Josephson effect in low-capacitance superconductor–normal-metal–superconductor systems

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(Received 27 July 1993)

The transport properties of a small superconductor–normal-metal–superconducting tunnel junction can be controlled by a gate electrode coupled capacitively to the central island. We evaluate the critical Josephson current I_c through such a system as a function of the gate voltage V_g taking into account parity effects in the superconductors. The dependence $I_c(V_g)$ shows resonant singularities and has a qualitatively different character for $\Delta < E_C$ and $\Delta > E_C$, Δ being the superconducting gap and $E_C = e^2/2C$ being the charging energy. Due to the sweeping of the gate voltage, the system can be driven to states which are metastable with respect to a change of the number of electrons on the central island. Since the lifetime of the metastable states can be macroscopically large, one can observe a remarkable bistability of the Josephson current. A weak magnetic field supresses the interference of the electrons propagating through the central electrode, and, consequently, the magnitude of the critical current.

I. INTRODUCTION

Parity effects predicted recently for small tunnel junction systems[1] are attracting the attention of a growing number of theoreticians[2,3] and experimentalists.[4–6] This interest is focused on the basic nature of the superconducting ground state. The experiments[4–6] have unambiguously demonstrated that the ground-state energy of a superconducting island depends on the parity of the number of electrons in it, and this even-odd difference in energy is just the superconducting gap Δ.

Another point of interest concerns the electron transport in the presence of both parity and charging effects. In this context, small NSN (Refs. 1 and 3) and SSS (Ref. 2) systems have been considered. The transport reflects the properties of the ground state that depend on the ratio between Δ and the Coulomb energy $E_C \equiv e^2/2C$ (here C is the capacitance of the central electrode). For $\Delta > E_C$ the number of electrons on the central electrode in the ground state is always *even* and all the characteristics are $2e$-periodic functions of the offset charge $Q_0 = C_g V_g$ (expressed in terms of the gate voltage V_g and the capacitance C_g between the central and the gate electrode). In contrast, for $\Delta < E_C$ the electron number can be *even or odd* depending on the value of Q_0. As a result, even-odd transitions show up in the transport characteristics of the system. For $\Delta/E_C \ll 1$ the characteristics tend to be quasi-e-periodic.

In the NSN system tunneling of one electron through both junctions[1] and tunneling of two electrons through each junction[3,5] were investigated. The latter process is analogous to the well-known Andreev reflection. If $\Delta > E_C$, this manifests itself in $2e$-periodic peaks of the dc conductance measured as a function of Q_0.[3] These peaks have been observed in a recent experiment.[5] A resonant behavior of the superconducting critical current I_c as a function of Q_0 has been predicted in Ref. 2 for a small SSS system.

In this paper we investigate the influence of the even-odd effects on the nondissipative electron transport through a small SNS system, Fig. 1(a). This system has two important features distinguishing it from those considered previously.

First, there should be no observable even-odd effect in equilibrium, since there is no small superconducting

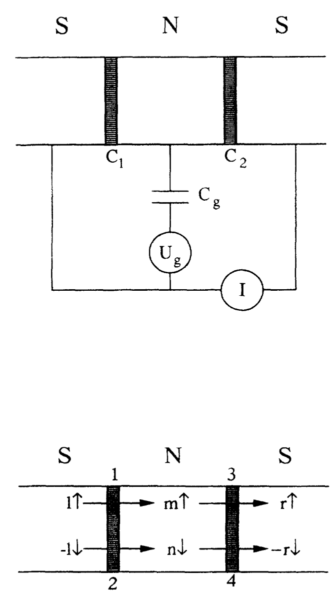

FIG. 1. (a) The SNS system under consideration. (b) A Cooper pair transfer, formed by four virtual single-electron tunneling processes. The terms in Eq. (10) can be obtained as permutations of the steps 1, ..., 4.

island. On the other hand, the parity of number of electrons on the normal island equilibrates very slowly at low temperatures (see Sec. V for details). By means of the gate voltage, the system can be driven to the state which is metastable with respect to the parity change. The Josephson current in this case will differ from its equilibrium value, showing a remarkable bistability. One can observe two different types of critical-current–gate-voltage characteristics. If the sweeping rate is larger than the equilibration rate, a $2e$-periodic dependence will be observed. In the opposite case, one-electron processes will restore the e-periodic dependence.

Second, the Josephson current can be thought of as a transfer of pairs of mutually coherent electrons through the central normal electrode. This makes important the interference between two electrons propagating in the normal metal. A relatively weak magnetic field destroys the interference and suppresses the Josephson current.

The paper is organized as follows. In Sec. II we develop a general formalism for the supercurrent in the presence of charging effects. In Sec. III the interference of two propagating electrons is considered in the absence of magnetic field and in a weak magnetic field. In Sec. IV, we evaluate the supercurrent as a function of the gate voltage neglecting the parity equilibration processes. We discuss the effect of these processes in Sec. V. The conclusions are drawn in Sec. VI.

II. METHOD

We study the Josephson effect in a SNS system and calculate the critical current at zero transport voltage and temperature. We describe the system by the Hamiltonian

$$\hat{H} = \hat{H}_{\rm el} + \hat{H}_T + \hat{H}_Q \ . \tag{1}$$

The term $\hat{H}_{\rm el}$ describes the superconducting electrodes and the middle island,

$$\hat{H}_{\rm el} = \sum_{l,\sigma} \epsilon_l \alpha^\dagger_{l\sigma}\alpha_{l\sigma} + \sum_{r,\sigma} \epsilon_r \gamma^\dagger_{r\sigma}\gamma_{r\sigma} + \sum_{m,\sigma} \xi_m b^\dagger_{m\sigma} b_{m\sigma} . \tag{2}$$

Here the operators α_l and γ_r result from the Bogoliubov transformation (see, e.g., Ref. 7) of the electron operators a_l and c_r for the left and right electrodes, e.g.,

$$\alpha_{l\uparrow} = u_l a_{l\uparrow} - v_l a^\dagger_{-l\downarrow} \ , \tag{3}$$

$$\alpha_{l\downarrow} = u_l a_{l\downarrow} + v_l a^\dagger_{-l\uparrow} \ , \tag{4}$$

and $\epsilon_i = \sqrt{\Delta^2 + \xi_i^2}$ $(i = l, r)$. The operators b_m correspond to the electron states on the middle electrode.

The tunnel Hamiltonian $\hat{H}_T$ is given by

$$\hat{H}_T = \sum_{l,m,\sigma} (T^{(1)}_{lm} b^\dagger_{m\sigma} a_{l\sigma} + \text{H.c.}) + \sum_{n,r,\sigma} (T^{(2)}_{nr} c^\dagger_{r\sigma} b_{n\sigma} + \text{H.c.}), \tag{5}$$

and $\hat{H}_Q$ stands for the charging energy of the system,

$$\hat{H}_Q = \frac{(\hat{Q} - Q_0)^2}{2C} . \tag{6}$$

Here $\hat{Q}$ is the operator of the charge on the middle island, $Q_0 = C_g V_g$ is the charge induced by the gate electrode, and C is a total capacitance of the island.

In the absence of electron tunneling, the states $|\psi_n\rangle$ with different number $n/2$ of Cooper pairs on the superconducting electrodes are degenerate (here n is an even integer). The electron tunneling lifts this degeneracy, and an energy band of width $2E_J$ appears. The critical current is related to this Josephson coupling energy by $I_c = 2eE_J/\hbar$. In lowest nonvanishing order in $\hat{H}_T$ we find (see, e.g., Ref. 8),

$$E_J = 2|\langle \psi_{n+2}|\hat{H}^{(4)}_T|\psi_n\rangle| , \tag{7}$$

with

$$\hat{H}^{(4)}_T = \hat{H}_T \frac{1}{E_0 - \hat{H}_0} \hat{H}_T \frac{1}{E_0 - \hat{H}_0} \hat{H}_T \frac{1}{E_0 - \hat{H}_0} \hat{H}_T . \tag{8}$$

Here E_0 is the energy of degenerate states $|\psi_n\rangle$, and $\hat{H}_0 = \hat{H}_{\rm el} + \hat{H}_Q$ is the unperturbed Hamiltonian.

The matrix element (8) describes the transfer of one Cooper pair from the left superconducting electrode to the right one. Figure 1(b) shows four virtual steps of this process. Independently of the sequence of the steps, the combination $T^{(1)}_{lm} T^{(1)}_{-ln} T^{(2)}_{mr} T^{(2)}_{n-r}$ of tunnel matrix elements and the combination $u_l v_l u_r v_r$ of superconducting coherence factors arise. The energy denominators correspond to the energies of the virtual intermediate states and hence do depend on the sequence of virtual steps.

The possible sequences can be divided into three groups depending on whether only the electron excitations, only the hole excitations, or both occur in the virtual intermediate states. This gives rise to the factors $[1-f(\xi_m)][1-f(\xi_n)]$, $f(\xi_m)f(\xi_n)$, or $[1-f(\xi_{m,n})]f(\xi_{n,m})$, respectively. We therefore write (7) in the form

$$E_J = 2 \sum_{l,m,n,r} T^{(1)}_{lm} T^{(1)}_{-ln} T^{(2)}_{mr} T^{(2)}_{n-r} u_l v_l u_r v_r \left\{ \frac{1}{\epsilon_l + \xi_m + \mathcal{E}(e)} \frac{1}{\xi_m + \xi_n + \mathcal{E}(2e)} \frac{1}{\epsilon_r + \xi_n + \mathcal{E}(e)} [1 - f(\xi_m)][1 - f(\xi_n)] + 23 \text{ other terms} \right\} . \tag{9}$$

The combination of energy denominators written explicitly above corresponds to the ordering 1234 of the virtual steps in the notations of Fig. 1(b). The quantity $\mathcal{E}(Q) = \langle Q|H_Q|Q\rangle - \langle 0|H_Q|0\rangle$ is the difference in charging energy (6) between the virtual and the initial states.

In order to replace the summation over the states by integrations over the energy we have to calculate the mean value of the combination of tunnel matrix elements on the corresponding energy surfaces. This can be done by introducing the function $F(\xi_1, \xi_2, \xi, \xi')$ given by

$$F(\xi_1,\xi_2,\xi,\xi') = \sum_{l,m,n,r} T^{(1)}_{lm} T^{(1)}_{-ln} T^{(2)}_{mr} T^{(2)}_{n-r} \delta(\xi_l - \xi_1)\delta(\xi_r - \xi_2)\delta(\xi_m - \xi)\delta(\xi_n - \xi'). \tag{10}$$

With this definition Eq. (9) is transformed into

$$E_J = 2\int_{-\infty}^{\infty} d\xi_1 d\xi_2 d\xi d\xi' F(\xi_1,\xi_2,\xi,\xi') \frac{\Delta}{2\epsilon_1}\frac{\Delta}{2\epsilon_2}\left\{ \frac{1}{\epsilon_1+\xi+\mathcal{E}(e)}\frac{1}{\xi+\xi'+\mathcal{E}(2e)}\frac{1}{\epsilon_2+\xi'+\mathcal{E}(e)} \times[1-f(\xi)][1-f(\xi')] + \cdots \right\}. \tag{11}$$

III. INTERFERENCE OF TUNNELING ELECTRONS

In this section we estimate the averages of matrix elements (10) using the standard quasiclassical approach[9] in the formulation described in Ref. 10. We rewrite Eq. (10) in the coordinate representation and relate contributions from the tunneling and propagation of electrons in the superconductors to the conductances of the tunnel junctions.[10] As a result we obtain that for $|\xi| \ll E_F$ the function F depends primarily on the energy difference $\hbar\omega = \xi - \xi'$ of two electron excitations in the central (normal) electrode;

$$F(\xi,\xi') = \frac{\hbar}{32\pi^3 e^4 \nu}\int d^2 r_{1,2}\, g_1(r_1) g_2(r_2) \times [P_\omega(r_1,r_2) + P_{-\omega}(r_1,r_2)]. \tag{12}$$

Here ν is the density of electron states in the normal metal and $g_i(r_i)$ are normalized conductances of the tunnel barriers per unit area, $\int d^2 r_i g_i(r_i) = G_i$ (G_i are the conductances of the junctions). The function $P_\omega(r_1,r_2)$ is the Fourier transform of the quasiprobability $P(r_1,r_2,t)$ which describes the propagation of the Cooperon from one junction to the other through the central electrode and satisfies the equation,[11,12]

$$\left[\frac{\partial}{\partial t} + D\left(-i\frac{\partial}{\partial r_2} - \frac{2eA(r_2)}{\hbar c}\right)^2\right] P = \delta(r_1 - r_2)\delta(t). \tag{13}$$

If there is no magnetic field, the solution of the diffusion equation at large times $t \gg \tau_d \equiv L^2/D$ is a constant $P = 1/V$ (here L and V are the size and the volume of the central electrode). Therefore, the function F reduces to a δ function at zero energy and a regular part. If the energy $\hbar/\tau_d$ exceeds the characteristic energy scale $\max(\Delta, E_C)$ in the expression (11) for the Josephson energy, the contribution to F from finite energies $\xi - \xi' \neq 0$ can be neglected. As a result, we obtain for a small central electrode, $L \ll [\hbar D/\max(\Delta,E_C)]^{1/2}$,

$$F(\xi,\xi') = \pi^{-2} J_0 \delta(\xi - \xi'),\ J_0 \equiv \frac{\hbar^2}{8e^4} G_1 G_2 \delta. \tag{14}$$

Here $\delta = 1/\nu V$ is the mean level spacing in the central electrode. This does not depend on the geometrical shape of the electrode. For larger system, the regular part of the function F becomes important and the result depends on the geometry. For still larger L, $L \gg \sqrt{\hbar D/\Delta}$, the regular contribution compensates the δ function almost exactly, and the supercurrent becomes exponentially small.[13] This is why we concentrate on the limiting case of a small island.

Let us consider now the effect of a magnetic field. To be closer to the experimental setup, we assume that the central electrode is made of a thin metal film and the magnetic field is applied perpendicularly to the film plane.

The quasiprobability P decays exponentially in time and one can seek a solution of Eq. (13) in the form of an expansion in eigenfunctions $P_m(r)$ of the operator on the left hand side. The necessary boundary condition is that the normal derivative of P be zero at the edges of the electrode. In a weak magnetic field $\Phi = BS \ll \Phi_0$ (S is the area of the central electrode) the lowest eigenvalue λ_0 can be estimated perturbatively in a magnetic field [unperturbed function $P_0(r)$ is obviously a constant]. For example, for a circular electrode with the radius R we obtain

$$\lambda_0 = \frac{\hbar D}{2R^2}\frac{\Phi^2}{\Phi_0^2}. \tag{15}$$

This lowest eigenvalue is by a factor $\sim (\Phi/\Phi_0)^2$ smaller than the next one. For a small electrode, $R \ll [\hbar D/\max(\Delta,E_C)]^{1/2}$, effectively only this eigenvalue contributes to the function F in the energy range of interest, $|\xi - \xi'| < \max(\Delta, E_C)$. As a result, the function $F(\xi,\xi')$ acquires the Lorentz shape

$$F(\xi,\xi') = \pi^{-3} J_0 \frac{\lambda_0}{\lambda_0^2 + (\xi - \xi')^2}. \tag{16}$$

Note that the function F is substantially smeared at $\lambda_0 \sim \max(\Delta, E_C) \ll D/R^2$, which corresponds to relatively small values of the magnetic flux $\Phi \ll \Phi_0$ [see Eq. (15)]. However, the corresponding magnetic field $B \simeq (\Phi_0/R)\sqrt{\Delta/D}$ may be as large as the critical field H_{c2} of the bulk superconductor.

To stress the importance of the interference, let us consider a ring-shaped central electrode. The spectrum of the eigenvalues λ_m is now a periodic function of the flux Φ through the ring,

$$\lambda_m = \frac{D}{R^2}\left(m - \frac{\Phi}{\Phi_0}\right)^2 . \tag{17}$$

This means that all the characteristics of the system will show a Φ_0 periodicity. In particular, when the flux Φ is close to an integer number of flux quanta, $\phi = \min_m |m - \Phi/\Phi_0| \ll 1$, the lowest eigenvalue $\lambda_{\min} = \phi^2 D/R^2$ is much less than all the others. Therefore, an expression for the function F has again the form (16) with λ_0 replaced by $\lambda_{\min}$.

So far we assumed that the phase breaking time τ_φ is infinitely long. It can be taken into account by adding the term $\tau_\varphi^{-1}P$ to the left hand side of Eq. (13). For a weak phase breaking $\tau_\varphi \gg L^2/D$ an additional broadening of the peak (16) arises, $\lambda_0 \to \lambda_0 + \tau_\varphi^{-1}$, even for zero magnetic field. In the opposite case, $\tau_\varphi \ll L^2/D$, the function F is suppressed by the exponential factor $\sim \exp[L/(D\tau_\varphi)^{1/2}]$.

IV. RESULTS

Inserting $F(\xi,\xi')$ into Eq. (11), we calculate the Josephson coupling energy E_J as a function of the offset charge Q_0. In the absence of magnetic field the δ function form (14) of $F(\xi,\xi')$ simplifies the calculations considerably. One can say that the two electrons involved in the tunneling have to pass through the same energy level on the island. In particular, this means that the contribution of processes which involve an electron and a hole is negligible.

We discuss now the features of the dependence $E_J(Q_0)$. The energy of the system with different numbers n of excess electrons on the island is plotted in Figs. 2(a,b) for $E_C < \Delta$ and $E_C > \Delta$, respectively. Since the energy spectrum and all characteristics of the system are $2e$-periodic functions of Q_0, we restrict our consideration to the interval $-e < Q_0 < e$. We assume that the initial state $n = 0$ is the ground state with even numbers of electrons in the superconductors. The parabolas $n = \pm 1, \pm 2$ correspond to the minimal energy of the intermediate states. At the points of intersection of these parabolas with the parabola for $n = 0$, the energy denominators (11) vanish (for zero electron energies) and E_J becomes large. This happens at $Q_0 = \pm e$ for $E_C < \Delta$ [Fig. 2(a)] and at $Q_0 = \pm Q_C$, $Q_C = e(1/2 + \Delta/2E_C) < e$, for $E_C > \Delta$ [Fig. 2(b)].

First, we consider the case $E_C < \Delta$ when the parity of the electron number n in the ground state does not change at any value of Q_0. We find that the Josephson energy diverges logarithmically for $Q_0 \to e$,

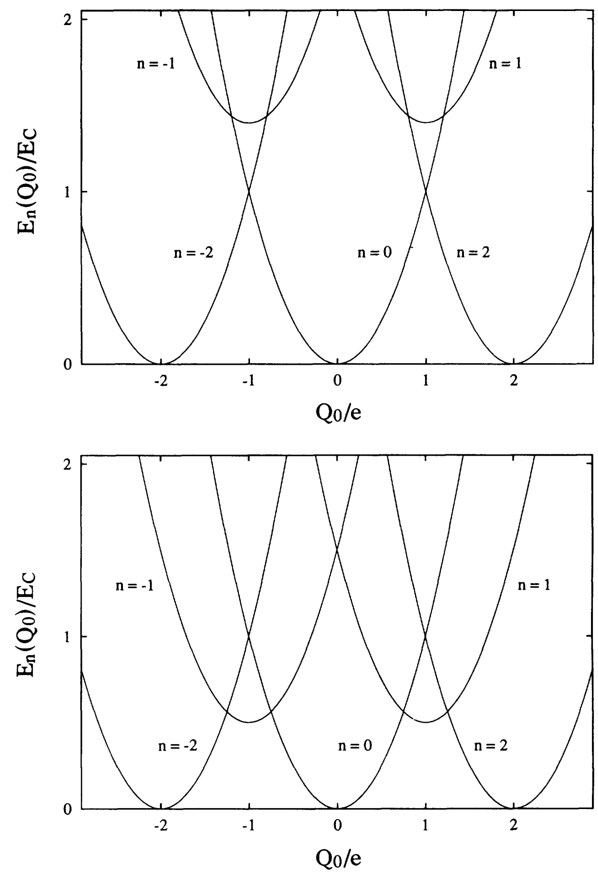

FIG. 2. The ground-state energy $E_n(Q_0) = \langle ne|H_Q|ne\rangle + \mathrm{mod}(n,2)\Delta$ as a function of the offset charge Q_0 for different numbers n of excess electrons on the normal island: (a) $E_C < \Delta$, (b) $\Delta < E_C$.

$$E_J = J_0 \ln \frac{\mathcal{E}(2e)}{2\Delta}. \tag{18}$$

This corresponds to the lifting of the Coulomb blockade for the processes in which a pair of electrons tunnels to the island [e.g., the process 1234 in Fig. 1(b)]. The dependence $E_J(Q_0)$ calculated numerically in the whole range of Q_0 is shown in Fig. 3(a).

It is instructive to compare the result (18) with the classical result[13] for SNS systems in the absence of charging effects ($E_C \to 0$). In the relevant limit $L^2 \ll D\Delta$ the classical result reads

$$E_J = 2J_0 \ln(\Delta/T) \tag{19}$$

in our notation.[14] The logarithmic divergence is regularized now by the temperature $T \simeq \mathcal{E}(2e)$. The difference in the numerical factor accounts for the fact that in our case only one of the two charging energies $\mathcal{E}(\pm 2e)$ becomes small.

When a magnetic field is applied, the δ function in $F(\xi,\xi')$ changes into a Lorentz function which allows the involved electrons to have different energies. As a result, the Josephson energy $E_J(Q_0)$ is no longer divergent at the resonance points [see dashed lines in Fig. 3(a)]. Its

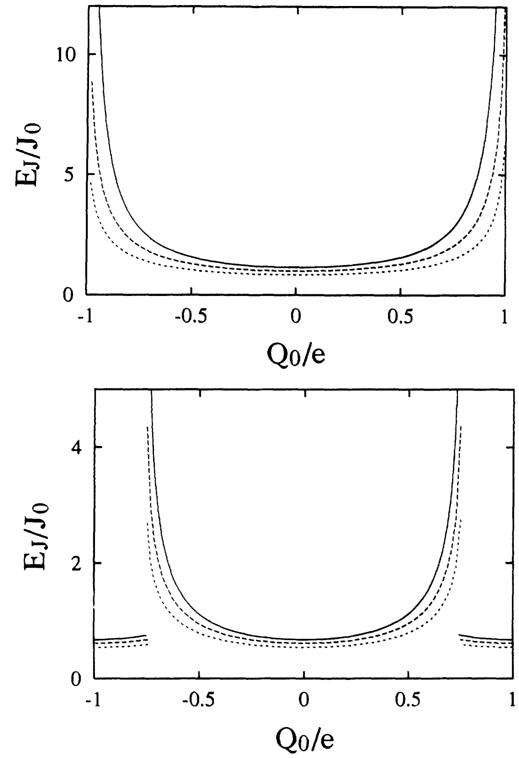

FIG. 3. Josephson coupling energy E_J as a function of the offset charge Q_0 for different magnetic fields $\lambda_0/\Delta = 0, 0.3, 1$ (from top to bottom). (a) $E_C < \Delta$ ($E_C/\Delta = 0.95$), (b) $\Delta < E_C$ ($E_C/\Delta = 2$).

value there can be found by replacing $\mathcal{E}(2e)$ in Eq. (18) by the "magnetic energy" λ_0.

We now turn to the case $E_C > \Delta$ [Fig. 3(b)], where the ground state changes at $|Q_0| = Q_C$. First we consider a domain $|Q_0| < Q_C$, in which there are no quasiparticle excitations in the ground state. One finds a logarithmic divergence of the energy E_J at $Q_0 \to Q_C - 0$,

$$E_J = 2J_0 \frac{E_C}{E_C - \Delta} \ln \frac{\Delta + \mathcal{E}(e)}{\Delta}. \tag{20}$$

The last equation is valid for $(Q_C - Q_0)/Q_C \ll 1$. A diverging contribution arises not only form the considered processes in which electrons tunnel in pairs [e.g., 1234 in Fig. 1(b)], but also from the processes in which two electrons tunnel from the left to the right electrode one by one (e.g., 1324). The latter processes dominate for $\Delta \ll E_C$.

For $|Q_0| > |Q_C|$ the ground state changes: One electron tunnels to the island, leaving a quasiparticle excitation in one of the superconducting electrodes. The conditions for the stability of such a state will be discussed in the next section. The contributions to E_J for $Q_C < Q_0 < e$ are the same as for $0 < Q_0 < e - Q_C$, which means that they remain finite [Fig. 3(b)].

As a result, the peak of $E_J(Q_0)$ at $Q_0 = Q_C$ is asymmetric. The magnetic field decreases the critical current and eliminates the logarithmic singularity [Fig. 3(b)]. The value of E_J at the resonance point Q_C can be obtained by changing $\Delta + \mathcal{E}(e)$ with λ_0 in Eq. (20).

For a system with the central electrode in shape of a ring a small magnetic field again suppresses the Josephson current. However, when the flux Φ through the ring is close to an integer number of flux quanta, the constructive interference of electrons [i.e., a sharp peak (16) of the function F] is restored. As a result, the Josephson current as a function of the flux shows sharp peaks at $\Phi = n\Phi_0$.

V. BISTABILITY

In the preceding section, the system was treated as if there were no quasiparticles in the superconducting leads and no room to create one without spending energy Δ. Theoretically, this is valid at dilution refrigerator temperatures, where the thermally activated processes require astronomical times.

Another possible mechanism for one-electron transfer[1] arises in the presence of normal inclusions in the superconducting material. A reasonable rate would arise only if the distance between the tunnel junction and the inclusion does not exceed much the superconducting coherence length. According to recent experiments,[5] this possibility can be excluded.

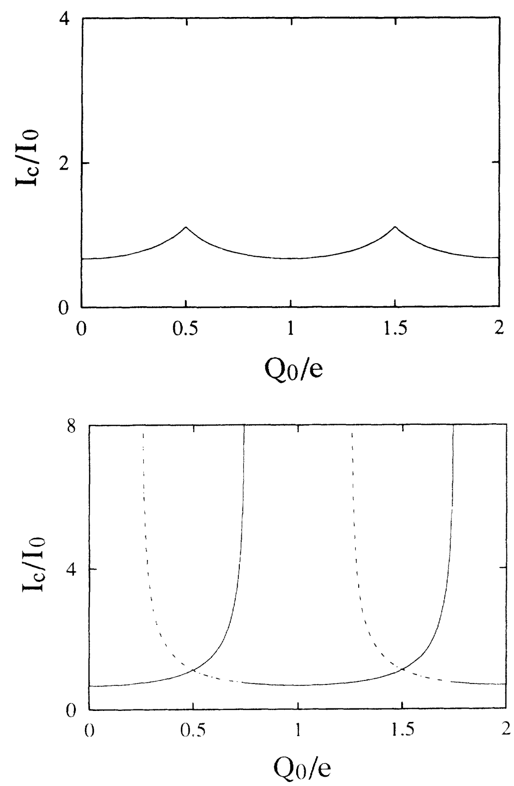

FIG. 4. The critical current I_c as a function of the offset charge Q_0, expected in experiments with different sweeping rates: (a) $\Gamma \ll \tau_r^{-1}$, (b) $\Gamma \gg \tau_r^{-1}$. $E_C/\Delta = 2$; $I_0 = 2eJ_0/\hbar$.

The genuine mechanisms for one-electron processes have not been revealed experimentally yet. It is not clear, for example, how many quasiparticles remain in a bulk superconductor at low temperatures. This relates to the question why the results of Refs. 15 and 6 look inconsistent. This is why we prefer not to review this subject but rather introduce a typical time scale τ_r for the parity relaxation process. According to Ref. 5, this time may be of order of hours.

Hence, the result of an experimental measurement of $I_c(Q_0)$ may depend on the sweeping rate $\Gamma = \dot{Q}_0/e$. For $\Delta > E_C$ a large sweeping rate $\Gamma \gg \tau_r^{-1}$ excludes the influence of the quasiparticles and thus leads to the ideal $2e$-periodic picture shown in Fig. 3(a). For slow sweeping, $\Gamma \ll \tau_r^{-1}$, the changes of the ground state are determined exclusively by the charging energy which means that the resulting picture is e periodic [see Fig. 4(a)].

An interesting behavior can be expected for $\Gamma \gg \tau_r^{-1}$ and $\Delta < E_C$. Assume that one increases Q_0 starting from $Q_0 = 0$. The ground-state energy follows the parabola $n = 0$ in Fig. 2(b) and the Josephson current I_c is the same as discussed in Sec. IV as long as $Q_0 < Q_C$ [compare Figs. 3(b) and 4(b)]. When the offset charge Q_0 exceeds Q_C, the ground state changes by creating one excitation which, however, relaxes immediately on the time scale of sweeping. Here, in contrast to the ideal case shown in Fig. 2(b), the energy of the ground state $n = 1$ is no longer shifted up by Δ. As a result, the critical current $I_c(Q_0)$ follows the solid line in Fig. 4(b) and shows e-periodic behavior. Sweeping now Q_0 in the opposite direction, one follows the dashed line and finds no divergency at Q_C. The divergency will be seen at $Q'_C = e - Q_C$. Hence, the critical current I_c shows a hysteresis.

VI. CONCLUSIONS

In conclusion, we have calculated the Josephson current through a small SNS system at $T = 0$ taking into account the charging energy. The current as a function of the gate voltage shows a resonant structure. In a typical experiment the shape of $I_c(Q_0)$ depends strongly on the sweeping rate of Q_0 compared with the rates of one-electron processes. This can be used to explore these processes.

The value of the Josephson energy $E_J = \hbar I_c/2e$ is of the order of $(\hbar/4e^2)^2 G_1 G_2 \delta$ (δ being the average level spacing in the normal island), which agrees with the known result for large SNS systems in the limiting case $E_C \to 0$. This can be compared with $E_J = (\pi\hbar/4e^2)G\Delta$ for a conventional Josephson junction. Hence, E_J in the SNS system is reduced not only by an additional factor $(\hbar/4e^2)G$ but also by the factor δ/Δ, which is of the order of 10^{-4} for a typical experimental setup. The latter reduction is due to the interference of the electrons in virtual states on the middle island. This interference and, consequently, the critical current can be influenced by the magnetic field.

ACKNOWLEDGMENTS

We wish to thank C. Bruder, F. W. J. Hekking, Yu. N. Ovchinnikov, A. D. Zaikin, and, especially, Gerd Schön for valuable discussions. The financial support of the Deutsche Forschungsgemeinschaft through Grant No. SFB 195 is gratefully acknowledged.

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