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Long-time dynamics of the infinite-temperature Heisenberg magnet

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Infinite-temperature long-time dynamics of Heisenberg model $\hat{H} = -\frac{1}{2}\sum_{i,j} J_{ij}\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$ is investigated. It is shown that the quantum-spin pair correlator is equal to the correlator of a classically evaluated vector field averaged over the initial conditions with respect to the Gaussian measure. In the continuous limit case, the scaling estimations allow one to find the one-point correlator that turns out to be $C(\mathbf{r}=0;t) \propto \text{const} \times t^{-6/7}$. All results are obtained by straightforward procedures without any assumptions of a phenomenological character.

INTRODUCTION

Because of the boundedness of the spin operators the problem of infinite-temperature dynamics of quantum Heisenberg model is well defined. For the equilibrium case it reduces to the computation of correlators as follows:

$$C^{\alpha\beta}(\mathbf{r};t) = \text{Tr}[\exp(it\hat{H})\hat{S}_i^\alpha \exp(-it\hat{H})\hat{S}_j^\beta]. \quad (1)$$

Here $\hat{H}$ is the Heisenberg exchange Hamiltonian

$$\hat{H} = -\frac{1}{2}\sum_{i,j} J_{ij}\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j, \quad (2)$$

$\hat{S}_i^\alpha$ is the αth spin operator component on the lattice site i, and $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$ is the distance between lattice sites i and j. The thermodynamic limit case ($\sum_i 1 = \infty$) is supposed.

For fixed $\mathbf{r}$ and $t \to 0$ the correlator $C^{\alpha\beta} = \delta^{\alpha\beta}C(\mathbf{r};t)$ can be calculated by the direct temporal expansion of the evolution operator $\exp(it\hat{H})$ (see the initial papers,[1–3] the review of the results obtained by this approach in the work,[4] and the more recent paper[5]). Attempts to investigate the long-time dynamics starting from expression (1) have been done by a number of authors.[3,6–8] The main tool used in these papers was the correlator decoupling method. However, it can only work in the case when the dynamics consists mainly of well-defined propagating excitations, but this is not correct in our case and may lead wrongly to obtain whatever is supposed. So, the most popular theory of such a so-called spin-diffusion theory[3,7] gives the spin autocorrelation function in the form $C(t) \propto t^{-3/2}$.

In Ref. 9 the functional integral representation for (1) has been derived. The small time t but large $\mathbf{r}^2 t$ limit has been calculated with the use of this representation. In the present paper we show that *under some controlled assumptions* it is possible to extend this approach to a long-time limit case ($Jt \gg 1$) . The problem reduces to the averaging of some classical equation over initial conditions [see Eqs. (27)–(29)]. In the continious-limit case scaling estimates allow one to find a one-point correlator (see the last section) which turns out to be

$$C(t) \propto \text{const} \times t^{-\frac{6}{7}}, \quad (3)$$

and to prove the validity of our approximation at large enough time.

FUNCTIONAL REPRESENTATION

We start from the Hubbard-Stratonovich transformations of the generating functional of the spin correlators

$$Q[\mathbf{h}] = \text{Tr}\left[T_c \exp\left(i\int_c dt(\hat{H} + \mathbf{h}_i \cdot \hat{\mathbf{S}}_i)\right)\right] = N\int D\boldsymbol{\varphi}^{(1)} D\boldsymbol{\varphi}^{(2)} \exp\left(\frac{i}{2}\sum_{i,j}\int_c dt' \boldsymbol{\varphi}_i(t') J_{ij}^{-1}\boldsymbol{\varphi}_j(t')\right) \times \prod_i T_c \exp\left(i\int_c dt'[\boldsymbol{\varphi}_i(t') + \mathbf{h}_i(t')]\cdot\hat{\mathbf{S}}_i\right), \quad (4)$$

where N is a normalization factor. Here, T_c indicates the ordering of operators along the contour c in the t plane and consists of two oppositely directed branches that pass in the vicinity of real axes.[9–12] $\mathbf{h}_i(t)$ is the auxiliary external field. The indices 1, 2 of the vector-valued fields indicate the upper and the lower branch of the c contour, respectively. If we define the external field as

$$\mathbf{h}_j^{(1)} = \mathbf{a}_j\delta(t) + \mathbf{b}_j\delta(t-T),$$
$$\mathbf{h}_j^{(2)} = -\mathbf{a}_j\delta(t) - \mathbf{b}_j\delta(t-T), \tag{5}$$

then the correlator (1) is

$$C^{\alpha\beta}(\mathbf{r};t) = -\frac{1}{4}\frac{\partial^2 Q(\mathbf{h})}{\partial a_i^\alpha \partial b_j^\beta}\bigg|_{\mathbf{a}=\mathbf{b}=0}. \tag{6}$$

Making the translation $\varphi \to \varphi - \mathbf{h}$ and neglecting the terms which do not contribute to (6) we get the expression

$$Q[\mathbf{h}] = \int D\boldsymbol{\varphi}^{(1)} D\boldsymbol{\varphi}^{(2)} \prod_i \mathrm{Tr}[\hat{A}_i(-\infty,+\infty)] \exp\bigg(\frac{i}{2}\sum_{i,j}\int_c dt'[\boldsymbol{\varphi}_i^{(1)}(t')J_{ij}^{-1}\boldsymbol{\varphi}_j^{(1)}(t') - \boldsymbol{\varphi}_i^{(2)}(t')J_{ij}^{-1}\boldsymbol{\varphi}_j^{(2)}(t')]$$
$$+\mathbf{b}_i J_{ij}^{-1}[\boldsymbol{\varphi}_j^{(1)}(t) + \boldsymbol{\varphi}_j^{(2)}(t)] + \mathbf{a}_i J_{ij}^{-1}[\boldsymbol{\varphi}_j^{(1)}(0) + \boldsymbol{\varphi}_j^{(2)}(0)]\bigg). \tag{7}$$

Here the operator $\hat{A}(-\infty,t)$ for a given lattice site is defined as

$$\hat{A}(-\infty,t) = T\exp\left(i\int_{-\infty}^t \varphi_i^{(1)}(t')\cdot\hat{\mathbf{S}}dt'\right) \times\tilde{T}\exp\left(-i\int_{-\infty}^t \varphi_i^{(2)}(t')\cdot\hat{\mathbf{S}}dt'\right), \tag{8}$$

and $\tilde{T}$ denotes antichronological ordering. This operator is determined by the differential equation

$$-i\frac{d\hat{A}(-\infty,t)}{dt} = [\varphi_i^{(1)}(t)\cdot\hat{\mathbf{S}}]\hat{A}(-\infty,t) - \hat{A}(-\infty,t)[\varphi_i^{(2)}(t)\cdot\hat{\mathbf{S}}], \tag{9}$$

and by the initial condition $\hat{A}(-\infty,-\infty) = 1$. Let us perform the ansatz (see also Refs. 9 and 10)

$$\hat{A}(t) = \exp\left(i\int_{-\infty}^t [\boldsymbol{\rho}^{(1)}(t') - \boldsymbol{\rho}^{(2)}(t')]\cdot\hat{\mathbf{S}}dt'\right), \tag{10}$$

where $\boldsymbol{\rho}^{(1)}(t), \boldsymbol{\rho}^{(2)}(t)$ are some new vector fields. Differentiating this equation with respect to time we obtain

$$-i\frac{d\hat{A}(-\infty,t)}{dt} = \int_0^1 d\tau\exp(i\tau\boldsymbol{\zeta}\cdot\hat{\mathbf{S}}) \times(\boldsymbol{\rho}^{(1)} - \boldsymbol{\rho}^{(2)})\cdot\hat{\mathbf{S}}\exp[i(1-\tau)\boldsymbol{\zeta}\cdot\hat{\mathbf{S}}] = \hat{U}_1(t)\hat{A}(t) - \hat{A(t)}\hat{U}_2(t). \tag{11}$$

Here we introduce notations of the following type:

$$\boldsymbol{\zeta}(t) = \int_0^t dt'[\boldsymbol{\rho}^{(1)}(t') - \boldsymbol{\rho}^{(2)}(t')], \tag{12}$$
$$\hat{U}_1(t) = \int_0^1 d\tau\exp(i\tau\boldsymbol{\zeta}\cdot\hat{\mathbf{S}})\boldsymbol{\rho}^{(1)}\hat{\mathbf{S}}\cdot\exp[i(1-\tau)\boldsymbol{\zeta}\cdot\hat{\mathbf{S}}],$$
$$\hat{U}_2(t) = \int_0^1 d\tau\exp(i\tau\boldsymbol{\zeta}\cdot\hat{\mathbf{S}})\boldsymbol{\rho}^{(2)}\cdot\hat{\mathbf{S}}\exp[i(1-\tau)\boldsymbol{\zeta}\cdot\hat{\mathbf{S}}]. \tag{13}$$

Noting that the operator of the spin rotations appears in the integrands of (13), we get immediately

$$\hat{U}_1(t) = \boldsymbol{\eta}_1\cdot\hat{\mathbf{S}}\ , \quad \hat{U}_2(t) = \boldsymbol{\eta}_2\cdot\hat{\mathbf{S}},$$

$$\boldsymbol{\eta}_1 = \boldsymbol{\rho}^{(1)}\frac{\sin|\boldsymbol{\zeta}|}{|\boldsymbol{\zeta}|} + \boldsymbol{\zeta}\frac{(\boldsymbol{\zeta}\boldsymbol{\rho}^{(1)})}{\boldsymbol{\zeta}^2}\left(1 - \frac{\sin|\boldsymbol{\zeta}|}{|\boldsymbol{\zeta}|}\right) + \frac{1}{\boldsymbol{\zeta}^2}[\boldsymbol{\zeta}\times\boldsymbol{\rho}^{(1)}](1-\cos|\boldsymbol{\zeta}|),$$

$$\boldsymbol{\eta}_2 = \boldsymbol{\rho}^{(2)}\frac{\sin|\boldsymbol{\zeta}|}{|\boldsymbol{\zeta}|} + \boldsymbol{\zeta}\frac{(\boldsymbol{\zeta}\cdot\boldsymbol{\rho}^{(2)})}{\boldsymbol{\zeta}^2}\left(1 - \frac{\sin|\boldsymbol{\zeta}|}{|\boldsymbol{\zeta}|}\right) - \frac{1}{\boldsymbol{\zeta}^2}[\boldsymbol{\zeta}\times\boldsymbol{\rho}^{(2)}](1-\cos|\boldsymbol{\zeta}|), \tag{14}$$

and

$$\mathrm{Tr}[\hat{A}(-\infty,+\infty)] = \exp\left[g_s\left(\left|\int_{-\infty}^{+\infty}(\boldsymbol{\rho}^{(1)} - \boldsymbol{\rho}^{(2)})dt\right|\right)\right],$$
$$g_s(x) = \ln\left(\frac{\sin[(S+1/2)x]}{\sin(x/2)}\right), \tag{15}$$

where S is the spin magnitude. Comparing Eq. (9) with Eqs. (11)–(14) we conclude that the parametrization

$$\boldsymbol{\varphi}^{(1)}(t) = \boldsymbol{\eta}_1(t)\ , \quad \boldsymbol{\varphi}^{(2)}(t) = \boldsymbol{\eta}_2(t) \tag{16}$$

gives the explicit form of Eq. (15) for $\mathrm{Tr}[\hat{A}(-\infty,+\infty)]$. It is more convenient to rewrite equalities (14) in terms of the $\boldsymbol{\psi}$, $\boldsymbol{\zeta}$ fields:

$$\boldsymbol{\varphi}^{(1)} + \boldsymbol{\varphi}^{(2)} = \boldsymbol{\psi},$$

$$\boldsymbol{\varphi}^{(1)} - \boldsymbol{\varphi}^{(2)} = \dot{\boldsymbol{\zeta}}\frac{\sin|\boldsymbol{\zeta}|}{|\boldsymbol{\zeta}|} + \boldsymbol{\zeta}\frac{(\boldsymbol{\zeta}\cdot\dot{\boldsymbol{\zeta}})}{\boldsymbol{\zeta}^2}\left(1 - \frac{\sin|\boldsymbol{\zeta}|}{|\boldsymbol{\zeta}|}\right) - \frac{1-\cos|\boldsymbol{\zeta}|}{|\boldsymbol{\zeta}|\sin|\boldsymbol{\zeta}|} \times\left([\boldsymbol{\zeta}\times\dot{\boldsymbol{\zeta}}] + \frac{1}{\boldsymbol{\zeta}^2}(1-\cos|\boldsymbol{\zeta}|)[\boldsymbol{\zeta}\times[\boldsymbol{\zeta}\times\dot{\boldsymbol{\zeta}}]]\right), \tag{17}$$

$$\zeta(-\infty) = 0, \tag{18}$$

where

$$\boldsymbol{\psi} = \boldsymbol{\rho}^{(1)} + \boldsymbol{\rho}^{(2)} \ , \quad \boldsymbol{\zeta} = \int_{-\infty}^{t} [\boldsymbol{\rho}^{(1)}(t') - \boldsymbol{\rho}^{(2)}(t')] dt'. \tag{19}$$

We can consider $\boldsymbol{\psi}, \boldsymbol{\zeta}$ as the new integration variables in Eq. (7) . This allows us to write down an explicit functional representation for the generating functional $Q[\mathbf{h}]$. Let us note that the essential difference with Ref. 9 is that the transformations (17) are nonlinear only in the subspace of field configurations. Indeed, one of the new integration variables, the field $\boldsymbol{\psi}$, is connected with the old integration variables $\boldsymbol{\varphi}^{(1)}$ and $\boldsymbol{\varphi}^{(2)}$ linearly. The Jacobian of transformation (18) has the "ultralocal" form

$$J = \text{const} \times \prod_t \left[1 + \frac{6}{|\boldsymbol{\zeta}|} \tan|\boldsymbol{\zeta}|/2 \left(\frac{2\tan|\boldsymbol{\zeta}|/2}{|\boldsymbol{\zeta}|} - 1 \right) \right]. \tag{20}$$

EFFECTIVE EQUATION OF MOTION

The effective action that is obtained after the substitution [(15) and (18)] into (7) is rather complicated and some approximations are necessary. It seems to be reasonable to assume that for the long-time dynamics the relevant field configurations obey the inequality

$$|\boldsymbol{\zeta}(t)| = \left| \int_{-\infty}^{t} [\boldsymbol{\rho}^{(1)}(t') - \boldsymbol{\rho}^{(2)}(t')] dt' \right| \ll 1. \tag{21}$$

This assumption is based on some facts. First of all, the action does not contain even with respect to $\dot{\boldsymbol{\zeta}}(t)$ terms. It leads, in particular, to the zero value of all the correlators containing field $\dot{\boldsymbol{\zeta}}(t)$ at least once. Second, all physical [not containing the field $\dot{\boldsymbol{\zeta}}(t)$] correlators must be close to zero as t goes to ∞ due to absence of the spreading excitations at infinite temperature. But in any case, to check the validity of assumption (21) we have to estimate the neglected terms (see the last paragraph of the next section).

Expanding the right-hand side of (17) in a series of $\boldsymbol{\zeta}(t)$ and keeping only the first nontrivial term, we write down the approximate form of the map (17)–(20)

$$\boldsymbol{\varphi}^{(1)} + \boldsymbol{\varphi}^{(2)} = \boldsymbol{\psi},$$

$$\boldsymbol{\varphi}^{(1)} - \boldsymbol{\varphi}^{(2)} \approx \dot{\boldsymbol{\zeta}} + \frac{1}{2}[\boldsymbol{\psi} \times \boldsymbol{\zeta}], \tag{22}$$

$$J \approx 1. \tag{23}$$

The approximation for $\mathrm{Tr}[\hat{A}(-\infty, +\infty)]$ is

$$\mathrm{Tr}[\hat{A}(-\infty, +\infty)] \approx \exp\left(-\frac{D}{2} \boldsymbol{\zeta}^2(+\infty) \right), \tag{24}$$

where $D = |\ddot{g}_s(0)|$ [see (15)]. Substituting (22) into (7) and introducing the field $\boldsymbol{\phi}(t)$,

$$\boldsymbol{\phi}_i = 2J_{ij}\boldsymbol{\psi}_j, \tag{25}$$

that directly corresponds to the spin on the lattice site i, we note that the $D\boldsymbol{\zeta}$ integration can be easily performed. The classical equation of motion for the field $\boldsymbol{\phi}$ averaged over the "initial" data at the far future is obtained

$$Q[\mathbf{a}, \mathbf{b}] = \int D\boldsymbol{\phi} \exp\left(-\frac{1}{2D} \sum_i \boldsymbol{\phi}_i^2(+\infty) - 2\mathbf{a}_i \boldsymbol{\phi}_i(t) - 2\mathbf{b}_i \boldsymbol{\phi}_i(0) \right) \prod_{i;t} \delta\left(\dot{\boldsymbol{\phi}}_i - \sum_j J_{ij} [\boldsymbol{\phi}_i \times \boldsymbol{\phi}_j] \right). \tag{26}$$

[Fourier-like integration with respect to the field $\boldsymbol{\zeta}(t')$ in an intermediate time moment gives the δ function product in Eq. (26), while the Gaussian integration with respect to $\boldsymbol{\zeta}(+\infty)$ produces a probability distribution for $\boldsymbol{\phi}(+\infty)$.] It is important to note that the $\boldsymbol{\phi}_i^2$ term defining the weight of averaging is the integral of motion of classical equations given in (26). Let us mention that from the ergodicity consideration it is natural to obtain the averaging over the distribution of the motion integral that is the only nonstochastical variable in the system. Thus in our approximation the quantum spin correlator (1) is equal to the correlator of the classically evaluated vector field $\boldsymbol{\phi}_i(t)$,

$$\dot{\boldsymbol{\phi}}_i = \sum_j J_{ij} [\boldsymbol{\phi}_i \times \boldsymbol{\phi}_j], \tag{27}$$

averaged over the initial conditions

$$\boldsymbol{\phi}_i(0) = \mathbf{p}_i \tag{28}$$

with respect to the Gaussian measure

$$\prod_i d\mathbf{p}_i \exp\left(-\frac{1}{2D} \sum_i \mathbf{p}_i^2 \right). \tag{29}$$

Here the conservation of the phase space volume element $d\boldsymbol{\phi}_i$ during the evolution (27) has been taken into account. Note that Eq. (27) is only the spin-operator equation of motion with the changing $\hat{\mathbf{S}}$ to the classical evaluated field $\boldsymbol{\phi}$.

SCALING ESTIMATIONS

It is natural to suppose that the long-time evolution is determined by the long-wavelength fluctuations. Thus we can use the continium version of (27),

$$\dot{\boldsymbol{\phi}}(\mathbf{r}; t) = \alpha[\Delta\boldsymbol{\phi} \times \boldsymbol{\phi}], \tag{30}$$

and (29),

$$Dp(\mathbf{r})\exp\left(-\frac{1}{2\tilde{D}}\int d^3r\mathbf{p}^2(\mathbf{r})\right). \tag{31}$$

Here $\tilde{D} = a^3D$, a is the lattice spacing, and α is defined in terms of the Fourier transform $J(\mathbf{k})$ of the exchange integral $J_{ij} = J(\mathbf{r}_i - \mathbf{r}_j)$ as follows:

$$J(\mathbf{k}) \approx J(0) - w_{\rm ex}(ak)^2\ , \quad \alpha = w_{\rm ex}a^{7/2}. \tag{32}$$

Equations (31) and (32) can be studied in principle with the use of the Wyld diagram technique.[13] However, all the terms of the perturbation theory suffer from infrared singularities. On the other hand, in the infrared region the scaling arguments can work. Indeed, Eq. (30) is invariant with respect to the following continuous set of the scale-transformation group:

$$\mathbf{r} \to \lambda\mathbf{r}, \quad t \to \lambda^\beta t,$$
$$\boldsymbol{\phi}(\mathbf{r};t) \to \lambda^{\beta-2}\boldsymbol{\phi}(\lambda\mathbf{r};\lambda^\beta t). \tag{33}$$

Here β is an arbitrary real number. The quantity

$$K(\mathbf{r};t) = \boldsymbol{\phi}(\mathbf{r};t)\cdot\mathbf{p}(\mathbf{r}) \tag{34}$$

obtained after averaging the desired one-lattice site spin-spin correlator transforms as

$$K(\mathbf{r};t) \to \lambda^{2(\beta-2)}K(\lambda\mathbf{r};\lambda^\beta t). \tag{35}$$

The requirement for the weight of averaging to be scaling invariant gives us the unique value of β:

$$\beta = \frac{7}{2}. \tag{36}$$

The invariance (33) means that if some initial conditions $\boldsymbol{\phi}(\mathbf{r};0) = \mathbf{p}(\mathbf{r})$ transform to $\lambda^{\beta-2}\mathbf{p}(\lambda\mathbf{r})$ then for any further moment $\boldsymbol{\phi}(\mathbf{r};t)$ transforms by (33). For $\beta = 7/2$ all points on each orbit generated in the functional phase space by the scaling group (33) have equal probabilities. Thus the correlator $C(\mathbf{r};t) = \langle K(\mathbf{r};t)\rangle$ should be invariant with respect to the transformations (33) and (36):

$$C(\mathbf{r};t) = \lambda^3 C(\lambda\mathbf{r};\lambda^{7/2}t). \tag{37}$$

Consequently, it has the form

$$C(\mathbf{r};t) = t^{-6/7}f(r/t^{2/7}) \tag{38}$$

and the one-point correlator is finally

$$C(t) \equiv C(\mathbf{r}=0;t) = \text{const}\times t^{-6/7}. \tag{39}$$

To estimate the contribution of the neglected terms let us return to the functional integral with respect to $\boldsymbol{\psi},\boldsymbol{\zeta}$ fields from the substitution (22). The suitable action is invariant with respect to the following scaling transformation:

$$\mathbf{r} \to \lambda\mathbf{r}\ ,\ t \to \lambda^{7/2}t\ ,\ \ \boldsymbol{\psi}(\mathbf{r};t) \to \lambda^{3/2}\boldsymbol{\psi}(\lambda\mathbf{r};\lambda^{7/2}t),$$
$$\boldsymbol{\zeta}(\mathbf{r};t) \to \lambda^{3/2}\boldsymbol{\zeta}(\lambda\mathbf{r};\lambda^{7/2}t). \tag{40}$$

So, we can compute scaling indices for all the corrections to the correlator $C(\mathbf{r};t)$. For the first nontrivial correction arising from the nonlinearities of the third-order nonlinearities in (18) with respect to the small parameter (21) we obtain

$$C^{(3)}(\mathbf{r};t) = t^{-12/7}f^{(1)}(r/t^{2/7}) \tag{41}$$

and it can be neglected comparing with Eq. (38) in the limit $t \to \infty$ and especially for $r = 0$. Every next order correction to the correlator gives an additional factor $t^{-3/7}$ and does not affect the long-time behavior. Here we have assumed that the ultraviolet divergencies cannot affect drastically the long-time behavior.

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[1] M.F. Collins and W. Marshall, Proc. Phys. Soc. **92**, 390 (1967).
[2] D.G. McFadden and R.A. Tahir-Kheli, Phys. Rev. B **1**, 3649 (1970).
[3] H.S. Bennett and P.C. Martin, Phys. Rev. **138**, 608 (1965).
[4] J. Labrujere, T.O. Klaasen, and N.J. Poulis, J. Phys. C **15**, 999 (1982).
[5] V.E. Zobov, Zh. Eksp. Teor. Fiz. **99**, 809 (1991) [Sov. Phys. JETP **72**, 449 (1991)].
[6] P. Resibois and M. de Leener, Phys. Rev. **152**, 305 (1966).
[7] M. Blume and J. Hubbard, Phys. Rev. B **1**, 3815 (1970).
[8] D. Ter Haar, *Lectures on Selected Topics in Statistical Mechanics* (Pergamon, New York, 1977).
[9] I.V. Kolokolov, Zh. Eksp. Teor. Fiz. **91**, 2313 (1986) [Sov. Phys. JETP **64**, 1373 (1986)].
[10] I.V. Kolokolov, Phys. Lett. A **114**, 99 (1986).
[11] I.V. Kolokolov and E.V. Podivilov, Zh. Eksp. Teor. Fiz. **95**, 211 (1989) [Sov. Phys. JETP **68** , 119 (1989)].
[12] I.V. Kolokolov, Ann. Phys. (NY) **202**, 165 (1990).
[13] V. Lvov, *Nonlinear Spin Waves* (Nauka, Moscow, 1987).