PHYSICAL REVIEW B VOLUME 49, NUMBER 3 15 JANUARY 1994-I

Effects of electron heating on conductance fluctuations in mesoscopic wires

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(Received 16 September 1993; revised manuscript received 12 November 1993)

We analyze the contributions of electron heating to the generation of time-independent conductance fluctuations as a function of voltage in mesoscopic wires. We discuss both harmonic generation and direct measurements of differential conductance versus voltage. Electron heating may easily dominate the properties of the fluctuations. We propose that observations of conductance fluctuations with a small correlation voltage may be a consequence of heating, rather than being related to a small intrinsic energy scale such as the energy-eigenvalue spacing.

Time-independent fluctuations in the low-temperature (T) electrical conductance of mesoscopic metal samples have been studied for several years.[1-3] Fluctuations as a function of magnetic field, carrier density, defect reconfiguration, and T are generally well explained as a direct consequence of the interference of electron waves.[4,5] However, aspects of fluctuations as a function of source-drain voltage (V) have remained a puzzle. The original theories[6,7] considered how V would modify the wavelength of electrons, thus changing the interference pattern. These theories predicted that the V correlation scale V_c of the conductance fluctuations in a diffusive sample would be determined by the Thouless energy,[8] $eV_{\rm Th} \equiv E_{\rm Th} \sim \hbar D/L^2$. Here $D = v_F l/3$ is the diffusion constant and L is the effective sample length. However, harmonic generation measurements on Sb and Au wires, performed by Webb, Washburn, and Umbach (WWU) (Ref. 9) at base T as low as 10 mK, found fluctuations on a much smaller V scale. WWU determined an upper bound of roughly 0.1 μV for V_c near $V=0$ in an Sb wire. This was a factor of 300 smaller than an estimate of $V_{\rm Th}$,[9] and was even a factor of 9 smaller than $k_B T/e$. Meanwhile, semiconductor samples at higher T have displayed different behavior. The V_c measured at 280 mK in GaAs-Al$_x$Ga$_{1-x}$As heterostructures[10] and at 450 mK in Si metal-oxide semiconductor field-effect transistors[11] (MOSFET's) were consistent with estimates of $V_{\rm Th}$. Attempts have been made to explain these observations by suggesting that V_c is determined by the spacing of energy eigenstates within the sample.[2,12]

In this paper we offer the alternative proposal that the small V_c measured in the experiments of WWU (Ref. 9) may originate in electron heating. We discuss several lines of evidence. (1) A simple estimate shows that even small currents may heat the electrons in mesoscopic wires well above the base T. (2) Direct measurements of differential conductance versus V show that the amplitude of fluctuations decreases and V_c increases as a function of excitation current over the entire range of excitation used in the harmonic generation experiments.[9] This behavior is a signature of heating.[2,7,9] (3) The behavior of the harmonic amplitudes, $V_n \propto I_0$, observed by WWU (Ref. 9) for small applied currents, I_0, may be explained by heating. (4) Conductance fluctuations measured in metal wires at very low T are qualitatively different from the fluctuations measured both in metal point contacts[13,14] (which are much less prone to electron heating than wires) and in semiconductor samples at higher T.[10,11]

(i) Theoretical treatments of heating[15] and also measurements by Roukes *et al.*[16] demonstrate why the effects of electron heating are difficult to avoid in four-probe experiments using mesoscopic wires. The V drop which acts to heat electrons is the V across a length of the current leads equal to the diffusion length, $L_{e\text{-ph}}$, over which an electron loses excess energy to phonons. At low T, $L_{e\text{-ph}}$ may be much longer than the distance between the voltage probes, L_m, so that the effective T of the electrons, T^*, may be many times greater than eV_m/k_B, where V_m is the V between voltage probes. ($L_{e\text{-ph}}$ may be ≥ 1 mm at 10 mK.[16]) The experiment of WWU (Ref. 9) corresponds to a slightly different case than the effectively infinite wires of Roukes *et al.*,[16] in that the distance $L_{\rm tot}$ between low-resistance contact pads may provide a cutoff for the length scale over which electrons may be heated. For the Sb wire used in the WWU experiment, $L_{\rm tot} \approx 12$ μm, 20 times L_m.[17] In Sb, $L_{e\text{-ph}} \approx (D\tau_{e\text{-ph}})^{1/2} \approx (D/\alpha)^{1/2}(T^*)^{-1}$ with $\alpha \sim 2 \times 10^8\,{\rm s}^{-1}\,{\rm K}^{-2}$ (Ref. 18) and $D \approx 650$ cm^2/s.[9] Consequently, $L_{e\text{-ph}} > L_{\rm tot}$ in the WWU sample for $T^* < 1.5$ K. In this regime, T^* is determined by a balance between heat deposition into the electrons and conduction into the contact pads. We have solved the thermal diffusion equation in this case for the sample T^* in the four-probe configuration, with all four leads identical and of length L_L ($L_{\rm tot} = 2L_L + L_m$). We assume uniform Joule heating along the current lead, and zero resistance in the contact pads. However, we note that any V drop or heating in the contact pads within $L_{e\text{-ph}}$ from the sample will raise T^* above our estimate. The solution is

$$(T^*)^2 \approx T_0^2 + \left[\frac{3}{8\pi^2}\right]\left[\frac{eV_{\rm tot}}{k_B}\right]^2\left[1+\left[\frac{L_m}{L_{\rm tot}}\right]^2\right], \quad (1)$$

where T_0 is the base T and $V_{\rm tot}$ is the *total* V drop along the current leads ($V_{\rm tot} \geq 20V_m$ for the WWU device). Strong heating occurs over the V range for which harmonic generation was measured by WWU.[9] For $V_m = 0.2$ μV ($V_{\rm tot} \geq 4$ μV), T^* is already raised at least 40% above T_0.

This heating may have a profound effect on the conductance fluctuations, because it will cause the dephasing length L_ϕ to decrease as V is increased. It is L_ϕ, not the lithographically defined dimension of the sample, which determines the length scale within which interference effects are generated.[19] Therefore, as V is increased, the effective sample length will decrease and the conductance pattern will be modified. In order to produce even-order harmonics, such as those observed by WWU,[9] heating must modify the conductance in a way that is not symmetric with respect to reversing V.[9] Such behavior would not be expected classically, but may occur if one takes into account thermoelectric effects in mesoscopic samples.[20–22] Fluctuations in the thermopower are predicted to be large (with relative fluctuations larger than in the conductance[21]) and for some samples the Peltier heat may approach the Joule heat.[22] These effects are greatest in low-carrier density materials[22] such as the Sb wire examined by WWU.[9] Consequently, there is no symmetry which guarantees that heating will be even under voltage reversal, and the presence of even-order harmonics in the conductance of a strongly heated sample may not by itself be used as an argument that the fluctuations do not originate in heating effects.

(ii) As noted by WWU,[9] the effects of heating can be seen directly in measurements of the differential conductance fluctuations versus V. V_c in a phase-coherent, unheated sample has been predicted to be independent of V, and the amplitude of the fluctuations has been predicted to grow as $V^{1/2}$ for $V > V_{\rm Th}$.[7] Recent calculations[23] and measurements[13,14] indicate that correlation properties of the fluctuations may slow the rate of amplitude growth, but the amplitude of the differential conductance fluctuations in an unheated sample is not predicted to decrease with increasing V.[24] In contrast, measurements by WWU (Ref. 9) on the Sb wire found fluctuations whose amplitude decreased and whose V_c increased over the entire V range of the measurements. The shrinking amplitude of the fluctuations indicates that the dephasing length was decreasing[2,9] and/or that energy averaging at elevated T^* was increasingly important.[4] The increase in V_c is directly visible in the differential conductance only for $V > 24$ μV, but this was the value at which V_c first grew beyond the size of the ac excitation used to perform the measurement, 0.5 μV. Measurements at smaller V were artificially broadened by the excitation. Based on the harmonic measurements, V_c near $V=0$ was less than 0.1 μV, so V_c also grew by at least a factor of 5 between 0 and 24 μV. The increase in V_c may be explained by the heating-induced decrease in the dephasing length, which will enlarge V_c in both the model of Larkin and Khmel'nitskii (LK) (Ref. 7) and the model of Tang and Fu (TF).[12] Alternatively, if the small value of V_c is a direct consequence of heating, the V dependence of V_c may be explained by the V dependence of T^*.

(iii) The experimentally observed heating-induced growth of V_c provides a straightforward explanation for the behavior of harmonic amplitudes measured by WWU,[9] $V_n \propto I_0$ at low I_0. Previously, it has been shown[17,1] that this behavior is due to the presence of a long-range tail in the current (I) autocorrelation function of the dc conductance fluctuations. However, long-range I correlations predicted to be caused by quantum interference[7] appear to be too weak to explain the observed harmonics, at least in one and two dimensionally shaped samples. Here we will show that heating is a separate mechanism which may produce a long-range I autocorrelation function, to give $V_n \propto I_0$. For pedagogical simplicity we will describe the effects of heating using energy-averaging arguments like those used to estimate the T dependence of conductance fluctuations.[4] Such arguments assume that the underlying energy (E) and V correlation functions for $S(E,V)$ (defined below) are short ranged [decaying at least as fast as $(\Delta E)^{-1}$ for large ΔE].[4] This condition is obeyed for wire and thin-film samples,[4] but fails in three dimensionally shaped samples[4] since there quantum interference results in slower than $(\Delta E)^{-1}$ decay in the E correlation function. Therefore, although the effects of heating do not depend upon the nature of the E correlations, our arguments will be applicable only to wires and thin films.

We will analyze harmonic generation both with and without heating, using both analytic estimates and numerical calculations. We consider devices with average conductance $G_0 \gg e^2/h$, under current bias. For $k_B T \ll E_S$, defined below, the dc conductance may be written[12,25]

$$G_{\rm dc}(I) = \frac{I}{V(I)} \approx G_0 \left[1 + \frac{e^2}{h}\frac{1}{I}\int_{E_F - eI/2G_0}^{E_F + eI/2G_0} \frac{dE}{e} S(E, V(I)) \right] . \quad (2)$$

The function $S(E,V)$ describes fluctuations in transmission probability due to quantum interference. For an applied current $I(t) = I_0 \cos(\omega t)$, the nth harmonic is

$$V_n = \left| \int_{-\pi/\omega}^{\pi/\omega} \exp(in\omega t) \frac{I(t)}{G_{\rm dc}(I(t))} \omega\, dt \right| . \quad (3)$$

Following LK,[7] we assume that $S(E,V)$ is a random function of both E and V with constant rms amplitude, $\delta S \sim 1$, in the absence of heating. We will consider the suggestion of TF (Ref. 12) that the correlation scales E_S and V_S may differ, with $eV_S \le E_S$. In both the models of LK and TF, without heating, the correlation scales are independent of I. We define $V_0 = I_0/G_0$. In the regime $V_0 \ll V_S, E_S/e$, we find $V_n \propto I_0^n$, in agreement with previous workers.[9,10,12]

In order to make further analytic estimates, it is convenient to manipulate Eqs. (2) and (3) to the form, for even-order harmonics,

$$V_n \approx \frac{e^2}{h}\frac{2}{G_0} \left| \int_{-1}^{1} \left\{ \frac{P_n(x)}{\sqrt{1-x^2}} \left[\int_{E_F - eV_0 x/2}^{E_F + eV_0 x/2} \frac{dE}{e} S(E, V_0 x) \right] \right\} dx \right| , \quad (4)$$

where $P_n(\cos(\theta))=\cos(n\theta)$. (Odd-order harmonics may be analyzed similarly.) For $V_0 \gg V_S$, the integral over E is a very rapidly varying function of x. In the integral over x, this is weighted by a slowly varying envelope function (we assume $n \ll V_0/V_S$). The weighting function emphasizes contributions from the extremes of the range of x integration, near 1 and -1, but one may gain physical insight by ignoring this weighting in a first-order nonrigorous estimate. The x integral can then be evaluated easily as a random sum.

We first consider the regime $V_S \ll V_0 \ll E_S/e$, with no heating. Then the integrand in the E integral in Eq. (4) is roughly constant as a function of E, so that the E integral is approximately $xV_0S(E_F,V_0x)$. The x integral may be estimated as the sum from $N=2V_0/V_S$ regions of width $\Delta x \sim V_S/V_0$, each contributing independently to the sum some value of order $\pm V_S\delta S$. The rms amplitude of V_n therefore grows roughly as $N^{1/2}$ times this value, or $V_n \sim (e^2/h)\delta S(V_SI_0)^{1/2}/G_0^{3/2}$. In this regime, $V_S \ll V_0 \ll E_S/e$, one should therefore expect $V_n \sim I_0^{1/2}$ as long as there is no heating. This agrees with an estimate by Imry and Washburn.[17]

The amplitude of V_n may also be estimated in the regime $V_S \leq E_S/e \ll V_0$, with no heating. We assume that fluctuations as a function of V and E in $S(E,V)$ are uncorrelated.[26] The integral over E in Eq. (4) is then a random function of x with correlation scale V_S/V_0 and rms amplitude $\delta S(E_S/e)(eV_0x/E_S)^{1/2}$. The x integral may be estimated in the same way as above, neglecting the weighting factor, to give $V_n \sim (e^2/h)\delta S(E_SV_S/e)^{1/2}/G_0$. In this case, V_n is independent of I_0 in the absence of heating.

Both of our estimates differ from the results of Tang and Fu, who claim that $V_n \propto I_0$ in the regime $V_S \ll V_0 \ll E_S/e$, and $V_n \propto I_0^{1/2}$ when $V_S \leq E_S/e \ll V_0$.[12] Numerical evaluation of Eq. (3) shows that the weighting function in Eq. (4) can produce a small change from our nonrigorous analytic estimates, but numerical results are in better agreement with our estimates than those of Tang and Fu. The top curve in Fig. 1(a) shows the rms value of V_2 for an ensemble of 20 random functions $S(E,V)$. We have used $G_0=1000e^2/h$, $V_S=1/G_0$, $E_S \gg eV_0$, and $\delta S=1$. The regime $V_0/V_S \gg 1$ corresponds to $V_S \ll V_0 \ll E_S/e$, and the result may be fit to the form $V_2 \propto I_0^{0.66}$. The exponent is statistically accurate to within ± 0.02. Similar calculations give $V_4 \sim I_0^{0.70}$. The bottom curve in Fig. 1(a) indicates that in the regime $V_S=E_S/e \ll V_0$, $V_2 \propto I_0^{0.15}$. Our nonrigorous analytic results ($V_n \propto I_0^{0.5}$ for $V_S \ll V_0 \ll E_S/e$ and $V_n \propto I_0^{0.0}$ for $V_S \leq E_S/e \ll V_0$) only slightly underestimate the correct strength of the I_0 dependence.

The presence of heating modifies the above arguments so that the behavior $V_n \propto I_0$ may occur in either the regime $V_S \ll V_0 \ll E_S/e$ or $V_S \leq E_S/e \ll V_0$. As discussed above, heating may reduce the amplitude of fluctuations and also increase the correlation scales, V_S and E_S.[7] These effects may be included in our nonrigorous estimates of V_n by allowing the average values of δS, V_S, and E_S to depend on the amplitude of the excitation current, rather than assuming they are constant. The re-

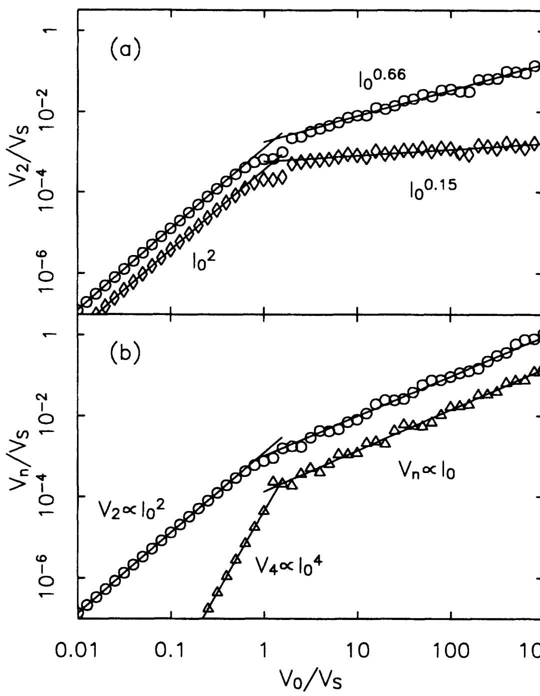

FIG. 1. (a) Models of second-harmonic generation without heating, as discussed in the text. The top curve corresponds to $E_S \gg eV_0$ and the bottom curve to $E_S=eV_S$. (b) Model of second- (circles) and fourth- (triangles) harmonic generation for $E_S \gg eV_0$ in the presence of heating. The lower curves in both plots are shifted down by half a decade.

quirement for $V_n \propto I_0$ in the regime $V_S \ll V_0 \ll E_S/e$ is that $[(V_S)^{1/2}\delta S] \propto I^{1/2}$, and for $V_S \leq E_S/e \ll V_0$ that $[(E_SV_S)^{1/2}\delta S] \propto I$. These conditions are approximate, since our estimates give power-law exponents which may be in error by 0.2 or so, as indicated by the numerical calculations. However, we have confirmed numerically that the requirement $[(V_S)^{1/2}\delta S] \propto I^{1/2}$ in the regime $V_S \ll V_0 \ll E_S/e$ does produce $V_n \propto I_0$ for $n=2$ and 4 [Fig. 1(b)]. Based on the direct measurements of V_S and δS versus V reported by WWU,[9] the condition $[(V_S)^{1/2}\delta S] \propto I^{1/2}$ is at least approximately correct for their sample.

(iv) In (ii) and (iii) we have argued that the conductance fluctuations in mesoscopic metal wires are strongly altered by heating, but we have not yet shown that their small V_c may be a consequence of heating. Evidence for this claim comes from comparison with fluctuations measured in disordered point contacts. Because point contacts are measured using low-resistance three-dimensional leads, almost all the V drop occurs across the contact, and the electron heating is greatly reduced relative to wires with high-resistance current leads.[13] The T dependence of the point-contact conductance fluctuations[13] is such that the V dependence of the conductance at high T is well described by the convolution of a low-T curve with the derivative of the high-T Fermi function. As T is raised, the fluctuations are thus broadened, so that we observe no fluctuations with V_c less than k_BT. This is in contrast to the harmonic measurements of WWU,[9] which revealed fluctuations with $V_c \ll k_BT$. We infer that the fluctuations measured in the wires of WWU (Ref. 9) are produced by a different mechanism than fluctuations in disordered point contacts. Heating in the wires may explain the difference. Our point-contact results are not in accord with the arguments of Tang and Fu[12] concerning the energy-eigenvalue spacing, as this mechanism was proposed to account for fluctuations with $V_c < k_B T/e$.

In summary, we have shown by a simple estimate that electron heating by an applied V is quite severe at low T in mesoscopic wires. We have discussed evidence that the amplitude and V_c of measured conductance fluctuations are strongly altered by heating. We propose that the existence of conductance fluctuations in mesoscopic wires with V_c much less than $k_B T$ may be the result of electron heating. Identification of the measured V_c with the energy-eigenvalue spacing in wires is premature unless the effects of heating can be dismissed.

We thank S. Washburn for discussions of unpublished results. This research was supported by the Office of Naval Research, Contract No. N00014-89-J-1692, and by the National Science Foundation through the Cornell Materials Science Center, DMR-9121654, and through use of the National Nanofabrication Facility at Cornell, ECS-86-19049.

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