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Reflection of ballistic electrons from diffusive regions

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We have investigated reflection of edge states from the boundary between a disordered region and a high-mobility region of a two-dimensional electron gas. α-particle bombardment has been employed to reduce selectively the mobility. We observe an exponential increase in reflection of the innermost edge state from the disordered region as the temperature decreases. The results are explained in terms of the Milne reflection of electrons which are elastically scattered back before thermalization can occur.

The Landauer-Buttiker formalism provides a powerful method of characterizing conduction processes in a quantum conductor. Although the formalism is widely accepted, it involves a number of subtle points which have recently been extensively discussed by Landauer.[1] The conductor is conceptually divided between regions of nondissipative transport in the bulk of the conductor and contact regions ("thermal reservoirs" acting as an electronic "black body") where electrons are completely thermalized.[1,2] The boundary between these two regions is not clearly defined in real experiments and may depend on external parameters such as temperature and magnetic field. As the temperature decreases, larger parts of the system must be considered as part of the quantum conductor and the effective positions of thermal reservoirs change. This change in geometry of the conductor may dramatically influence the resistance measurements.[3,4]

In this paper we report an experiment which addresses the problem of Landauer thermal reservoirs and clearly demonstrates the fundamental importance of temperature in quantum resistance measurements. We have employed two-dimensional electron gas (2DEG) devices with long and narrow diffusive leads where the mobility was reduced with respect to the rest of the conductor by He-ion bombardment. Measurements were carried out in the quantum Hall effect (QHE) regime where the influence of the disordered contacts is dramatic. The devices exhibit a rapid quenching of the high-field Shubnikov–de Haas oscillations (SdHO) and a significant shrinking of the width the quantum Hall plateaus as the temperature decreases, in contrast to the behavior of conventional QHE devices and devices with unimplanted leads. We explain the results by changes in the effective geometry of the conductor as temperature varies. At high temperatures, electrons entering the disordered region from the edge states in the high-mobility region are able to thermalize rapidly and the implanted areas act as the reservoirs. At low temperatures the electrons are mostly backscattered by elastic collisions in the disordered regions before equilibration can occur. We present a model of the process which is based on the conventional model of adiabatic edge-state transport[4-6] and, in addition, which assumes that the effective position of the thermal reservoirs is determined by some thermalization (equilibration) length L_{th} of the edge states. We note that the experiment is closely related to the Milne problem, known in astrophysics and neutron physics,[7] where ballistic particles penetrating into a strongly scattering medium are mainly reflected back without being thermalized if the inelastic scattering length l_i is much longer than the elastic length l_e.

Three types of devices have been used in our experiment. Hall bar structures with several voltage probes were fabricated using electron-beam lithography and photolithography from a modulation-doped GaAs/(AlGa)As heterostructure with 2D electron concentration $\cong 4.5\times10^{15}$ m^{-2} and mobility $\mu \cong 20$ m^2/V s. The devices differ in the fabrication of the contact leads as shown schematically in the insets to Fig. 1. The first two types of Hall bar [Figs. 1(a), and 1(b); insets] have a conducting width $w \cong 1$ μm of all 2DEG sections and the distance between adjacent voltage probes is between 10 and 20 μm. All contact leads have a relatively long length of $\cong 50$ μm terminated by conventional AuGe contact pads. Type-A devices [see Fig. 1(a)] were exposed to bombardment by 50-keV α particles of a total dose 3×10^{14} m^{-2} with the central part of the device, including the first 5 μm of the contact leads, protected by a thick metal mask. The bombardment dramatically reduced the mobility of the exposed 2DEG (the measured 2DEG mobility of an unmasked device is $\mu_{\text{im}} \simeq 2$m^2/V s) while the electron concentration remained practically unchanged.[8] Details of the He-ion implantation procedure and characteristics of implanted 2DEG's can be found elsewhere.[9] We refer to unimplanted Hall bars of the same geometry as type-B devices. For completeness, we have also fabricated control samples (type C) from the same heterostructure with the contact lead geometry as shown in Fig. 1(c) (inset). The width of the leads rapidly increases from $\cong 1$ to 10 μm where they are terminated by the alloyed contact pads.

In Fig. 1(a) the longitudinal magnetoresistance R_{xx} of a type-A device is shown for three different temperatures. It is clearly seen that, as temperature decreases from 5 to 0.3 K, the amplitude of SdH oscillations above 3 T decreases considerably and the last peak in R_{xx} at about 13 T ($\nu=1.5$) virtually disappears. The quenching of R_{xx} at

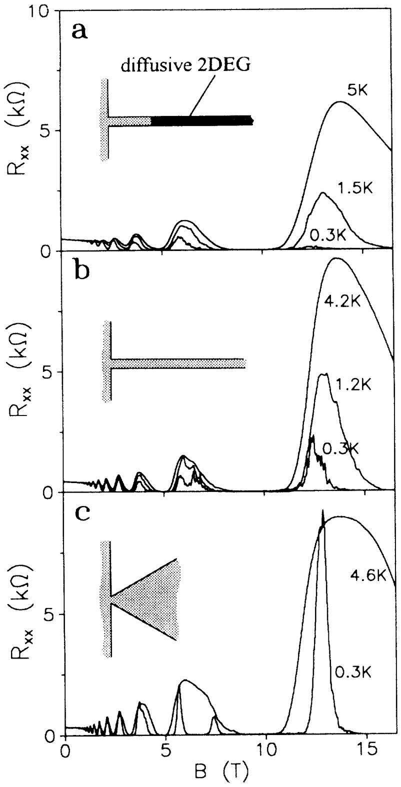

FIG. 1. Longitudinal magnetoresistance at different temperatures in QHE devices with various geometries of contact leads as shown in the insets.

low temperatures is in stark contrast to the usual behavior of QHE devices[4,10] and to theory (see, e.g., Ref. 11) which both give nearly constant amplitude of Shubnikov–de Haas peaks as temperature decreases. Figure 1(c) shows the behavior of R_{xx} in the control device. The high-field SdH peaks become narrower at low temperatures but their amplitudes remain nearly constant as temperature decreases. Type-*B* devices exhibit an intermediate behavior as shown in Fig. 1(b) and the peak at $\nu=1.5$ decreases by a factor of 4 with decreasing temperature from 4.2 to 0.3 K. The amplitude of this peak is found to remain nearly constant below 0.6 K. Note that two other peaks at about 3 and 7 T in samples *A* and *B* [Figs. 1(a) and 1(b)] are also suppressed as temperature decreases although the effect is not as pronounced as for the peak at 13 T. In addition, different degrees of spin-splitting of the peak at about 7 T make it rather difficult to compare the temperature dependences of R_{xx} for the two devices at this magnetic field. We note that in a number of previous experiments (see, e.g., Refs. 12 and 13) a similar behavior of the high-field SdHO has also been observed. This behavior has not been explained but may be understood in terms of the present work.

Figure 2 shows the Hall resistance R_{xy} in a type-*A* device at different temperatures. The width of the quantum Hall plateau at $\nu=2$ decreases at both high and low temperatures and the plateau is widest at about 5 K. The lower plateau at $\nu=4$ also has some tendency to shrink at low temperatures (not visible in the figure). This behavior is in contrast to the behavior of conventional QHE devices[4,10,11] and our control (*C*) samples where the plateaus are *always* wider at lower temperatures. Type-*B* devices exhibit a slight shrinking of the Hall plateau at $\nu=2$ at low temperatures. Note that for the *A* devices there is an interval of magnetic field, say from 10 to 12 T, where the longitudinal resistance goes to zero as temperature decreases while the Hall resistance goes up and away from its quantized value (compare Figs. 1 and 2).

Considering the behavior of R_{xx} with temperature we first address the temperature dependence in magnetic fields far away from the magnetoresistance maxima. In these fields the dominant effect is the narrowing of the R_{xx} peaks with decreasing temperature (θ). The changes in this regime are exponential and can be described for all the devices by the functional form $R_{xx} \propto \exp(-\alpha\theta^{-\beta})$ where α is a field-dependent constant. We find $\beta=0.5\pm0.2$ for all the devices and on both sides of the high-field magnetoresistance peak. This behavior is consistent with previous experiments on the QHE (Refs. 10 and 12–14) and with theory.[15] We avoid detailed considerations for this regime and concentrate on the behavior in the immediate vicinity of the SdH maximum. Figure 3 shows the temperature dependence of R_{xx} in the magnetic field of 13 T for all the devices. For *C* devices (crosses) the magnetoresistance changes are weak, corresponding to a nearly constant height of the R_{xx} peak. In contrast, for type-*A* devices (closed circles) the changes at the maximum are exponential. The best fit to the experimental data over the whole temperature interval in Fig. 3 yields the activation dependence $R_{xx} \propto \exp(-c/\theta)$ with $c \cong 2$ K which is shown by the solid line. The activation behavior also applies to $R_{xx}(\theta)$ in the immediate vicinity of the maxima (within $\cong 0.5$ T) although the coefficient c may vary slightly. We note that a similar temperature dependence was also reported in one of the early QHE experiments where quenching of the SdH peaks for $\nu<3$ was observed in a device with an unreported geometry of the contact leads.[12] Near the magnetoresistance peak type-*B* devices exhibit the initial decay which can be described by the same exponent as in the *A* device but the amplitude levels off at lower temperatures (see Fig. 3).

Our explanation is based on the known fact that the presence of scattering in the contact regions yields unequal populations of different in-going and out-going electron channels[16] and, at sufficiently low temperatures, this may even lead to complete decoupling of a channel

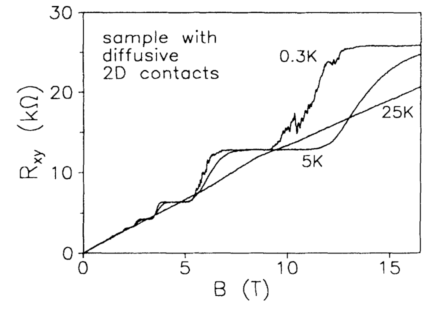

FIG. 2. The Hall resistance in a type-*A* device [see Fig. 1(a)] for three different temperatures.

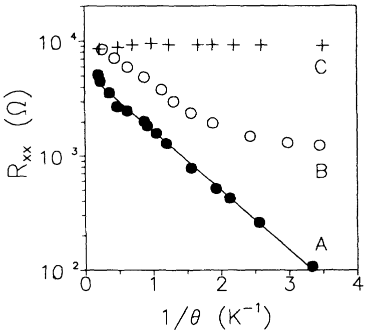

FIG. 3. Changes of the longitudinal magnetoresistance with temperature at 13 T for type-A (●), type-B (○), and type-C (+) devices. The solid line corresponds to $R_{xx} \propto \exp(-c/\theta)$.

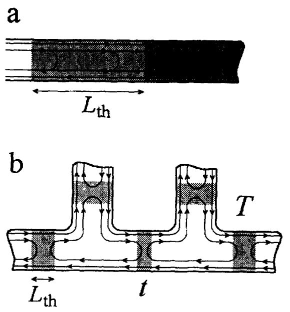

FIG. 4. (a) Schematic illustration of how the effective position of the thermal reservoirs varies with the equilibration length L_{th} (b) Model of the contacts and the edge-state reflection from the disordered regions in type-A devices.

from the effective thermal reservoirs. An electron entering the diffusive region may either be scattered back elastically or be thermalized in the lead which then effectively acts as a contact. As the temperature decreases, the inelastic scattering rate also decreases so that the electrons can spend more time in the lead before undergoing an inelastic event. This increases the probability of backscattering by involving a longer section of the lead in the process and effectively moving the position of the boundary between the quantum conductor and the contact further inside the contact leads [see Fig. 4(a)]. For an infinitely long lead and no inelastic scattering there would be 100% probability of backscattering (the full Milne reflection).

To describe the results we consider the case of adiabatic edge-state transport in high magnetic fields and a model of the contact shown schematically in Fig. 4(a). Only two occupied Landau levels are taken into account, corresponding to the experimental situation for the high-field peak at 13 T. Note that the edge states remain well defined in the disordered 2DEG regions, where $\mu_{\text{im}}B \gg 1$ at these magnetic fields. We assume that it is the innermost occupied edge state which is mostly influenced by the disorder and strongly scattered back by the low-mobility contact leads.[4–6] Referring to the qualitative considerations of the previous paragraph and Fig. 4(a) and to avoid an involved analysis for the distributed QHE networks,[6] we assume that the equilibration of the edge states occurs at a distance of $L_{\text{th}}(\theta)$ inside the disordered regions.[17] Following Ref. 5, we express the transmission probability T of the inneredge state into the thermal reservoir placed at the distance L_{th} as

$$T = 1/[1+\sigma_{xx}^{L}(h/e^2)(L_{\text{th}}/w)] , \tag{1}$$

where σ_{xx}^{L} is a parameter which can be interpreted as the electrical conductivity of the uppermost Landau level.[5] The temperature dependence of σ_{xx}^{L} is metallic near half-integer filling factors when the electron states are extended.[4,11,16] For other magnetic fields, it is expected that $\sigma_{xx}^{L} \propto \exp(-\alpha\theta^{-1/2})$.[14,15]

We model our type-A devices as shown in Fig. 4(b). The backscattering is described by the transition coefficient t in the central part and the coefficients T which are assumed to be the same for all the contact leads. For simplicity, we neglect the presence of other leads in our devices which may be important for a detailed description[16,18] but does not change our major conclusions. Following the standard procedure for QHE networks,[2,4,5] we obtain for the geometry in Fig. 4(b).

$$R_{xx} = h/e^2 \frac{T}{4} \frac{t(1-T)^2+2T(1-t)}{t(1+T^2)+T(1-t)} . \tag{2}$$

Note that when $T \ll 1$, the behavior of R_{xx} depends on this coefficient alone, i.e., $R_{xx} \propto T$ and the quantum resistance is determined by the properties of the contacts. For the A devices, the implantation in the contact regions implies a much larger value of σ_{xx}^{L} than that in the central part. This means that in the considered range of magnetic fields near the magnetoresistance maximum and at low temperatures, when L_{th} is not negligibly small, $T < t < 1$ [see Eq. (1)]. In this limit Eq. (2) can be simplified to $R_{xx} \cong h/e^2 T/4$. Therefore, the temperature dependence of R_{xx} in Fig. 3 indicates that the transmission coefficient T decays exponentially with decreasing temperature and the diffusive regions behave as nearly ideal reflectors of the edge states at low temperatures. Deviations from the straight line in Fig. 3 for the A device above 2 K (a more rapid increase in R_{xx}) probably indicate the transition to the case $t < T$, where $R_{xx} \simeq h/e^2 T/2$.

The anomalous behavior of R_{xy} in Fig. 2 can be easily understood within the same model. We consider the Hall cross with transition probabilities T into the contacts and neglect the backscattering in the small central section of the cross. The analysis for the Hall geometry gives

$$R_{xy} = h/e^2\{1-[T/(1+T^2)]\} . \tag{3}$$

When T equals unity, R_{xy} corresponds to its normal quantized value $h/2e^2$. When T goes to zero with decreasing temperature, R_{xy} increases away from $h/2e^2$ to h/e^2 in agreement with the experimental behavior.

The temperature dependence of the transmission coefficient is determined by both σ_{xx}^{L} and L_{th} [see Eq. (1)] which have competing temperature dependences, i.e., L_{th} generally increases with decreasing temperature while σ_{xx}^{L} decreases or remains constant. However, near the R_{xx} maxima, the changes in the conductivity σ_{xx}^{L} are slow and we infer from $R_{xx} \propto T$ and Eq. (1) that the thermalization length can be described by an activated behavior, i.e., $L_{\text{th}} \propto \exp(c/\theta)$. This dependence is in agreement with experiment, where the interedge scattering rate has been found to depend exponentially on temperature,[19] and also with theory which predicts exponential dependences for both interedge and intraedge scattering.[19–21] Far away from the resistance maxima, the temperature dependence of R_{xx} is determined by the competition between σ_{xx}^{L} and $L_{\text{th}}(\theta)$.

Our results have significant implications for quantum Hall effect experiments where low-mobility alloyed AuGe contacts are usually considered to act as the thermal reservoirs. This simplistic idea that AuGe contacts act as ideal thermal reservoirs appears to work surprisingly well for interpreting the majority of experimental results.[4] However, at *sufficiently* low temperatures, the inevitable presence of scattering in the contact regions must lead to different populations of different edge states[16] and therefore to the anomalous quantum Hall effect as shown in the present experiment. Indeed, in a number of QHE experiments anomalous behavior has been observed which is attributed to "nonideal" contacts.[4,19,22] It is curious that the nonideal contacts are considered as anomalous when it is the ideal behavior which is harder to understand. The condition for the complete Milne reflection $l_i \gg l_e$ is always valid in metals at low temperatures and therefore it seems surprising that alloyed contacts are able to act as "ideal" Landauer thermal reservoirs.

Three additional remarks may be helpful. First, our experiment is somewhat similar conceptually to the experiments with gated contact leads[2,4] but in our case the reflection of the edge-state electrons is controlled by temperature rather than by the gate voltage. Second, one may find it useful to consider the case $T \rightarrow 0$ as low-temperature localization of electrons within the topmost Landau level in the contact leads.[23] Third, we note that the proposed model can also explain the high-temperature nonlocal magnetoresistance oscillations which have been observed in mesoscopic n^+ GaAs wires,[24] although the mechanism of conductance through the bulk is different in that case.

Finally, we discuss the observed difference in the behavior of R_{xx} and R_{xy} for the different devices. The difference is due to the relationship between L_{th} and the length of the 2DEG contact leads for the different devices. When the contact leads are shorter than L_{th}, their length should be substituted into Eq. (1) instead of L_{th} since the edge-state electrons are more rapidly thermalized in the alloyed contacts.[7] This situation is expected to be the case for type-*C* devices at helium temperatures. In the type-*B* sample, the increase of L_{th} can describe the initial decay of the SdH peak with decreasing temperature while, at 0.6 K, L_{th} becomes comparable with the length of the leads and the amplitude of the peak saturates at lower temperatures. In type-*A* devices, the thermalization length in the leads is suppressed with respect to the *B* devices by the He-ion bombardment[19,20] and we can expect that it is always shorter than the length of the leads for the temperature interval investigated.

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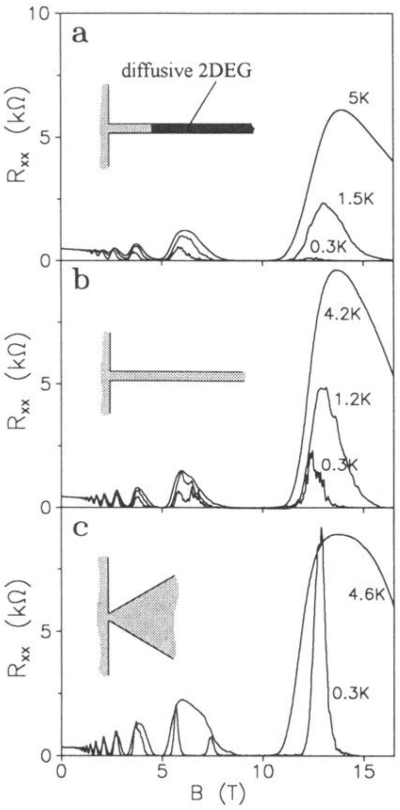
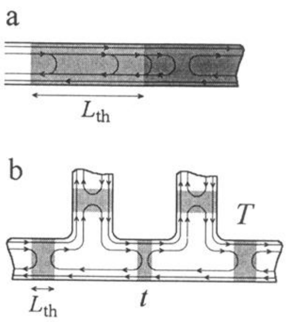


FIG. 1. Longitudinal magnetoresistance at different temperatures in QHE devices with various geometries of contact leads as shown in the insets.



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