As shown in Sec. III the latter brings about a dramatic reduction of the decay rate just below the critical temperature.

III. RELAXATION AND RECOMBINATION RATES

Before we discuss the experimentally more interesting situation of an adsorbed atomic hydrogen gas at constant chemical potential, we first consider a gas at fixed density to bring out more clearly the intrinsic magnitude of the degeneracy effects. In Fig. 1 we present the decrease of the recombination and relaxation rates as a function of temperature for a density of 1×10^{13} cm^{-2} and compare it to the decrease due to the renormalized correlator alone. At low temperatures the many-body T matrix is approximately equal to $T^{2B}(0,0;-\hbar\Sigma)$ and independent of temperature. Under these conditions the temperature dependence of the effect is primarily determined by the correlator but the magnitude is considerably larger, since the ratio $T^{2B}(\mathbf{0},\mathbf{0};-2\hbar\Sigma)/T^{2B}(\mathbf{0},\mathbf{0};-\hbar\Sigma)$ is about 1.16 at this density. The recombination rate is, therefore, at zero temperature, reduced by more than a factor of 14 instead of by a factor of 6. It is important to note that this additional reduction follows also from the Popov theory. Nevertheless, it was not found by Kagan, Svistunov, and Shylapnikov, who neglected the change in the two-body scattering probability due to the presence of the quasicondensate.

At higher temperatures and, in particular, near the critical temperature ($T_c \simeq 56$ mK) the influence of the increasing number of quasiparticles on the collision of two atoms is greatly enhanced, leading to a sharp decrease of the decay rate in contrast with the prediction from the correlator alone. Clearly, there is a discontinuous behavior at the critical temperature. Although there is no doubt about this discontinuity itself, which is a result of a similar behavior of the quasicondensate density, its magnitude might be overestimated in the ladder approximation. As we have argued before, higher-order fluctuations are of importance near the critical temperature but can, in the equation of state, be approximately accounted for by adjusting the infrared cutoff.[11] Unfortunately, the same does not have to be true for the decay rates. However, it is not difficult to see from the inequality $T^{MB}(\mathbf{0},\mathbf{0},\mathbf{0};0) \leq T^{2B}(\mathbf{0},\mathbf{0};2\mu')$ that the results presented in Fig. 1 are qualitatively correct and, in particular, that the magnitude of the discontinuity is much larger than expected on the basis of the correlator alone.

Bearing these considerations in mind we now turn to an adsorbed atomic hydrogen gas at a fixed chemical potential, which in practice is determined by the density of the three-dimensional buffer volume. An important quantity in these circumstances is the adsorption isotherm, which also shows a discontinuity at the critical point of the Kosterlitz-Thouless transition.[10] The reason for this is that in the superfluid phase the chemical potential obeys $\mu=(2n-n_0)T^{MB}(\mathbf{0},\mathbf{0},\mathbf{0};0)$, whereas for the normal phase we find $\mu=\hbar\Sigma=2nT^{2B}(\mathbf{0},\mathbf{0};-2\hbar\Sigma)$ in the degenerate regime. As a result the chemical potential is continuous at the transition if the density in the superfluid phase is about a factor of 3 larger than in the normal phase. More precise values for the density ratio are shown in Fig. 2.

Hence, increasing the density of the buffer volume beyond its critical value leads to a discontinuity in the rate constants that is different from the one depicted in Fig. 1. Qualitatively it diminishes the effect, because in the normal phase the density and consequently also the rate constant is reduced. In Fig. 3 we present our results for the reduction of the recombination- and relaxation-rate constants under these conditions. Although there is some temperature dependence we see that in the superfluid phase the recombination-rate constant is de-

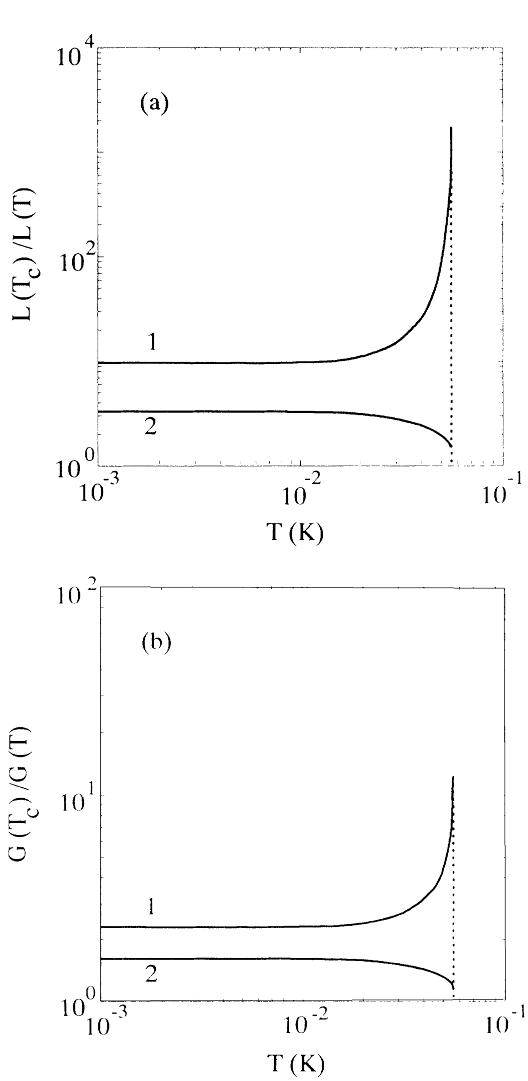

FIG. 1. Factor by which (a) the recombination- and (b) the relaxation-rate constant decreases in the superfluid phase of atomic hydrogen at a fixed density of 1×10^{13} cm^{-2}. In both cases the full result is given in curve 1, whereas curve 2 shows the decrease due to the renormalized correlator only.

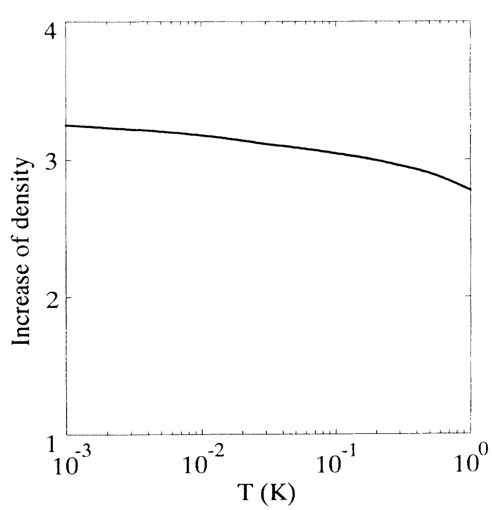

FIG. 2. Factor by which the two-dimensional density increases at the critical point of an adsorbed hydrogen gas in thermal contact with a three-dimensional buffer volume.

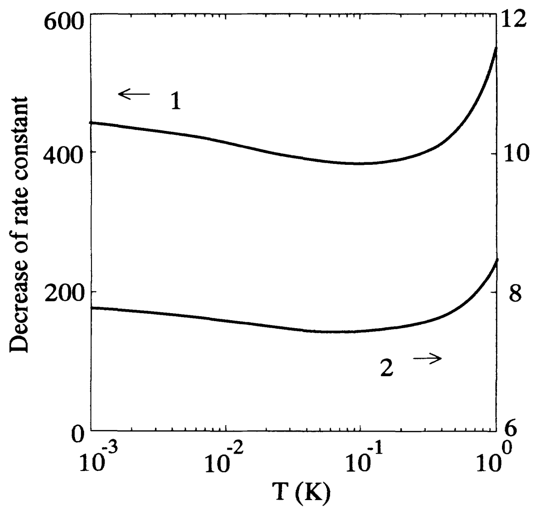

FIG. 3. Factor by which (1) the recombination- and (2) the relaxation-rate constant decreases at the critical point of an adsorbed hydrogen gas in thermal contact with a three-dimensional buffer volume.

creased by a factor of around 400, whereas the relaxation-rate constant is decreased by a factor of about 8. Notice also that if we are interested in the decay rates we must take the discontinuity in the two-dimensional density into account again and roughly divide the results for the recombination-rate constant by nine and the results for the relaxation-rate constant by three, respectively.

Finally, we discuss the relative importance of the recombination and relaxation processes. Using the experimental values $L \simeq 1.4 \times 10^{-24}$ cm^4/s for the three-body[21] and $G \simeq 2.5 \times 10^{-14}$ cm^2/s for the two-body[22] rate constant, we estimate with the help of Fig. 1, that at a density of 1×10^{13} cm^{-2} and near the critical temperature, a typical time scale for the decay due to recombination is $\tau_{\rm rec} \simeq 10$ s and due to relaxation $\tau_{\rm rel} \simeq 40$ s. The dominant decay channel, therefore, remains the three-body dipolar recombination even though the rate constant is reduced considerably. From an experimental point of view this is an interesting result, because it simplifies the analysis of the decay kinetics. In particular, it thus seems that no corrections are needed to incorporate the fact that the magnetic-field direction is not everywhere perpendicular to the helium surface. An average over the density profile may be necessary, however, to account for the inhomogeneity of the system. This situation is not considered here, because it depends too much on the details of the experimental setup.

IV. CONCLUSIONS

In summary, we have shown that the Kosterlitz-Thouless transition has a profound effect on the stability of adsorbed doubly polarized atomic hydrogen. Not only are the relaxation- and recombination-rate constants reduced substantially, but the phase transition also leads to a discontinuity in these rate constants at the critical point. This is true in the case of adsorbed atomic hydrogen at a fixed density but also in the experimentally more important case of an adsorbed gas in thermal equilibrium with a large buffer volume. Considering both two-body relaxation as well as the three-body recombination, we have found that the three-body process primarily determines the lifetime of the system even if we take the reduction of the decay rates into account.

With respect to the latter, it should be remembered that our results are valid in lowest order of $k_B T/\Delta$, where Δ is the energy released in the inelastic collision. For the recombination process this is an excellent approximation, because the energy released is several tens of a kelvin. However, for the relaxation this is not the case and the first-order corrections are of importance in principle. They can be evaluated by a more advanced theory, which is similar in spirit to the one used to discuss the influence of Bose-Einstein condensation on the decay of a three-dimensional gas, and are expected to further reduce the rate constant.[20] Therefore, our conclusion that recombination is the most important decay mechanism remains valid for the conditions envisaged in future experiments and it does not seem necessary at present to develop also the theory for the calculation of the first-order corrections.

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