Quantum tunneling across a domain-wall junction

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In this article, we discuss a system that is appropriately called a domain-wall junction (DWJ). The DWJ is a close analog of the Josephson junction with respect to the usefulness of both systems in studying switching by thermal activation (TA) and by quantum tunneling (QT). We will show through theoretical analysis that it is within our means to fabricate a DWJ that should exhibit switching via quantum tunneling of magnetization. We further lay out experimental procedures for looking for quantum tunneling of magnetization in a DWJ; these procedures are based upon those used in observing macroscopic quantum tunneling in Josephson junctions. Two examples of DWJ's have been recently made that apparently show a crossover from TA to QT.

I. INTRODUCTION

Interest in looking for evidence for quantum tunneling of magnetization (QTM) has increased recently for a number of reasons: (i) the theoretical basis for this phenomenon has gained understanding and acceptance,[1,2] (ii) the possibility of observing manifestations of QTM experimentally has been realized,[3-6] (iii) the march towards ever decreasing sizes of computer elements is proceeding to such an extent that it is becoming more conceivable that, in a generation or so, the regime wherein quantum tunneling will become the limiting factor in the stability of computer elements will be realized.[7]

Numerous reports of the possible observation of QTM have appeared.[3-6] To our knowledge, all the systems that have thus far been studied have the shortcoming that they consist of an ensemble of many subsystems, which leads to a distribution of energy barriers. The parameters that determine the transition rate over the relevant energy barrier, whether it be by thermal activation or by quantum tunneling, is exponentially dependent upon, and therefore extremely sensitive to, certain characteristic parameters—such as particle volume and anisotropy—of each subsystem. The systems therefore have a distribution of transition rates. This situation makes it difficult to gain strong confidence that manifestations of QTM are indeed not associated with purely classical yet anomalous processes. The next generation of experiments should consist of studying QTM effects in a single particle.

A number of researchers have called attention to the attractive characteristics of quantum tunneling of domain walls.[8-11] In this work we will focus our attention on what we will refer to as a domain-wall junction (DWJ). What makes this system especially attractive is that we do not have to deal with an ensemble of subsystems. It is a single system, with a single set of parameters that characterize its behavior. The DWJ is the prototype analog of the Josephson junction (JJ), for which quantum tunneling in its switching, from the zero voltage state to the voltage state upon application of a basis current close to the critical current, has long been firmly established.[12] As in the case of the JJ, switching of the DWJ can be studied in the classical, high-temperature regime, wherein switching takes place via thermal activation (TA) and the theory is firmly established. Also, as in the case of the JJ, the parameters characterizing the DWJ can be determined from the DWJ's behavior in the classical regime and then used to analyze the low-temperature regime wherein QTM is expected to be dominant.

A DWJ is essentially a small region along a narrow channel of magnetic material that is so fabricated as to provide a barrier for the motion of a planar domain wall (DWJ). A DWJ is depicted schematically in Fig. 1, below as a local barrier. One could just as well use a potential

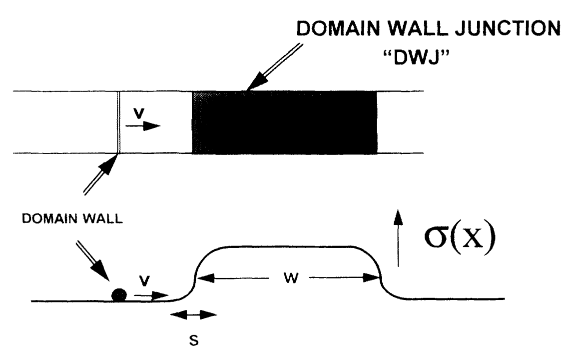

FIG. 1. A depiction of a domain wall moving along a narrow channel of a sample of magnetic material encountering a domain junction (DWJ). The potential energy $\sigma(x)$ in the absence of an applied field is depicted below. Outside the DWJ, $\sigma(x)=\sigma$, while inside the DWJ, $\sigma(x)=\sigma(1+\eta)$. Thus, η is the relative jump in the potential energy. The width s of the transition region determines the coercive field H_c, being given by $\eta\sigma/Ms$.

well. See Ref. 13. We will refer to a DWJ with a barrier as a "*b*-DWJ" and a DWJ with a well as a "*w*-DWJ." The barrier (or well) consists of a sandwich layer with a larger (or smaller) domain-wall energy σ. The anisotropy direction lies in the plane of the film.

It has long been recognized that the equation of motion of a DWJ is the same as that of a Josephson junction, with the phase difference of the JJ replacing the position coördinate x of the DWJ. Close to the critical "force" (the critical current and the coercive field, respectively), both systems are governed by a potential energy that is *cubic* in the coördinate.[14] The purpose of this work is manyfold: First, we demonstrate the formal analogy between the JJ and the DWJ. Then, we seek to present results previously obtained in a form that can be related more closely to junction fabrication. Next we extend the theory so as to cover the regime such that the parameters that characterize the DWJ vary over a length scale greater than the domain-wall thickness. Finally, we lay our experimental procedures for carrying out a search for QTM in a DWJ.

II. THEORY

We describe here a *b*-DWJ. (The analysis of a *w*-DWJ is very similar.) Our DWJ is characterized entirely in terms of six parameters: two geometrical parameters (the width w of the barrier layer and the width s of the transition region) and the values of the exchange constant J and the anisotropy energy K in the barrier and outside of it. The latter two parameters are extremely uniform along the length of the sample (so as to avoid random pinning sites), except in the region of the DWJ.

The full width w must be greater than s, ranging from s to a value much greater than s. On the other hand, in principle, the DW thickness $\delta=(J/K)^{1/2}$ is not so restricted. The simplest case to deal with theoretically is when $\delta \ll s$; we have slow spatial energy variation. We will refer to this regime as "case *I*." Then, it can be shown that the dynamics of the DW is governed by a potential-energy function $U(x)$. It is the width s of the transition regions that is crucial to the switching rate, rather than the width w of the barrier as a whole.

If $\delta \gg s$, a thorough theoretical analysis is more complicated. The DW cannot be treated as an entity having a fixed structure. As the DW tunnels through the barrier, one must find the fully evolving spatial variation of the DW, a task that does not lend itself to an analytical solution; in principle, one would have to rely on numerical techniques as were used for the problem of quantum nucleation.[15] In Sec. II, we show that the expected results can be obtained for $\delta \gg s$ to within purely numerical factors which are expected to be on the order of unity.

A. The regime $\delta \ll s$

In this case it can be shown (see the Appendix for the case when transverse anisotropy is omitted), that as long as the DW velocity is much less than the limiting velocity [see Eq. (3)], the domain wall moves according to the equation of motion:

$$\frac{d[m(x)v]}{dt}=-\frac{d\sigma}{dx}+2M_sH \ . \tag{1}$$

In this equation, M_s is the magnetization, v is the velocity of the DW, and x is its position. The mass $m(x)$ per area and the wall energy $\sigma(x)$ per area are both dependent upon position due to the spatial variation of the exchange energy $J(x)$ and the parallel anisotropy energy $K(x)$. The domain-wall energy inside the barrier can be written as $\sigma(x)=(1+\eta)\sigma$, where σ is the wall energy outside the barrier. The parameter η can be easily modified through the chemical composition of the barrier. A *b*-DWJ has $\eta>0$, while a *w*-DWJ has $\eta<0$. In the following, we shall always assume that $\eta>0$.

The local energy, width, and mass of the DW are given, respectively, by

$$\sigma(x)=4\sqrt{J(x)K(x)}\ ,$$
$$\delta(x)=\sqrt{J(x)/K(x)}\ , \tag{2}$$
$$m(x)=\frac{\sigma(x)}{v_0(x)^2}\ ,$$

where $v_0(x)$ is the local limiting velocity of a DW, given by[16]

$$v_0=\left[\left(1+\frac{K'}{K}+\frac{1}{Q}\right)^{1/2}-1\right]\frac{\gamma\sigma}{2M_s}\equiv f\frac{\gamma\sigma}{2M_s}\ . \tag{3}$$

Here, γ is the gyromagnetic ratio and $Q=K/2\pi M_s^2$ is the "quality factor." Also, it is assumed that the easy axis lies along the z axis in the plane of the DW, being represented by the anisotropy energy KM_z^2/M_s^2, and that the hard axis is reflected by the transverse anisotropy energy $K'M_x^2/M_s^2$. From Eqs. (2) and (3) we find $m=4M_s^2/\gamma^2\sigma f^2$. The spatial dependence of the mass in the equation of motion will be neglected for simplicity in what follows.

The tunneling rate of the DW through the barrier is given by the expression

$$\Gamma=\Gamma_0\exp(-B)\ . \tag{4}$$

The WKB exponent B can be obtained simply from the equation

$$B=\frac{2A}{\hbar}\int_0^{x_0}[2m(x)U(x)]^{1/2}dx\ , \tag{5}$$

where A is the area of the DWJ, $\hbar$ is Planck's constant, and x_0 is the exit point of the barrier. The potential energy $U(x)$ per area is given by $\sigma(x)-2M_sHx$. The function $U(x)$ is depicted in Fig. 2, wherein we note that in Eq. (5), we must shift our origins so that $U(0)=U(x_0)=0$.

Now, in order to be concrete, we have assumed that the profile for the local wall energy near the barrier edge is given by

$$\frac{\sigma(x)}{\sigma}=\left[1+\frac{\eta}{2}\right]+\frac{\eta}{2}\frac{x}{(x^2+s^2)^{1/2}}\ . \tag{6}$$

In terms of the above parameters, we define the *coercive field* H_c as that field above which the potential-energy

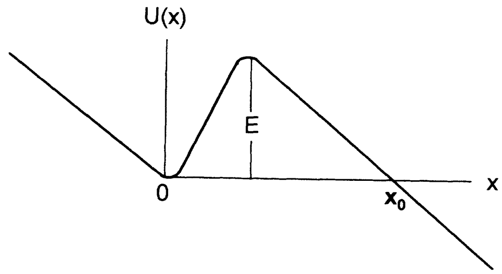

FIG. 2. A depiction of the potential $U(x)$ in the presence of a field very close to the coercive field, so that the function is essentially cubic in x.

function $U(x)$ has no barrier. For the model of Eq. (6), it is given by

$$H_c = \frac{\eta\sigma}{4sM_s} . \tag{7}$$

The specific choice (6) for $\sigma(x)$ merely pins down the definition of the parameter s. (Any other choice would give the same result to within an order of magnitude.) It is interesting to note that the ratio of the coercive field to the anisotropy field $H_A = 2K/M_s$ can be expressed simply as $h_c \equiv H_c/H_A \sim \eta(\delta/s)$.

Domain-wall switching across the barrier will be possible when the applied field is close to the coercive field, i.e., $\varepsilon \equiv (1 - H/H_c) \ll 1$. Then, the potential is a cubic in x, given by

$$U(x) = \frac{27}{4} E \left[\frac{x}{\Delta} \right]^2 \left[1 - \frac{x}{\Delta} \right] , \tag{8}$$

where the energy barrier per area E is given by[17]

$$E \equiv E_0 \varepsilon^{3/2} = (\tfrac{2}{3}) \eta\sigma \varepsilon^{3/2} \cong 0.54 \eta\sigma\varepsilon^{3/2} , \tag{9}$$

and the width of the barrier Δ is given by

$$\Delta = s(6\varepsilon)^{1/2} . \tag{10}$$

Once we recognize that the potential is cubic, with an $\varepsilon^{3/2}$ dependence, we can use the results determined long ago for the Josephson junction.[18]

The WKB exponent B, on the order of $(mE)^{1/2}\Delta$, is given by

$$B = \left[\frac{96}{5} \right] \left[\frac{2}{3} \right]^{3/4} \frac{M_s As}{f\hbar\gamma} \eta^{1/2} \varepsilon^{5/4}$$

$$\cong 7 \left[\frac{M_s}{\mu_B} \right] As \frac{\eta^{1/2}}{f} \varepsilon^{5/4} , \tag{11}$$

where A is the area of the DWJ and μ_B is the Bohr magneton.

The WKB exponent is seen to be proportional to the tunneling magnetic moment contained in the volume (As) of the barrier. It is noteworthy that except through the parameter ε, the WKB exponent is independent of the exchange energy. The exchange energy as well as other parameters contained in H_c determine the scale of the applied field.

The crossover temperature can be estimated from the relation

$$kT_c = \frac{EA}{B} , \tag{12}$$

where k is Boltzmann's constant. This relation leads to

$$kT_c = \frac{5}{96} \frac{f\hbar\gamma\sigma}{M_s s} \eta^{1/2} \varepsilon^{1/4} \cong 0.074 \frac{f\mu_B}{M_s} \frac{\sigma}{s} \eta^{1/2} \varepsilon^{1/4} . \tag{13}$$

As usual, the crossover temperature is independent of the size of the system. The prefactor frequency is given by[19]

$$\Gamma_0 = \left[\frac{30}{\pi} \right]^{1/2} B^{1/2} \omega_0 , \tag{14}$$

where ω_0 is the oscillation frequency of the DW at the bottom of the well. This frequency is given explicitly by $[U''0)/m]^{1/2}$, which turns out to be expressible as $2(\frac{3}{2})^{1/4} f(\varepsilon^{1/4}/\eta^{1/2})\omega_L$, where $\omega_L \equiv \gamma H_c$ is the Larmor frequency at the coercive field H_c. Alternatively, ω_0 is given by $(\frac{72}{5})kT_c/\hbar$.

In order to observe QTM, we desire B to be as small as possible and the crossover temperature to be as large as possible. Equations (12) and (14) show us that it is desirable to have the width s, the magnetization M_s, and the anisotropy energy K as small as possible and the transverse anisotropy K' as large as possible. We must keep in mind that the width s is limited from below in our analysis and approximation by having to be much larger than the wall thickness δ.

In Table I, we list an example of a possible set of values of parameters. We list in Table II a set of values for the derived parameters. The control needed for the coercive field, $\varepsilon H_c = 25G$, is quite easy to manage and the tunneling rate, $\Gamma_0 \exp(-B) = 17\ \text{sec}^{-1}$, is quite high.

The next important question is how the basic parameters in Table I are to be determined. Methods exist[20] for determining the parameters γ, J, K, and M_s, from which one can obtain σ and δ, and the area A can be known accurately. We assume that techniques are available for determining the area A. Experiments in the TA regime can be used to reliably determine H_c and the energy $E_0 \equiv 0.54\eta\sigma$, so that η can be determined. Finally, the parameter s can be determined from the relation $s = \eta\sigma/4M_sH_c$. In a later section we will discuss experimental methods for studying DW tunneling through a DWJ.

B. The regime $\delta \gg s$

In this regime, the coercive field is still well defined, since it depends only upon a static configuration of the

TABLE I. Values of the basic parameters characterizing a domain-wall junction.

$A = 100$ Å $\times 100$ Å	$K = 6\times10^6$ ergs/cm^3	$K' = 10^8$ ergs/cm^3
$s = 100$ Å	$J = 10^{-6}$ ergs/cm	$\varepsilon = 0.01$
$\eta = 0.1$	$M_s = 100$ emu/cm^3	

TABLE II. Values of the derived parameters characterizing a domain-wall junction.

$\delta=40$ Å	$H_c=2500$ Oe	$\omega_0=3.0\times10^{11}$ Hz
$s/\delta=2.5$	$H_0\equiv 2K/M_s=120\,000$ Oe	$E_0A=5.3\times10^{-13}$ ergs
$Q=95$	$v_0=2.8\times10^{6}$ cm/sec	$B=24$
$\sigma=9.8$ ergs/cm^2	$m=1.3\times10^{-12}$ g/cm^2	$T_c=140$ mK

DW. Recall that in this case, strictly speaking there is no potential-energy function for the dynamics of the DW. However, we expect that the dynamics can be approximated by the use of a quasistatic potential-energy function $U(x)$, which in this case is *not* given by $\sigma(x)-2M_sHx$. In our approximation, we imagine the DW positioned at various locations x and estimate the energy associated with that position, neglecting variations in the shape of the domain wall and the domain-wall thickness parameter $\delta(x)$. Let us suppose that the width s of the transition region is fixed and that the width w of the barrier is varied from being $\ll\delta$ to being $\gg\delta$. We discuss cases below.

1. *Case II-a: $s \ll w \ll \delta$*

In this case, the characteristic length of the function $U(x)$ is δ and we write

$$U(x)=V(x/\delta)-2M_sHx \; . \tag{15}$$

Clearly, the function $V(y)$ looks like a broadened version of $\sigma(x)$. Its width is δ, its height is the same as in the regime $\delta \ll s$ except that because only a fraction (w/δ) of the DW is in the region where the wall energy $\sigma(x)$ is increased by the factor $\eta\sigma$, the barrier energy of $V(x/\delta)$ is now given by $(w/\delta)\eta\sigma$. The order of magnitude of the coercive field is determined by setting the barrier energy equal to $2M_sH\delta$ and is therefore given by

$$H_c \sim \frac{\eta\sigma}{M_s}\frac{w}{\delta^2} \quad \text{or} \quad h_c \sim \eta\frac{w}{\delta} \; . \tag{16}$$

This approach reproduces the result of Friedberg and Paul[21] and Barbara and Uehara[22] to within a factor of order unity.

Next, we expand $U(x)$ about the minimum next to the barrier. The resulting potential is cubic as before. The barrier width is given by $\Delta\sim\delta\varepsilon^{1/2}$, so that δ replaces s in our previous expression for case I ($\delta \ll s$). We find the following results:

$$E\sim\eta\sigma\left[\frac{w}{\delta}\right]\varepsilon^{3/2}\; , \quad B\sim\frac{M_sA(w\delta)^{1/2}}{f\mu_B}\eta^{1/2}\varepsilon^{5/4}\; ,$$
$$kT_c\sim\frac{f\mu_B}{M_s}\left[\frac{w}{\delta}\right]^{1/2}\eta^{1/2}\varepsilon^{1/4}\; . \tag{17}$$

According to the expression for B and T_c, the number of spins that tunnel is given by $M_sA(w\delta)^{1/2}\mu_B$, instead of that case I, M_sAs/μ_B.

2. *Case II-b: $s \ll \delta \ll w$*

In this case, considerations similar to those for case II-a show that w is irrelevant, the characteristic length of the function V is again δ, but the barrier height of the function is that of $\sigma(x)$, namely, $\eta\sigma$. As a result $\Delta\sim\delta\varepsilon^{1/2}$ as in case I above, while $h_c\sim\eta$. We find[23]

$$E\sim\eta\sigma\varepsilon^{3/2}\; , \quad B\sim\frac{M_sA\delta}{f\mu_B}\eta^{1/2}\varepsilon^{5/4}\; ,$$
$$kT_c\sim\frac{f\mu_B}{M_s}\frac{\sigma}{\delta}\eta^{1/2}\varepsilon^{1/4}\; . \tag{18}$$

We see that all three parameters, E, B, and T_c, *saturate* as w approaches infinity at a value corresponding to case I with w replaced by δ.

In both cases II, small M_s and large K' lead to small B and large T_c. However, for small B we need small K, while for large T_c we need large K. Therefore, a compromise must be made with regards to the magnitude of K.

C. Variations of the predicted behavior of a *b*-DWJ with respect to its topological characteristics

All of the results we have obtained thus far in this paper are summarized in Table III. We have included numerical coefficients for case I since they are not all on the order of unity. While they pertain to the specific model of Eq. (7), we expect them to reflect the general situation.

TABLE III. Values of the coercive field h_c, energy barrier E, WKB exponent B, and crossover temperature T_c for various ranges of domain-wall thickness δ.

Case	$h_c/[\eta]$	$E/[\eta\sigma\varepsilon^{3/2}]$	$B/[(M_sA/f\mu_B)\eta^{1/2}\varepsilon^{5/4}]$	$T_c/[(f\mu_B\sigma/M_s)\eta^{1/2}\varepsilon^{1/4}]$
I: $\delta \ll s$	δ/s	0.5	$7s$	$0.07/s$
II-a: $s<w\ll\delta$	w/δ	w/δ	$(w\delta)^{1/2}$	$(w/\delta)^{1/2}/\delta$
II-b: $s\ll\delta\ll w$	1	1	δ	$1/\delta$

It is interesting to note that at fixed w and s, the coercive field has a peak with respect to varying wall thickness δ at $\delta \sim s$. On the other hand, the energy barrier E and the crossover temperature T decrease monotonically with respect to δ, for $\delta > s$, while the WKB exponent increases monotonically with respect to δ. It is therefore best to have the smallest possible wall thickness allowable by constraints on measurability of the switching field. Systems with $\delta \sim s$ seem most appropriate.

III. DISSIPATION

Our analysis has so far neglected the effects of dissipation. Therefore, for our results to be valid, dissipation must be weak. The effect of dissipation on QTM in the switching of single-domain particles has been analyzed[24] with respect to eddy currents when the particles and/or particle environment is a conductor material and with respect to photon emission. In the first case, the damping is linear ("Ohmic"), with a rate γ' proportional to the square of the radius of the particle and is typically on the order of 10^{-6}–$10^{-4}\omega_0$. [The result in cgs units is $\gamma'/\omega_0 \approx 32\pi^{2'} R^2 \gamma M_s/\rho c^2$, where ρ is the resistivity of the material, R is the particle radius, and c is the speed of light.] The damping is thus extremely weak. In the second case, the damping is nonlinear; the damping can be characterized by an effective γ'/ω_0 that is proportional to the cube of the radius [$\sim \gamma^3 K^2 R^3 / M_s c^3$] and is on the order of 10^{-20} for a particle of 30-Å radius. Garg and Kim[25] have shown that γ'/ω_0 is typically on the order of 10^{-4} in the case of phonon emission mediated by the magnetoelastic interaction (a nonlinear process).

Dissipation in quantum tunneling of domain walls has been discussed in Refs. 2, 9, and 10 with respect to all three of the above mechanisms. Here, we will merely discuss *explicit* results for damping of DW motion via eddy currents. The seminal paper on this subject is that of Williams, Schockley, and Kittel.[26] Since the damping is linear, it contributes a term $-m\gamma' v$ to our Eq. (1). According to Ref. 26, for a wire of rectangular cross section, with a dimension d in the direction of the easy axis and length L that is much greater than d, in cgs units,

$$\gamma' \approx 98 \frac{M_s^2 d}{\rho m c^2} . \tag{19}$$

Using the relation for the DW mass $m = 4M_s s^2/\gamma^2\sigma$, this equation reduces to

$$\gamma' \cong \frac{\gamma^2 \sigma d}{\rho c^2} . \tag{20}$$

Letting $\rho = 10^{-16}$ sec and using the values of Tables I and II, we obtain a value $\gamma' = 1.4 \times 10^5$ sec^{-1}. In this case, $\gamma'/\omega_0 \equiv 2.9 \times 10^{-7}$. As has been surmised in Refs. 2, 9, and 10, damping from eddy currents is extremely weak.

IV. EXPERIMENTAL METHODS

A basic procedure for studying DW motion across the DWJ runs as follows: We first create a DW on the side of the DWJ. We then apply an oscillatory field, which is essentially linear in the range zero to H_c, noting precisely at what field the DW passes across the DWJ. Or, the passage of the DW past the DWJ can be recorded as a switching of the direction of the magnetization of the sample as a whole. The basic quantity determined is the distribution function $P(H)$ of fields at which this passage takes place. This quantity is closely related to the irreversible susceptibility of a hysteresis loop and is analogous to the distribution of switching currents used to study switching of JJ's via TA,[27] and via MQT in JJ's (Ref. 28) and SQUID's,[29] as discussed below.

We first note that the passage rate *at fixed applied field* H is a function of the applied field, writing $\Gamma = \Gamma_0 \exp(-EA/kT)$. At fixed applied field, the probability that the DW does *not* pass the DWJ at time t, given that the DW had *not* yet passed at time $t=0$, is given by $\exp[-\Gamma(H)t]$. However, if the field is swept at a rate dH/dt, that same quantity is given by

$$W[H(t)] = \exp\left[-\int_0^H \frac{\Gamma(H')}{dH'/dt} dH'\right] , \tag{21}$$

where we regard H and H' as functions of t and can regard W as a function either of H or of t. The distribution function $P(H)$ is then given by

$$P(H) = \frac{\Gamma(H)}{r} W(H) , \tag{22}$$

where $r = dH/dt$ is the sweep rate. For simplicity, we will consider the case of constant r.

To appreciate the meaning of this equation, note that the first factor $\Gamma(H)/r$ is the probability that the DW will pass the DWJ in the range H to $H + dH$ (at time t), given that it has not yet passed the DWJ. The distribution function $P(H)$ has the shape exhibited in Fig. 3 below, wherein it should be noted that $P(H)$ vanishes at H_c. We can relate $P(H)$, measured accurately as a func-

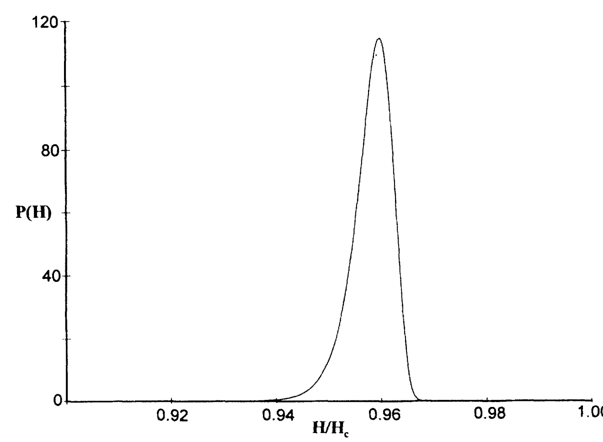

FIG. 3. The distribution function $P(H)$ of passages via TA as a function of swept applied field H, plotted as $H_c P(H)$ vs H/H_c. We have used $E_0 A/kT = 1000$, which according to Table II corresponds to a temperature of 3.8 K. In addition, we have used a value $H_c \Gamma_0 / r = 10^6$. In the TA regime $\Gamma_0 = \omega_0 / 2\pi$. With the values $\omega_0 = 3.0 \times 10^{11}$ Hz and $H_c = 2500$ Oe from Table II, along with $r/H_c = 1$ sec^{-1}, $H_c \Gamma_0 / r = 4.8 \times 10^{10}$ and the peak lies at around $H/H_c = 0.93$.

tion of the field H and temperature T, to theory, in a few ways, as follows.

(1)Method I. We plot the field H_p, at the maximum of $P(H)$, vs temperature. For large E/kT,

$$H_p \cong \left\{ 1 - \left[\frac{kT}{E_0} \ln \left[\frac{rE_0}{kTH_c\Gamma_0} \right] \right]^{2/3} \right\} , \quad (23)$$

where Γ_0 is the prefactor for passage of the barrier under TA. Neglecting the slow varying logarithmic factor, a graph of H_p vs $T^{2/3}$ should produce a straight line. QTM is indicated if, instead, H_p levels off—at low temperatures, below T_c—with a zero slope to a value below H_c [given approximately by Eq. (24), with T replaced by T_c].

(2)Method II. We can follow the Fulton-Dunkelberger[27] and Voss-Webb[28] procedure, plotting the *width* ΔH of the distribution function as a function of temperature. Analysis shows that, for large E_0/kT,

$$\Delta H \cong H_c \left[\frac{kT}{E_0} \ln \left[\frac{rE_0}{kTH_c\Gamma_0} \right] \right]^{2/3} . \quad (24)$$

Under TA, the width should go to zero as $T^{2/3}$. QTM is indicated if, instead, ΔH exhibits a cross-over behavior (presumably at T_c) and levels off to a nonzero value [$\approx$ Eq. (25) with T replaced by T_c]. See Fig. 4.

(3)Method III. We extract the passage rate $\Gamma(H)$ from $P(H)$, using the relation

$$\Gamma(H) = \frac{rP(H)}{\left[1 - \int_0^H P(H')dH' \right]} . \quad (25)$$

We note that

$$\ln(\Gamma) = -\frac{E_0(1-H/H_c)^{3/2}}{kT} + \ln(\Gamma_0) . \quad (26)$$

The advantage of this method is that one does not have to deal with the logarithmic factors present in the expressions for the other two methods.[30] It also provides us with a direct way of extracting the value of E_0. One should plot $\ln(\Gamma)$ as a function of $(1/T)$ at fixed H as well as $(1-H/H_c)^{3/2}$ at fixed T for various T. QTM is indicated by the independence of $\ln(\Gamma)$ with respect to T at low temperatures, with a crossover temperature predicted at around T_c.

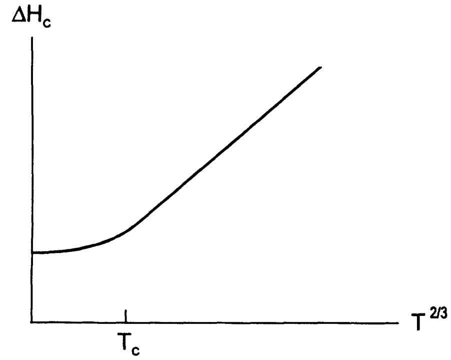

FIG. 4. The width ΔH_c of the distribution function $P(H)$ as a function of temperature $T^{2/3}$. T_c is the crossover temperature, below which QTM is manifest.

V. ON THE NEED TO CONTROL THE DW VELOCITY

Recall that Eq. (1) is valid only if the velocity v of the DW is much less than the limiting velocity v_0 [see Eq. (4)]. We will show below that if the DW is allowed to accelerate over the length L of the sample under the force present when the applied field is comparable to the coercive field, the DW velocity would exceed v_0, thus violating the above restriction. In this case, our theoretical analysis would have to be modified so as to deal with this "relativistic" regime. This situation can be avoided experimentally by simply nucleating the DW under a weak field and allowing it to accelerate slowly towards the DWJ. Only subsequently would the field be raised to a value close to but below the coercive field so as to bring about passage of the DW past the DWJ.

First we show that we need *not* take into account damping during this period of acceleration. This is so because the typical length L is much smaller than the "damping distance" v_t/γ' needed to reach the terminal velocity v_t of the DW. Generally, we have

$$v_t = \frac{2M_sH_c}{m\gamma'} = f^2 \frac{\gamma^2\sigma H_c}{2M_s\gamma'} . \quad (27)$$

Using the relation $v_0 = f\gamma\sigma/2M_s$, we have

$$\frac{v_t}{v_0} = f\frac{\gamma H_c}{\gamma'} \equiv f\frac{\omega_L}{\gamma'} . \quad (28)$$

With $H_c = 2500$ Oe, $\omega_L = 4.4\times10^{10}$ Hz. Assuming that γ' is as large $10^{-2}\omega_0$ and using the figures of Table II, we find a damping distance $v_t/\gamma' \sim 430\mu$, which is much larger than the length L one expects to use. Thus, finally, neglecting damping, the DW will accelerate along the length L under a force $2M_sH_c$, from rest to a velocity such that $v^2 = 2(2M_sH_c/m)L$, so that

$$\frac{v}{v_0} = \left[\eta\frac{L}{s} \right]^{1/2} . \quad (29)$$

With $\eta = 0.1$, $s = 100$ Å, and $L = 1\mu$, we find $v/v_0 = 3$, in violation of the requirement $v/v_0 \ll 1$.

VI. DISCUSSION

We have shown that the formal analogy existing between the equations of motion of a DWJ and a JJ allows for a very simple description of the dynamics of a DWJ for a whole range of domain-wall thicknesses. As in the case of a JJ, the attractiveness of a DWJ lies in its characterizability. It appears that the observation of quantum tunneling in a DWJ will be easiest when the DW thickness δ is of the order of magnitude of (and eventually thinner than) the width s of the interface between the DWJ and the surrounding material. We have also discussed the role played by other parameters, such as longitudinal and transverse anisotropy, spontaneous magnetization, and barrier width, on the behavior of the DWJ. The analogy with the JJ also suggests the most appropriate experimental procedures to display MQT in a DWJ. Numerical calculations indicate that the observation of quantum tunneling in a DWJ is currently accessible to experimental methods.

APPENDIX

In this appendix, we provide a derivation of Eq. (1) of the paper, neglecting transverse anisotropy. We begin with the action of Enz,[31] modified so as to include a spatial dependence of the various parameters that characterize the sample.

$$S=-A\int\int dx\,dt\left\{J(x)\left[\left[\frac{\partial\theta}{\partial x}\right]^2-\frac{1}{c(x)^2}\left[\frac{\partial\theta}{\partial t}\right]^2\right]+K(x)\sin^2\theta-M_sH\cos\theta\right\},\tag{A1}$$

where A is the area of domain wall, J is the exchange energy, c is the maximum speed of the domain wall, and θ is the angle of rotation of the magnetization in the plane of the domain wall. (See Enz for the details.) The equation of motion that follows from this action is given by

$$\frac{\partial^2\theta}{\partial u^2}-\tau(u)^2\frac{\partial^2\theta}{\partial t^2}=\sin\theta\cos\theta+f(u)\frac{\partial\theta}{\partial u}+h(u)\sin\theta\ ,\tag{A2}$$

where $u(x)$ is a dimensionless position variable, determined by the equation

$$\frac{du}{dx}=\frac{1}{\delta(x)}\ ,\tag{A3}$$

$\tau(u)$ is a local time coordinate, given by

$$\tau(u)=\frac{1}{2\gamma[2\pi K(u)]^{1/2}}\ ,\tag{A4}$$

and $f(u)$ is the force on the DW due the spatial variation of the wall energy $\sigma(u)$:

$$f(u)=-\frac{d\ln\sigma(u)}{du}\ .\tag{A5}$$

We next assume that the domain wall has the form

$$\theta(u,t)=2\tan^{-1}\exp[\varphi(u,t)]\ ,\tag{A6}$$

where

$$\varphi(u,t)k=\gamma(t)[u-q(t)]\ .\tag{A7}$$

The function $q(t)$ is the position of the center of the domain wall at time t. Substituting Eqs. (A3) and (A4) into Eq. (A2) leads to

$$\sin\theta\cos\theta\left[\left[\frac{\partial\varphi}{\partial u}\right]^2-\tau(u)^2\left[\frac{\partial\varphi}{\partial t}\right]^2\right]+\sin\theta\left[\frac{\partial^2\varphi}{\partial u^2}-\tau(u)^2\frac{\partial^2\varphi}{\partial t^2}\right]=\sin\theta\cos\theta+\sin\theta\left[\frac{\partial\varphi}{\partial u}f(u)+h(u)\right]\ ,\tag{A8}$$

where $h(u)\equiv MH/2K(u)$ is the local anistropy field. If we now equate the respective corresponding factors of $[\sin\theta\cos\theta]$ and $[\sin\theta]$, we obtain

$$\left[\frac{\partial\varphi}{\partial u}\right]^2-\tau(u)^2\left[\frac{\partial\varphi}{\partial t}\right]^2=1\tag{A9}$$

and

$$\frac{\partial^2\varphi}{\partial u^2}-\tau(u)^2\frac{\partial^2\varphi}{\partial t^2}=\frac{\partial\varphi}{\partial u}f(u)+h(u)\ .\tag{A10}$$

Using Eqs. (A6) and (A9) we find exactly

$$\left[\frac{\partial\theta}{\partial u}\right]^2-\tau(u)^2\left[\frac{\partial\theta}{\partial t}\right]^2=\sin^2\theta\ .\tag{A11}$$

Thus, the action is given by

$$S=-\frac{A}{2}\int\int du\,dt\,\sigma(u)[\sin^2\theta-h(u)\cos\theta]\ .\tag{A12}$$

Next, we assume that the domain wall has a thickness δ much less than the length scale of variation of the parameters $\sigma(x)$ and $h(x)$. Since

$$\int du\ \sin^2\theta\equiv du\ \mathrm{sech}^2\varphi=2/\gamma(t)\ ,\tag{A13}$$

in the integral (A9) we can let

$$\sin^2\theta\rightarrow\frac{2}{\gamma(t)}\delta[u-q(t)]\tag{A14}$$

and replace $(1-\cos\theta)/2$ by the unit step function, equal to zero for $u<q(t)$ and unity for $u>q(t)$. We then integrate (A12) over u, and, aside from an irrelevant constant, obtain

$$S=A\int dt\left\{\sigma(x)\left[1-\frac{v^2}{c^2}\right]^{1/2}-2M_sHx(t)\right\},\tag{A15}$$

where $v=dx/dt$ is the velocity of the domain wall. For small velocities, $v\ll c$, we obtain

$$S=\int dt\left\{\tfrac{1}{2}m(x)v^2-[\sigma(x)-2M_sHx]\right\},\tag{A16}$$

where $m(x)$ is the local mass per area, given by

$$m(x)=\sigma(x)/c(x)^2\ .\tag{A17}$$

The equation of motion for the variable $x(t)$ that locates the center of the domain wall is gotten from the variational equation $\delta S/\delta x=0$ and is given by

$$\frac{d[m(x)v]}{dt}=-\frac{\partial\sigma}{\partial x}+2M_sH+\frac{v^2}{2}\frac{dm}{dx}\ .\tag{A18}$$

For small velocities we can neglect the last term on the RHS, thus obtaining Eq. (1) of the text.

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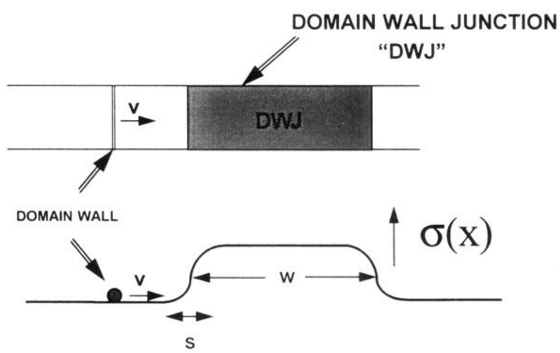

FIG. 1. A depiction of a domain wall moving along a narrow channel of a sample of magnetic material encountering a domain junction (DWJ). The potential energy $\sigma(x)$ in the absence of an applied field is depicted below. Outside the DWJ, $\sigma(x)=\sigma$, while inside the DWJ, $\sigma(x)=\sigma(1+\eta)$. Thus, η is the relative jump in the potential energy. The width s of the transition region determines the coercive field H_c, being given by $\eta\sigma/Ms$.