Electron correlations in a double-quantum-well structure

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The electron correlation effect in a double-quantum-well structure is investigated. The local-field correction and static structural factors are calculated for these structures. It is found that the static structural factors show an anomalous behavior at small q due to the coupled quantum-well-plasma excitation and the interlayer structural factor is negative at large q. We also found that the long-wavelength acoustic plasma mode is further softened due to short-range electron correlations. The minimum critical well separation for this mode to exist increases by an amount $\sim 1/k_{f1}$, where k_{f1} is the Fermi wave vector of the well with higher electron concentration.

Recently, there has been considerable interest, both theoretically and experimentally, on the coupling effects of two isolated, parallel quasi-two-dimensional electron gases.[1-9] For the simplest such structure, namely, the double quantum well (DQW), recent experiments[1] have suggested that interwell interaction can dramatically alter the single-electron levels in samples with thin tunneling barriers. Even in a DQW with negligible tunneling, fractional filling states in the extreme quantum limit have been investigated based on the interlayer electron-electron interaction.[2,3] Such direct electron interaction is also believed to be the dominant mechanism responsible for the interlayer momentum relaxation.[7-9] While the random-phase approximation (RPA) is generally satisfactory in these structures, the deviation from the RPA can sometimes be quite significant. In this paper, we shall study the short-range correlation effect in such a DQW structure. We are mainly interested in the correlation effect on those physical properties related to the interwell coupling. For example, the high-frequency acoustic plasma mode. Our result shows that the existence of this mode depends on the correlation effect very significantly. On the other hand, the dielectric response of the coupled system also affects the intrawell static structural factor. The formalism used here was developed by Singwi *et al.*[10] and was later generalized to the two-dimensional system[11] and the layered superlattice structure.[12] The correlation effect in the presence of a perpendicular magnetic field has also been investigated recently.[13]

The system we are considering is a double-quantum-well structure, where electrons are confined in either left or right quantum wells separated by a barrier region. Let well 1 have electrons of density n_1 and mass m_1 and well 2 have electrons of density n_2 and mass m_2 on layer 2. Our main concern is the intrawell and interwell short-range correlation effects among the mobile electrons. Therefore we shall adopt a simplified model with the following approximations: (i) the interwell tunneling shall be neglected and the only interwell coupling between the electrons on the different layer is the dynamically screened Coulomb interaction; (ii) the finite spread of the electronic wave function within the well shall be neglected, i.e., we will restrict ourselves to the case where the Fermi energy on either layer is smaller compared to the subband spacing and only the lowest subband is partially occupied. In this case, the electronic wave function on each well can be approximated as $\phi(\mathbf{p},\mathbf{r},z)=e^{i\mathbf{p}\cdot\mathbf{r}}\xi(z-z_{1,2})$, where $\xi(z)$ is defined in such a way that it gives δ-function-like distribution, $|\xi(z)|^2=\delta(z)$. Here $\mathbf{p},\mathbf{r}$ are, respectively, the two-dimensional momentum and position vector along the plane, $z_1=0$ and $z_2=d$. The Hamiltonian of our two-layer electronic system is given as

$$H=H_0+V\ , \tag{1}$$

where

$$H_0=\sum_{\mathbf{p}}\sum_{i=1,2}E_{p,i}a^{\dagger}_{\mathbf{p},i}a_{\mathbf{p},i} \tag{2}$$

and V consists of electron-electron interactions, given by

$$V=\tfrac{1}{2}\sum_{\mathbf{p},\mathbf{p}',\mathbf{q}}\sum_{i,j=1,2}V_q(ij)a^{\dagger}_{\mathbf{p}+\mathbf{q},i}a^{\dagger}_{\mathbf{p}'-\mathbf{q},j}a_{\mathbf{p}',j}a_{\mathbf{p},i}\ . \tag{3}$$

Here $E_{p,i}=p^2/2m_i$ is the kinetic energy of an electron having momentum $\mathbf{p}$. $a^{\dagger}_{\mathbf{p},l},a_{\mathbf{p},l}$ represent, respectively, the electron creation and destruction operators with momentum $\mathbf{p}$. The coupling term $V_q(ij)$ is the Fourier transform of the Coulomb interaction for planar electrons,

$$V_q(11)=V_q(22)=\frac{2\pi e^2}{q},\quad V_q(12)=V_q(11)e^{-2qd}\ .$$

The density operator is given as

$$\rho_i(q)=\sum_{\mathbf{p}}a^{\dagger}_{\mathbf{p}+\mathbf{q},i}a_{\mathbf{p},i}\quad(i=1,2)\ .$$

Let us consider the second time derivative of the density operator

$$\frac{\partial^2\rho_i(q)}{\partial t^2}=-\sum_{\mathbf{p}}\left[\frac{\mathbf{p}\cdot\mathbf{q}}{m_i}+\frac{q^2}{2m_i}\right]^2 a^{\dagger}_{\mathbf{p}+\mathbf{q},i}a_{\mathbf{p},i}$$
$$+\frac{q^2n_i}{m_i}\sum_j V_{ij}(q)\rho_j(q)$$
$$+\frac{1}{m_i}\sum_{\mathbf{q}'\neq\mathbf{q},j}V_{ij}(q)\mathbf{q}\cdot\mathbf{q}'\rho_j(q')\rho_i(\mathbf{q}-\mathbf{q}')\ . \tag{4}$$

We now employ the following approximation,[10,12] in which the summation over the electron coordinates is replaced by its static average, i.e.,

$$\begin{aligned}\rho_j(q')\rho_i(\mathbf{q}-\mathbf{q}') &= \sum_{\mu} \exp[i\mathbf{q}\cdot\mathbf{r}_j(\mu)] \sum_{\nu} \exp\{i(\mathbf{q}-\mathbf{q}')\cdot[\mathbf{r}_j(\mu)-\mathbf{r}_i(\nu)]\} \\ &= \sum_{\mu} \exp[i\mathbf{q}\cdot\mathbf{r}_j(\mu)] \frac{1}{n_i} \left\langle \sum_{\nu} \exp\{i(\mathbf{q}-\mathbf{q}')\cdot[\mathbf{r}_j(\mu)-\mathbf{r}_i(\nu)]\} \right\rangle \\ &= \sum_{\mu} \exp[i\mathbf{q}\cdot\mathbf{r}_j(\mu)] S_{ij}(\mathbf{q}-\mathbf{q}') , \end{aligned} \tag{5}$$

where $\mathbf{r}_i(\mu)$ denotes the position of μth electron on layer i and $S(q)$ is the static structural factor. Equation (4) can now be written as

$$\frac{\partial^2 \rho_i(q)}{\partial t^2} = -\sum_{\mathbf{p}} \left[\frac{\mathbf{p}\cdot\mathbf{q}}{m_i} + \frac{q^2}{2m_i} \right]^2 a^{\dagger}_{\mathbf{p}+\mathbf{q},i} a_{\mathbf{p},i} + \frac{q^2 n_i}{m_i} \sum_j U_{ij}(q)\rho_j(q) , \tag{6}$$

where the effective potential U_{ij} is given as

$$U_{ij}(q) = V_{ij}(q)[1-G_{ij}(q)] . \tag{7}$$

The local-field correction G_{ij} is

$$G_{ij}(q) = \frac{1}{n_i} \int \frac{d^2q}{(2\pi)^2} \frac{\mathbf{q}\cdot\mathbf{q}'}{\mathbf{q}^2} V_{ij}(q')[S_{ij}(\mathbf{q}-\mathbf{q}')-\delta_{ij}] . \tag{8}$$

The static structural factor is also related to the dynamical dielectric-response function $\epsilon(q,\omega)$ through the fluctuation-dissipation theorem, which has the following form for a two-dimensional two-component electronic system:

$$S_{ij}(q) = -\frac{\hbar}{2\pi^2 e^2 n_i} \int_0^{\infty} d\omega \coth\left[\frac{\beta\omega}{2}\right] \mathrm{Im}[\epsilon_{ij}^{-1}(q,\omega)] . \tag{9}$$

The dielectric function is a 2×2 matrix which can be obtained by considering the density fluctuation in an external potential,

$$\rho_i(q) = \sum_j \epsilon_{ij}^{-1}(q,\omega) Q_j(q,\omega) V_j^{\mathrm{ext}}(q,\omega) ,$$

where Q_i is the electronic polarizability in the absence of electron-electron interaction. Explicitly, each component of ϵ_{ij} and its inverse can be written as

$$\begin{aligned} &\epsilon_{ij} = \delta_{ij} - U_{ij} Q_i , \\ &\epsilon_{ij}^{-1} = \epsilon_{jj}/D, \quad \epsilon_{ij}^{-1} = -\epsilon_{ji}/D \quad (i \neq j) , \end{aligned} \tag{10}$$

where $D = \det|\epsilon_{ij}|$, whose roots determine the dispersion of collective excitations in a two-component system. The long wavelength collective excitation modes for such systems can be written as

$$\omega_{\pm}^2 = \frac{q}{2} \left[\frac{r_{s1}k_{f1}^3}{m_1^2}[1-G_{11}(q)] + \frac{r_{s2}k_{f2}^3}{m_2^2}[1-G_{22}(q)] \right] [1 \pm \sqrt{w}] ,$$

$$w = 1 - \frac{4r_{s1}r_{s2}k_{f1}k_{f2}v_{f1}^2 v_{f2}^2 P - 3q[(1-G_{11})r_{s1}k_{f1}v_{f1}^4 + (1-G_{22})r_{s2}k_{f2}v_{f2}^4]}{(1-G_{11})r_{s1}v_{f1}^2 k_{f1} + (1-G_{22})r_{s2}v_{f2}^2 k_{f2}} ,$$

$$P = [(1-G_{11})(1-G_{22}) - (1-G_{12})^2 \exp(-2qd)] ,$$

where $r_{si} = m_i e^2/\hbar^2 k_{fi}$ is the plasma parameter for the ith layer and v_f is the Fermi velocity.

When evaluating S_{ij}, there are contributions due to both particle-hole continuum and plasma excitation,

$$\mathrm{Im}[\epsilon_{ij}^{-1}] = (\epsilon_{ij}^I D^R - \epsilon_{ij}^R D^I)/|D|^2 + \pi\epsilon_{ij}^R \delta(D^R) ,$$

where the superscript R (I) denotes the real (imaginary) part. The static structural factor can be written as $S_{ij} = S_{ij}^{\mathrm{ph}} + S_{ij}^{\mathrm{pl}}$. The coupled Eqs. (7)–(10) were solved by iteration. The results for the static structural factors for some sample parameters are shown in Fig. 1. There are several interesting features in $S_{ij}(q)$. At low q, both particle hole and plasma excitation contribute to S_{ij}. The contribution due to the plasma excitation is only nonzero when $\omega_{\pm} > qv_f + q^2/2m$ (for $n_1 > n_2$). In a single-component system, usually one observes a smooth disappearance $S_{ij}^{\mathrm{pl}}(q)$. However, for a two-component system, $S_{ij}(q)$ exhibit a sharp cutoff. This behavior can be understood as follows. (i) The particle-hole continuum boundary for each layer remains unchanged when the two-layers coupled together, but the plasma frequency is enhanced by a factor around $\sqrt{2}$ (for $n_1 \approx n_2$). Therefore S_{ij}^{pl} is also enhanced. But this enhanced contribution should still be cut at the unchanged continuum boundary. (ii) Furthermore, the plasma contribution is weighted by the matrix elements ϵ_{ij}^R which has a zero (i.e., has a sign change) around the frequency corresponding to the single-layer plasma excitation. As a consequence of (i) and (ii), the merge of plasma frequency with the continuum results in a sharp drop in S_{ij}. It is also interesting to note that the interlayer component S_{12} does not have a definite sign. At small q, S_{12}^{pb} is positive but S_{12}^{pl} is negative. There is only a small regime immediately after the

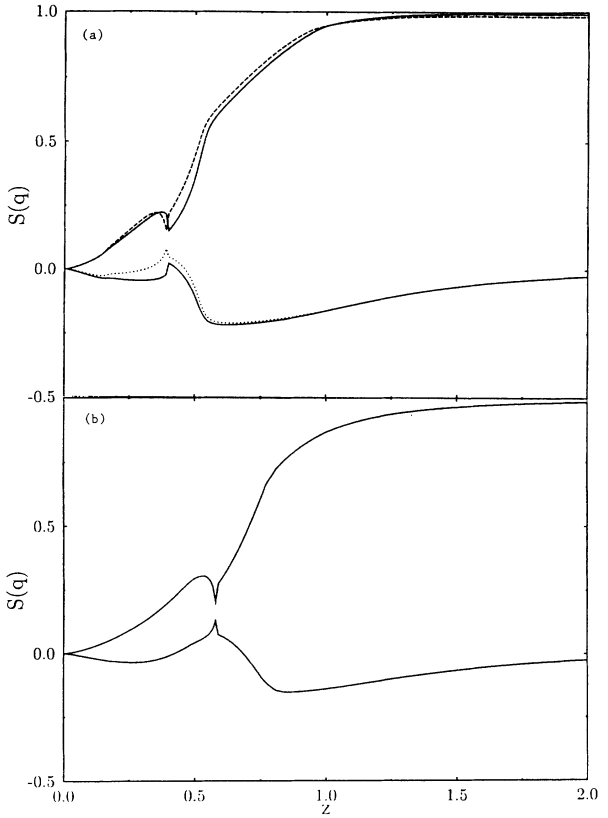

FIG. 1. Plots of the static structural factor for a double-well structure as a function of $z=q/2k_{f1}$ for $m_1=m_2$ and $k_{f1}d=1.0$. (a) $r_{s1}=1.0$, $n_1/n_2=1.1$, the solid lines are for S_{11} and S_{12}, the broken line is for S_{22}, and the dotted line is for S_{21}. (b) $r_{s1}=2.0$, $n_1/n_2=1.0$, the upper curve is for S_{11}, and the lower curve is for S_{12}.

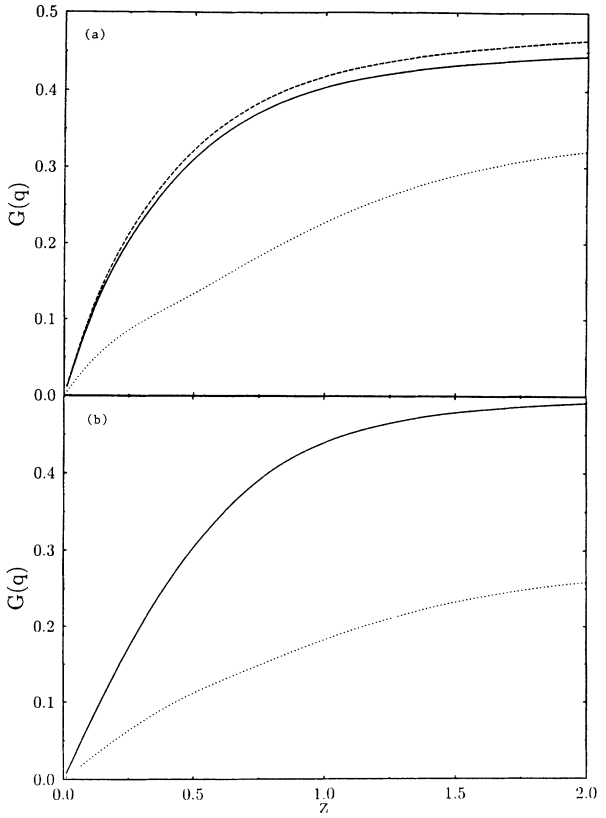

FIG. 2. Plots of the local-field correction for a double-well structure as a function of $z=q/2k_{f1}$ for $m_1=m_2$ and $k_{f1}d=1.0$. (a) $r_{s1}=1.0$, $n_1/n_2=1.1$, where the broken line is for G_{22}, the solid line is for G_{11}, and the dotted line is for G_{12}. (b) $r_{s1}=2.0$, $n_1/n_2=1.0$, where the solid line is for G_{11} and the dotted line is for G_{12}.

disappearance of $S_{12}^{\rm pl}$ where S_{12} is positive. At large q, S_{12} is negative and approaches zero asymptotically.

In Fig. 2, we present the result of local-field correction G_{ij}. All components are linear in q at small q. The intralayer components approach $\frac{1}{2}$ at large q and the lower-density layer has a larger G. The interlayer component ($G_{12}=G_{21}$) has a much smaller value due to the spatial separation. Since the layer separation is fixed, G_{12} will not approach $\frac{1}{2}$ asymptotically. When the density is reduced, the G increases as expected but the change becomes less noticeable if q is scaled in k_f. Finally, we discuss the effect of correlation on the plasma excitation in a double-well structure. It is known that there can be two plasmon modes in the two-layer two-component system. The high-frequency mode is $\sim\sqrt{q}$ at low q and is free of Landau damping. The effect of correlation on this mode is to bend it downward with a factor $[1-G_{ii}(q)]$ at finite q. The lower-frequency mode ($\omega_-\sim q$) can only be undamped as the layer separation exceeds a critical value.[14] If one neglects the short-range correlation effect, the critical separation is given by

$$d_c=\frac{1}{8}\left[\frac{7v_{f1}^2m_2}{r_{s2}v_{f2}}+m_1\frac{4v_{f1}^2+2v_{f2}^2}{r_{s1}v_{f1}}\right].$$

We now consider the correlation effect on this mode. The leading-order terms in all G_{ij} are linear in q, i.e., $G_{ij}(q)\approx A_{ij}q$. The factor P in ω_- can be approximated as $2[d+A_{12}-(A_{11}+A_{22})/2]q$. Therefore, the critical separation changes to $d_c'=d_c+(A_{11}+A_{22})/2-A_{12}$. Though the value of A's depends on the sample parameter, A_{ii} is always greater that A_{ij}. For $k_{f1}d=1$, $\Delta d_c\approx k_f^{-1}$, which is about 10^{-6} cm for typical sample parameters. While it is difficult to detect directly the short-range correlation effect, measurement of this large change in the critical separation should be experimentally feasible.

In conclusion, we have investigated the short-range correlation effect in a double-layer structure. The static structural factors and the local-field corrections have been calculated. Some features (e.g., low-q behavior of S_{ij} and ω_-) may be investigated under a suitable experimental situation.

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