# Electron-phonon interaction and transport properties in the quantum-Hall-effect regime

Ya. M. Blanter* and D. V. Livanov
*Moscow Steel and Alloys Institute, Leninskiy prospect 4, 117936 Moscow, Russia*
(Received 14 April 1993; revised manuscript received 17 September 1993)

The effect of electron-phonon interaction on the transport properties of two-dimensional electron gas in a strong transverse magnetic field is investigated. Corrections to nondiagonal (Hall) components of electrical and heat conductivities and thermopower tensors are obtained in the context of the diagrammatic technique and are shown to be essential in a wide range of temperatures and magnetic fields. Experimental conditions for the observation of electron-phonon interaction effects in the electronic transport in the quantum-Hall-effect regime are discussed.

Two-dimensional (2D) electron systems in quantized magnetic fields have drawn considerable attention of both theorists and experimentalists since the discovery of the quantum Hall effect (QHE) (for an extensive review, see Ref. 1). The advancement in the theory of this phenomenon has followed two main directions. The first one lies in refining the theory involving effects of electron-electron interaction and in understanding the nature of the fractional quantum Hall effect. The second direction (which the present work follows) is the consideration of the quantum Hall behavior in models of materials more realistic than the 2D ideal electron gas, which is necessary for a better understanding and interpretation of experimental data.

One of the most important properties of real materials is the disorder due to impurities. The influence of impurity disorder on kinetic characteristics in the quantum-Hall-effect regime has been extensively studied theoretically. It was found that disorder alters the shape of dependencies of off-diagonal components of the electrical[1] and heat[2,3] conductivity tensors as functions of the chemical potential, and leads to the appearance of nonzero diagonal components of these tensors. As for the thermopower tensor, disorder changes drastically both components.[4] All these results are in rather good qualitative agreement with experimental data (see Ref. 1 for a review for conductivity and Refs. 5–7 for thermopower), but sometimes the essential deviations from theoretically predicted dependencies has been observed.[7,8] These deviations are often attributed to experimental errors and other unphysical reasons. Nevertheless these discrepancies between experimentally observed and "ideal" behavior can be connected with some other features of real materials which usually are not taken into consideration. Among them the inelastic scattering of electrons, for which the electron-phonon interaction is to a large extent responsible, seems to be the most important. The study of electron-phonon effects in the electronic transport in the QHE regime is the subject of the present paper. For this purpose we consider the 2D electron system interacting with 3D phonon field in strong transverse magnetic field. We argue that the electron-phonon scattering leads to deviations from their "ideal" forms of kinetic-coefficients dependencies on the chemical potential.

As is well known (see, e.g., Refs. 9 and 10), the electron transport in a strong magnetic field is characterized by the fact that the Kubo formalism breaks down due to the screening of transverse currents by currents flowing on the surface of the sample. If this is the case, the transverse components of kinetic-coefficient tensors can be expressed through the thermodynamic potential.[11] So our purpose is to calculate the correction in the thermodynamic potential of the 2D electron system due to electron-phonon interaction.

The interaction term in the Hamiltonian has a form[12]

$$\hat{H}_{\rm int}(\tau) = g \int d^2r\ d^3\xi\ K(\mathbf{r}-\boldsymbol{\xi})\psi^+(\mathbf{r},\tau)\psi(\mathbf{r},\tau)\phi(\boldsymbol{\xi},\tau) \tag{1}$$

(here and below $\hbar = 1$), where $g$ is the electron-phonon coupling constant, $\psi(\mathbf{r},\tau)$ and $\phi(\boldsymbol{\xi},\tau)$ are electron and phonon field operators, respectively, and $K(\mathbf{r})$ is a screening function [$\mathbf{r} = (x,y)$ and $\boldsymbol{\xi} = (x,y,z)$]. The integration in Eq. (1) is performed over electron (2D) and phonon (3D) coordinates. To make further calculations tractable we adopt below the most simple form of the screening function: $K(\mathbf{r}) = \delta(\mathbf{r})$. Physically this assumption means that screening takes place at distances shorter than a lattice period. This speculation is always adequate for metallic systems, but for semiconductors it is in doubt and may be considered as the simplest model assumption only.

Using the standard diagrammatic technique[12] one may express the correction to the thermodynamic potential in the lowest order in electron-phonon interaction in terms of Green's functions. In order to avoid difficulties associated with analytical continuation from Matsubara to real frequencies we use the Keldysh formalism with electron $\hat{G}$ and phonon $\hat{D}$ Green's functions represented by matrices. For the thermodynamic potential we have

$$\Omega_{\rm int} = \frac{i}{2}\ T\ g^2 \int d\tau_1\ d\tau_2\ d^2r_1\ d^2r_2\ \hat{G}(\mathbf{r}_1,\mathbf{r}_2,\tau_1-\tau_2) \times\hat{\sigma}_x\ \hat{A}(\mathbf{r}_1,\tau_1;\mathbf{r}_2,\tau_2)\ \hat{\sigma}_x\ \hat{G}(\mathbf{r}_2,\mathbf{r}_1,\tau_2-\tau_1), \tag{2}$$

where $\hat{\sigma}_x$ is the Pauli matrix and we have introduced

$$\hat{A}(\mathbf{r}_1,\tau_1;\mathbf{r}_2,\tau_2)=\int dz_1\,dz_2\,\hat{D}(\mathbf{r}_1-\mathbf{r}_2,z_1-z_2,\tau_1-\tau_2) \tag{3}$$

(we assume magnetic field $B$ along the $z$ axis and Landau gauge for vector-potential: $A_y = Bx$). Then we perform the required Fourier transformations over spatial and temporal variables to express the thermodynamic potential (2) in terms of well-known electron and phonon Green's functions in the momentum-frequency representation. Expanding the electron Green's functions over the eigenfunctions of the electron in a magnetic field we find after straightforward summations and integrations

$$\Omega_{\rm int}=\frac{g^2}{32\pi^2}\frac{eB}{c}\left(\frac{\omega_c}{u}\right)^3\beta\sum_{n=1}^{\infty}\frac{1}{n}\left[\tanh\left(\frac{\epsilon_n-\mu}{2T}\right)-\tanh\left(\frac{\epsilon_{n-1}-\mu}{2T}\right)\right]\times\int_0^{\pi/2}d\phi\,\cos^3\phi\,e^{-\beta\cos^2\phi}\left[L_{n-1}^1\left(\beta\cos^2\phi\right)\right]^2. \tag{4}$$

Here $l_H = (c/eB)^{1/2}$ is the magnetic length, $\beta = \omega_c/mu^2$ ($u$ is the sound velocity), and $L(x)$ are Laguerre polynomials. When deriving Eq. (4) we have taken into account the interaction of electrons with acoustic longitudinal phonons in the context of the Debye model. We have introduced two more assumptions to get Eq. (4). First, we adopted that $q_D l_H \gg 1$ ($q_D$ is the Debye momentum) to expand the upper limit of momentum integration to infinity. Second, for further simplifications we assume also that only electron transitions between neighboring Landau levels are essential, which is justified in sufficiently high magnetic field $\omega_c \gg T$. The consequences of these restrictions for the limits of validity of the theory proposed will be discussed below.

As it was mentioned above, transverse components of kinetic coefficients in the presence of a quantized magnetic field may be expressed through the thermodynamic potential. We begin from the phonon contribution to the Hall component of the electrical conductivity tensor, which may be obtained in terms of the thermodynamic potential (4) as

$$\Delta\sigma_{xy}=\frac{ec}{B}\left(\frac{\partial\Omega_{\rm int}}{\partial\mu}\right)_{T,B}=\frac{g^2e^2}{64\pi^2T}\left(\frac{\omega_c}{u}\right)^3\times\beta\,\cosh^{-2}\left(\frac{\omega_c(N+1/2)-\mu}{2T}\right)I_N(\beta), \tag{5}$$

where the number of Landau levels $N$ is such that $|\,\omega_c(N+1/2)-\mu\,|\leq\omega_c/2$, and we define

$$I_N(\beta)=\int_0^{\pi/2}d\phi\cos^3\phi e^{-\beta\cos^2\phi}\left[\frac{1}{N+1}\{L_N^1\left(\beta\cos^2\phi\right)\}^2-\frac{1}{N}\{L_{N-1}^1\left(\beta\cos^2\phi\right)\}^2\right]. \tag{6}$$

Note that for $N = 0$ the second term in square brackets in Eq. (5) has to be omitted. One can evaluate the value of the integral $I_N(\beta)$ in the high-field approximation $\beta \gg 1$ by expansion in powers of the parameter $\beta^{-1}$. By means of the standard relations with the Laguerre polynomials we have

$$I_0=\frac{1}{2\beta^2},\qquad I_{N\geq1}=\frac{1}{2\beta^3}. \tag{7}$$

Equations (5) and (7) define the correction to the Hall conductivity due to electron-phonon interaction. It is seen that this correction has a series of maxima near Landau-level positions. It is worth mentioning that the height of the peak near the zero Landau level varies quadratically in magnetic field while others peak heights vary linearly.

For off-diagonal component of the thermoelectric coefficient $\hat{\beta}$ we obtain

$$\Delta\beta_{xy}=-\frac{c}{B}\left(\frac{\partial\Omega_{\rm int}}{\partial T}\right)_{\mu,B}=-\frac{eg^2}{64\pi^2T^2}\left(\frac{\omega_c}{u}\right)^3\beta\,\frac{\omega_c(N+1/2)-\mu}{\cosh^2\left(\frac{\omega_c(N+1/2)-\mu}{2T}\right)}\,I_N(\beta). \tag{8}$$

For transverse heat-conductivity we find

$$\Delta\kappa_{xy}=-\frac{cT}{eB}\int_{-\infty}^{\mu}d\mu'\left(\frac{\partial^2}{\partial T^2}\right)_{\mu',B}\Omega_{\rm int}(\mu',T)=-\frac{g^2}{64\pi^2T^2}\left(\frac{\omega_c}{u}\right)^3\beta\,\frac{[\omega_c(N+1/2)-\mu]^2}{\cosh^2\left(\frac{\omega_c(N+1/2)-\mu}{2T}\right)}\,I_N(\beta). \tag{9}$$

Dependencies of corrections to the transverse thermoelectric coefficient and heat conductivity on the chemical potential also have the shape of sets of peaks localized near the Landau-level positions. However the shape of these peaks differs considerably from those of the Hall conductivity, indicating that the Wiedemann-Franz law is violated in the case under consideration. Dependencies of the corrections (5) and (9) on chemical potential are shown in Fig. 1.

Now we have to compare the magnitude of the corrections to the electrical and heat conductivities (5) and (9) obtained above with the free-electron and disorder-induced conductivities. (A problem of thermopower is more complicated and is discussed separately.) For

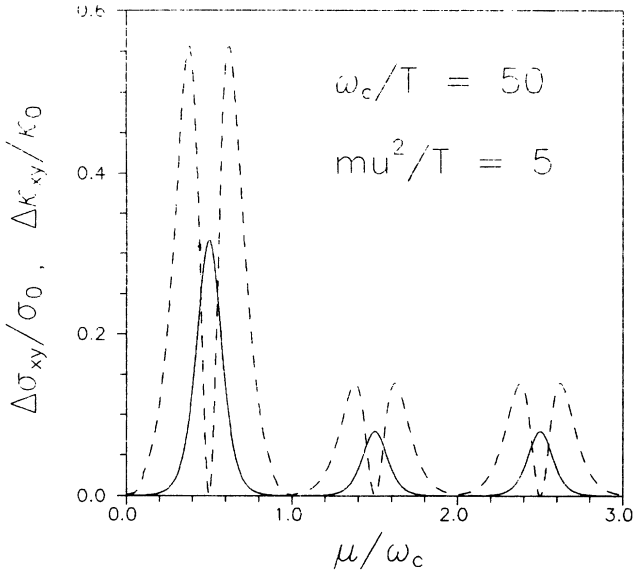


FIG. 1. Correction to off-diagonal components of electrical conductivity (solid line) and heat conductivity (dashed line) tensors as functions of chemical potential ($\sigma_0 = e^2g^2mT/u$, $\kappa_0 = g^2mT^2/u$).

this purpose we estimate the value of the electron-phonon coupling constant as[12] $g^2 \sim (e^2a^4Z^2\rho)/(Mu^2) \sim (av_F^2)/(Mu^2)$, where $a$ is the lattice constant and $Z$, $\rho$, and $M$ are the valence, density, and mass of ions, respectively. Comparing the maximal value of the correction to the Hall conductivity (5) for the zero Landau level with zero-temperature value of the free-electron Hall conductivity $\sigma_{xy}^0 = e^2/2\pi\hbar$ we obtain

$$\frac{\sigma_{xy}^{e\text{-}ph}(N=0)}{\sigma_{xy}^0} \approx 10^{-2}\,\frac{\omega_c}{T}\,\frac{\omega_c}{\omega_D}. \tag{10}$$

Here we introduce Debye frequency $\omega_D$ and used the relation $Mu^2 \approx mv_F^2$. We see that the correction due to the electron-phonon interaction may be essential in the case of low temperatures and rather strong fields. At field $B \approx 5$ T correction (5) may reach 10% of free-electron conductivity at temperatures $T \leq 1$ K. In other words, the electron-phonon interaction may lead to essential distortions of the Hall conductivity dependence in the vicinity of the Landau levels (near the steps). Naturally, it does not affect the plateau height far from steps. Note that, as it was shown above, the height of the second and others peaks in the phonon corrections to the Hall conductivity are linear in magnetic field. For those we have an estimation,

$$\frac{\sigma_{xy}^{e\text{-}ph}(N>0)}{\sigma_{xy}^0} \approx 10^{-2}\,\frac{\omega_c}{T}\,\frac{mu^2}{\omega_D} \ll \frac{\sigma_{xy}^{e\text{-}ph}(N=0)}{\sigma_{xy}^0}. \tag{11}$$

So an effect of the electron-phonon interaction on other Landau levels is that it is weaker than for the zero Landau level. Similar estimations for the heat conductivity (9) lead to the same results.

Let us now turn to the correction to the thermopower tensor $\hat{S} = -\hat{\beta}\hat{\sigma}^{-1}$. As it is seen from Eqs. (10) and (11) the Hall conductivity for $N \geq 1$ may be assumed simply to be $Ne^2/2\pi\hbar$, and we have Eq. (8) divided by this factor:

$$S_{xx}^{e\text{-}ph}(N \geq 1) \approx -\frac{\beta_{xy}^{e\text{-}ph}}{\sigma_{xy}^0} \approx \frac{1}{16\pi eN}\,\frac{g^2m}{u}\,\frac{\omega_c^2}{T}\,\frac{\omega_c/2-\mu}{\cosh^2\left(\frac{\omega_c/2-\mu}{2T}\right)}, \tag{12}$$

where we have taken into account the fact that the diagonal component of $\hat{\beta}$ due to electron-phonon interaction is much smaller than the off-diagonal one. One can see that only the diagonal component of tensor $\hat{S}$ exists under our assumptions. Estimation (12), however, is not valid for $N = 0$ and this case should be treated separately with the temperature dependence of the Hall conductivity being taken into account. As a result we obtain

$$S_{xx}^{e\text{-}ph}(N=0) = \frac{g^2}{8\pi eT^2}\,\frac{m\omega_c^2}{u}\,\frac{\omega_c/2-\mu}{1+\exp\left(-\frac{\omega_c/2-\mu}{2T}\right)}. \tag{13}$$

Dependence of the thermopower diagonal component on the chemical potential is shown in Fig. 2.

A direct comparison with the disorder-induced contributions to the thermopower[4] gives finally the same estimations (10) and (11) that take place for the conductivities. One circumstance is remarkable, however. The diagonal thermopower is very sensitive to the presence of disorder,[4] contrary to off-diagonal electrical and heat conductivities, where disorder results in small corrections of order $(\omega_c\tau)^{-2}$ ($\tau$ being the electron-impurity relaxation time). So the case of strong disorder ($\hbar/\tau \gg T$) needs a separate treatment. As it is known,[2] for this case the thermopower increases in comparison with the

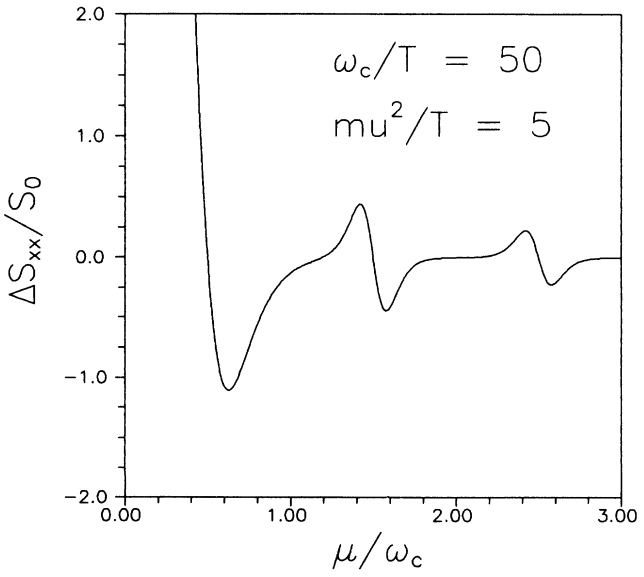


FIG. 2. Dependence of correction to diagonal component of thermopower tensor on the chemical potential ($S_0 = -g^2mT/ue$).

clean limit by a factor $\sim \hbar/\tau T$ and hence the ratio of the electron-phonon correction to the thermopower diagonal component and the value due to electron-impurity scattering is $10^{-2}\tau\omega_c^2/\hbar\omega_D$ (for $N = 0$). As a result one can see that the relative magnitude of the maximum in the thermopower dependence on the chemical potential may become comparable with the maximum of the impurity-induced thermopower even for $T \sim 3$ K when magnetic field is of order of several T.

When the above-discussed results were obtained, we introduced a number of assumptions: $\omega_c \gg T$, $q_D l_H \gg 1$, and $\omega_c \gg mu^2$. This set of assumptions restricts the magnetic field both from the bottom and from the top. The first one is also the restriction on temperature and may be easily satisfied for strong magnetic fields ($\sim 5-20$ T) and for rather low temperatures. Two other conditions may be satisfied also for this range of magnetic fields, because under real experimental conditions $ms^2 < 0.1$ K while $q_D^{-1} < 100$ Å. So all these conditions may be satisfied simultaneously. Note also that the condition $\omega_c\tau \gg 1$ is needed for the possibility of QHE observation. In the case when impurity scattering is negligible the finite electron state lifetime $\tau$ is connected with the electron-phonon interaction. Since for rather low temperatures electron-phonon relaxation time $\tau_{e\text{-ph}} \sim T^{-1}$,[13] the condition in question reduces to a form $\omega_c \gg T$, which was already discussed.

Two more points require comment. First, as it was mentioned, the screening of the phonon field was assumed to be full in calculations above, which is not valid for semiconductor structures. The second is that the free phonon Green's function was employed in the calculations above. This means that phonon drag effects are omitted. These effects can essentially affect the thermopower at low temperatures (in zero magnetic field at least) and require special consideration. However, it is believed that the main conclusion of the present paper—strong phonon renormalization of kinetic coefficients in the vicinity of the Landau-level positions—retains its validity for a more widespread spectrum of models than was assumed in the consideration above. Note that the importance of electron-phonon interaction effects on thermopower in the quantum-Hall-effect regime was pointed out in Refs. 7 and 8.

In conclusion, we have shown that the effect of electron-phonon interaction on transport properties in the quantum-Hall-effect regime may be essential in a rather wide range of temperatures and magnetic fields. For transverse components of electric and heat conductivities tensors this effect leads to deviations from "ideal" free-electron behavior near Landau-level positions, but does not affect the height of the plateau. On the contrary, for diagonal thermopower, correction due to electron-phonon interaction may dominate impure and free-electron thermopower for rather strong magnetic fields and low temperatures. Corrections due to electron-phonon interaction may be observed experimentally against the background of effects of impurity disorder with respect to the characteristic magnetic field dependence.

* Electronic address: blanter@s43.msk.su

[1] *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1987).
[2] H. Oji, Phys. Rev. B **29**, 3148 (1984); J. Phys. C **17**, 3059 (1984).
[3] Ya. M. Blanter, D. V. Livanov, and M. O. Rodin (unpublished).
[4] M. Jonson and S. M. Girvin, Phys. Rev. B **29**, 1939 (1984).
[5] H. Obloh, K. von Klitzing, and K. Ploog, Surf. Sci. **142**, 236 (1984).
[6] R. P. Smith, H. Closs, and P. J. Stiles, Surf. Sci. **142**, 246 (1984).
[7] J. S. Davidson, E. Dan Dahlberg, A. J. Valois, and G. Y. Robinson, Phys. Rev. B **33**, 8238 (1986).
[8] M. R. Cimberle, C. Ferdeghini, M. Putti, and A. S. Siri, Int. J. Mod. Phys. B **3**, 327 (1989).
[9] Yu. N. Obraztsov, Fiz. Tverd. Tela (Leningrad) **6**, 414 (1964) [Sov. Phys. Solid State **6**, 331 (1964)].
[10] H. Oji and P. Středa, Phys. Rev. B **31**, 7291 (1985).
[11] P. S. Zyryanov and V. P. Silin, Fiz. Met. Metalloved. **17**, 934 (1964) [Phys. Met. Metallogr. **17**, 130 (1964)].
[12] A. A. Abrikosov, L. P. Gor'kov, and I. Ye. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics* (Pergamon Press, New York, 1965).
[13] B. L. Altshuler, Zh. Eksp. Teor. Fiz. **75**, 1330 (1978) [Sov. Phys. JETP **48**, 610 (1978)].