Conduction of a weakly interacting one-dimensional electron gas through a single barrier

Dongxiao Yue, L. I. Glazman, and K. A. Matveev*
Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, 116 Church Street SE, Minneapolis, Minnesota 55455
(Received 9 September 1993)

Scattering from a localized potential of arbitrary strength is found for weakly interacting electrons. Poor man's renormalization group procedure allows us to calculate the renormalized transmission amplitudes at any energies. Simple formulas that describe the conductance at any temperature are derived. In the presence of the electron-electron backscattering, the low-temperature conductance deviates from the results of Luttinger liquid theory. In particular, the temperature dependence of the conductance may become nonmonotonic. In the presence of a magnetic field, backscattering gives rise to a peak in the differential conductance at bias equal to the Zeeman splitting.

I. INTRODUCTION

Recent advances in semiconductor technology have renewed interest in the transport properties of one-dimensional (1D) electron systems. By confining a high mobility two-dimensional (2D) electron gas within a narrow channel, a quasi-1D electron system can be obtained. It is one-dimensional in the sense that the motion of the electrons is restricted to be along the channel. The transverse degree of freedom is quantized as a standing wave. The transverse energy levels are well separated due to the quantization in the confining potential well, while the longitudinal motion along the channel has a continuous spectrum. For a system with low Fermi energy, only the lowest level is populated.

When a clean short channel is connected to two external leads, the two-terminal conductance is found to be quantized in units of $e^2/\pi\hbar$.[1,2] This is due to the fact that the electron transport is ballistic and the transverse mode number is conserved as electrons move through the channel. A calculation of the conductance of a channel smoothly connected to the 2D leads with variable width shows that the adiabatic transport of electrons along the channel is a very good approximation.[3] Deviations of conductance from the quantized values due to the tapered ends are exponentially small.

In longer channels, however, ballistic transport is destroyed by the impurities.[4] Quantization of conductance breaks down due to the backscattering caused by the random potential.[5] It is reasonable to expect that a long 1D channel with a single impurity can be formed in a sufficiently clean system. The transport properties of the channel are then determined by electron scattering on the impurity. For noninteracting electrons the conductance of such a system can be related to the corresponding transmission coefficient by the Landauer formula.[6]

Electron-electron interaction alters the properties of the 1D system qualitatively. To explain the properties of some organic quasi-1D conductors, extensive studies of transport properties of 1D electron systems were carried out in the 1970s, and yielded a number of interesting results. It was found that the Drude conductivity of a 1D metal varies as a power of temperature.[7–9] Recently, Kane and Fisher,[10] and subsequently Furusaki and Nagaosa,[11] investigated the transport of a 1D interacting electron gas in the presence of a single scatterer within the framework of the Luttinger model. The limit of strong interaction was considered in Ref. 12. It was shown that the conductance varies as a power of temperature at $T \to 0$. The corresponding exponent is positive for repulsive interaction. This implies perfect reflection at zero temperature, no matter how strong or weak the barrier is. Theory[13] shows that tunneling into a ballistic wire with an arbitrary number of transverse modes is also characterized by power-law asymptotics of tunnel density of states at low energies. The corresponding exponents were expressed explicitly in terms of microscopic electron-electron interaction. All these and other results (e.g., Refs. 14 and 15) have provided further insight into the understanding of the interacting 1D electron system.

The purpose of this paper is to study the transport properties of 1D interacting electron gas in the presence of a single scatterer at any temperature. Our approach for studying the transport properties will be based on the Landauer formula[6] relating conductance and the transmission coefficient. We show that the scattering on the impurity is renormalized by the electron-electron interaction due to the formation of a Friedel oscillation near the barrier. These renormalizations are treated using a simple renormalization-group (RG) method similar to the poor man's scaling method proposed by Anderson[16] for the Kondo problem. This method is equivalent to the summation of infinite subsequence of the most divergent terms in the perturbation expansion in the electron-electron coupling constants. This certainly restricts the applicability of the method to the case of weak interaction. However, in this limit, we are able to obtain a number of features that cannot be found in the framework of Refs. 10 and 11. In the Luttinger-liquid theories, barriers were treated only in two limiting cases: weak barrier and strong barrier. The temperature dependence of conductance was found only near the fixed points corresponding

to perfect transmission or perfect reflection. The technique does not allow us to describe the crossover between these two limits. Also, electron-electron backscattering which turns out to be responsible for qualitative modification of the system properties was neglected in the Luttinger-liquid theory. In our theory, scattering on the barrier is treated exactly for arbitrary barrier strength. The crossover behavior of the conductance between the fixed points is explicitly expressed in a simple formula. The backscattering is taken into consideration. It renormalizes the electron-electron interaction, which results in deviation from the Luttinger-liquid behavior and a possible nonmonotonic temperature dependence of the conductance. Backscattering is also responsible for a peak in the differential conductance of the 1D system in the presence of a magnetic field. This peak occurs at bias equal to the Zeeman splitting.

In the following sections, we present in detail the aforementioned results and techniques. In Sec. II we give an illustrative derivation of RG equations for spinless electrons that allow us to find the transmission amplitudes. In Sec. III a rigorous derivation of RG equations is given for spin-$\frac{1}{2}$ electrons. In Sec. IV the temperature dependence of the linear conductance and the nonlinear I-V characteristics at zero magnetic field are presented. In Sec. V the singularities in I-V characteristics in the presence of a magnetic field are predicted.

II. TRANSMISSION AMPLITUDE IN A SPINLESS 1D ELECTRON GAS

In this section, we consider a 1D gas of spinless electrons scattering on a potential $U(x)$ localized at the origin. In the absence of interactions, the Hamiltonian is $H_0 = p^2/2m + U(x)$ and asymptotic wave function far from the barrier $U(x)$ has the form

$$\phi_k(x) = \frac{1}{\sqrt{2\pi}} \begin{cases} e^{ikx} + r_0 e^{-ikx}, & x < 0 \\ t_0 e^{ikx}, & x > 0, \end{cases} \tag{1}$$

$$\phi_{-k}(x) = \frac{1}{\sqrt{2\pi}} \begin{cases} t_0' e^{-ikx}, & x < 0 \\ e^{-ikx} + r_0' e^{ikx}, & x > 0. \end{cases} \tag{2}$$

The wave vector k is defined to be positive. For a symmetric barrier we have $r_0 = r_0'$ and $t_0 = t_0'$.

Scattering on the barrier is modified by the electron-electron interaction. Below we treat these corrections using perturbation theory.

A. Born approximation

In the lowest-order perturbation theory in the interaction potential, we can neglect inelastic processes in which electrons above the Fermi level lose coherence by exciting electron-hole pairs. Within this approximation, the many-body electron state can be described by the Slater determinant of single-electron wave functions. Each electron is affected by an extra average potential produced by other electrons in the Fermi sea. This potential and the barrier potential $U(x)$ act together as an effective barrier for electron scattering. The single-electron wave function can be found as a solution of the Schrödinger equation with this effective barrier. Then the transmission coefficient can be calculated.

The extra potential consists of two parts: the Hartree potential $V_H(x)$ determined by the electron density in the system, and a nonlocal exchange potential $V_{\rm ex}(x,y)$, which is due to the identity of electrons. For a weak interaction one can treat the additional potential as a small perturbation. Its effect on the single-electron wave function can be found by the Green-function method. The equation for a single-electron state ψ_k is

$$\psi_k(x) = \phi_k(x) + \int dy G_k(x,y) \times \int dz \{V_H(z)\delta(y-z) + V_{\rm ex}(y,z)\}\psi_k(z). \tag{3}$$

The Hartree and exchange potentials are defined as:

$$V_H(x) = \int dy V(x-y) n(y), \tag{4}$$

$$V_{\rm ex}(x,y) = -V(x-y) \sum_{|q|<k_F} \psi_q^*(y)\psi_q(x), \tag{5}$$

where $V(x-y)$ is the electron-electron interaction potential, $n(y) = \sum_{|q|<k_F} |\psi_q(y)|^2$ is the electron density.

In the first-order Born approximation, ψ_k in the right-hand side of Eq. (3) and in Eqs. (4) and (5) is replaced by the unperturbed wave function ϕ_k. The unperturbed electron density has the form

$$n(x) = \begin{cases} n_0 + \frac{1}{\pi}\int_0^{k_F} dk \mathrm{Re}\{r_k e^{-2ikx}\}, & x < 0 \\ n_0 + \frac{1}{\pi}\int_0^{k_F} dk \mathrm{Re}\{r_k'^* e^{-2ikx}\}, & x > 0. \end{cases} \tag{6}$$

From Eq. (6), at large distances $|x| \gg k_F^{-1}$ the disturbance of density $\delta n(x) = n(x) - n_0$ caused by a symmetric barrier decays as

$$\delta n(x) \simeq \frac{|r_0|}{2\pi|x|} \sin(2k_F|x| + \arg r_0). \tag{7}$$

It follows from Eq. (4) that the oscillations of density (6) produce an oscillating Hartree potential. It is commonly referred to as the *Friedel oscillation*; see Fig. 1. In contrast to the three-dimensional case, where the density oscillation around an impurity decays as $1/R^3$ (where R is the distance from the impurity), in 1D it decays only as $1/|x|$. This asymptotics of the Hartree potential obviously leads to a logarithmic divergence of the reflection amplitude at $k \to k_F$ found in the Born approximation.

To find the correction to t_0, we consider a wave incoming from the left with wave vector k. The modified wave function $\psi_k(x)$ must have the following asymptotics:

$$\psi_k(x) \simeq \frac{1}{\sqrt{2\pi}} t_k e^{ikx}, \quad x \to +\infty, \tag{8}$$

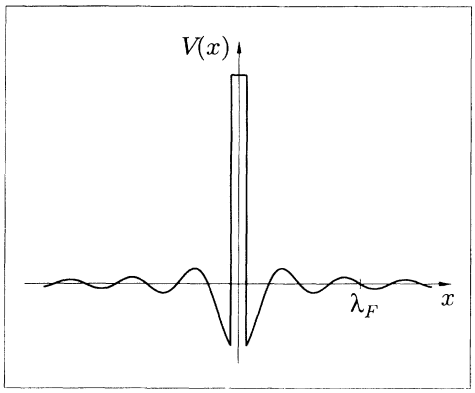

FIG. 1. Total scattering potential. The central peak is the bare potential of the barrier. The wings represent the Friedel oscillation induced by the barrier.

where t_k is the modified transmission amplitude. Thus to find the correction to the transmission amplitude, we only need the asymptotic form of the Green function $G_k(x,y)$ at $x \to +\infty$. Calculating it with free wave functions, we find

$$G_k(x,y) = \frac{1}{i\hbar v_k} \begin{cases} t_0 e^{ik(x-y)}, & y < 0 \\ e^{ik(x-y)} + r_0' e^{ik(x+y)}, & y > 0. \end{cases} \tag{9}$$

The transmission amplitude resulting from Eq. (3) is then

$$t_k = t_0 - \frac{1}{2}\alpha\left(r_0 r_0^* t_0 + t_0 r_0'^* r_0'\right) \ln\left|\frac{1}{(k-k_F)d}\right|, \tag{10}$$

where d is the characteristic spatial scale of the interaction potential $V(x)$. We introduced a dimensionless parameter α that characterizes the strength of the interaction:

$$\alpha = \alpha_2 - \alpha_1, \tag{11}$$

$$\alpha_1 = \frac{V(2k_F)}{2\pi\hbar v_F}, \qquad \alpha_2 = \frac{V(0)}{2\pi\hbar v_F}. \tag{12}$$

Here $V(q)$ is the Fourier transformation of the interaction potential, and v_F is the Fermi velocity.

The zero-momentum Fourier component $V(0)$ originates from the exchange term, while $V(2k_F)$ arises from the Hartree term. In the limiting case of a short-range interaction, since the electrons with the same spin cannot occupy the same position according to the Pauli principle, the interaction should have no effect. So in this case, contributions from the Hartree and exchange terms should exactly cancel. This agrees with our result, since $V(q)$ is constant for a short-range potential. We also notice that repulsive interaction [positive $V(q)$] suppresses transmission.

The first-order result (10) for the transmission amplitude has a simple physical meaning. The term containing $r_0 r_0^* t_0$ can be interpreted as follows. A plane wave coming from the left is reflected by the barrier with amplitude r_0. It is then scattered back to the barrier by the Friedel oscillation on the left-hand side with amplitude $-\frac{1}{2}\alpha r_0^* \ln(1/|k-k_F|d)$. Finally the electron penetrates the barrier with amplitude t_0. Similarly, the $t_0 r_0'^* r_0'$ term is the product of the amplitudes of the following processes: an electron first penetrates the barrier with amplitude t_0, then it is reflected back to the barrier by the Friedel oscillation on the right-hand side with amplitude $-\frac{1}{2}\alpha r_0'^* \ln(1/|k-k_F|d)$, and eventually reflected by the barrier to the right with amplitude r_0'. The total first-order correction to the transmission amplitude is the sum of the amplitudes of the two coherent first-order processes described above.

Our first-order result (10) has a logarithmic divergence at $k \to k_F$, no matter how small the coupling constants (12) are. This is a typical infrared divergency in 1D. It indicates the inadequacy of the first-order calculation at small $|k-k_F|$.

We calculate the correction to the transmission amplitude up to the second order in α, that is we use $\psi_k(x)$ obtained in the first-order calculation (8) as our new wave function in the right-hand side of Eq. (3) and repeat our previous calculation. The result is

$$t_k = t_0 - t_0|r_0|^2\alpha \ln\left|\frac{1}{(k-k_F)d}\right| - \frac{1}{2}t_0|r_0|^2(2|t_0|^2 - |r_0|^2) \times \left[\alpha \ln\left|\frac{1}{(k-k_F)d}\right|\right]^2. \tag{13}$$

In the above formula, we have only kept the most divergent terms. In the second order the most divergent term has the form $[\alpha \ln(1/|k-k_F|d)]^2$. Calculation up to this order does not remove the logarithmic divergence. Moreover, we expect that the most divergent term in the nth order has the form $[\alpha \ln(1/|k-k_F|d)]^n$, which means that a perturbative calculation to any finite order would not solve the problem. Thus, we will sum up the most divergent terms in all orders. This can be done by the renormalization-group method developed below.

B. RG in real space

As found in Sec. II A, there is a logarithmic divergence even in the first order correction to the transmission amplitude. Such a nonphysical divergence demonstrates the inadequacy of a simple perturbative calculation, which does not take into account the renormalization of the effective barrier. Since the Hartree and exchange potentials depend on the reflection amplitudes, they are modified along with these amplitudes. In a region $(-l,l)$ close to the origin, the electrons are scattered by the bare barrier with transmission amplitude t_0 and produce an extra potential that is proportional to $|r_0|$. Perturbative calculation for the transmission amplitude is carried out with the bare amplitudes. Such a calculation is justifiable as long as the correction from the perturbative calculation is indeed small, which is true for not too large l, such that $\alpha \ln(l/d) \ll 1$. Beyond this distance, the whole region $(-l,l)$ enclosed should be considered as an effective barrier to the electrons outside. This effective barrier is characterized by the now renormalized amplitudes r

and t. With these amplitudes, we can find the Hartree and exchange potentials in the outer region. Then the perturbative calculation can be carried out for a larger spatial scale. In order to ensure that perturbation theory is valid in every step, the above renormalization procedure is done repeatedly for larger and larger scales.

This idea leads to the following formulation of the problem. We start with a region of length $2l$ centered around the barrier. The scale l is chosen in such a way that it is much larger than d but not too large so that $1 \ll \ln(l/d) \ll \alpha^{-1}$. The modified transmission amplitude due to the electron-electron interaction in the region $(-l, l)$ can then be found by perturbation theory,

$$t_1 = t_0 - \alpha t_0(1 - |t_0|^2) \ln \Lambda, \tag{14}$$

with $\Lambda = l/d \gg 1$.

We then go to a larger scale, taking the region $(-l, l)$ as a composite scatterer. Using the renormalized transmission amplitude and correspondingly renormalizing the additional Hartree and exchange potentials, we can repeat the above calculation for this next scale Λl. We then go to the next larger scale, which is $\Lambda^2 l$, and so on. In general, the iterative renormalization of the transmission amplitude after n steps of scaling to larger distances can be found from

$$t_{n+1} = t_n - \alpha t_n(1 - |t_n|^2) \ln \Lambda. \tag{15}$$

This iteration procedure should be stopped at length scale $1/|k - k_F|$, beyond which the scattered electron loses phase coherence with the Friedel oscillation, and the transmission amplitude is not renormalized any further.

In the continuous limit, Eq. (15) becomes

$$\frac{dt}{d\ln(L/d)} = -\alpha t(1 - |t|^2), \tag{16}$$

where L is the current length scale. Integrating Eq. (16) from $L = d$ to $L = 1/|k - k_F|$ and using boundary condition $t|_{L=d} = t_0$, we find the renormalized transmission amplitude

$$t_k = \frac{t_0|(k - k_F)d|^\alpha}{\sqrt{|r_0|^2 + |t_0|^2|(k - k_F)d|^{2\alpha}}}. \tag{17}$$

The expansion of this formula up to the second order in α coincides with formula (13).

III. ENERGY SPACE RENORMALIZATION GROUP FOR SPINLESS AND SPIN-$\frac{1}{2}$ ELECTRONS

A. Spinless electrons

In Sec. II B, the RG is formulated as a scaling procedure in real space. In this section, we will demonstrate an equivalent RG procedure in energy space. This method allows generalization to the case of spin-$\frac{1}{2}$ electrons.

The renormalization of the bare transmission amplitude t_0 is caused by the interaction with the Fermi sea electrons. Since the maximum momentum transfer in a scattering event is determined by the spatial scale d of the interaction, only electrons in the energy strip of half-width $D_0 = \hbar v_F/d$ near the Fermi level contribute to the renormalization of t. In the finite energy range $(E_F - D_0, E_F + D_0)$, the electron dispersion relation can be approximated to be linear:

$$\epsilon(k) = \hbar v_F(|k| - k_F), \tag{18}$$

States outside this range are unimportant and neglected. The bandwidth cutoff D_0 defines the energy range for relevant electron states.

We now transform our problem to a similar one with smaller bandwidth cutoff, $D = D_0/\Lambda$, where $\Lambda \gg 1$. The two problems are equivalent if we simultaneously renormalize t in order to take into account the interaction with the states excluded by this RG transformation. The renormalization of t_0 found in the Born approximation is

$$\delta t(\epsilon) = -\alpha t_0(1 - |t_0|^2) \ln \Lambda, \tag{19}$$

where $\epsilon = E - E_F$ is the electron energy measured from the Fermi level.

We apply the RG transformation again, reducing the bandwidth $D \to D/\Lambda$ step by step until all the intermediate states outside the energy range $(-|\epsilon|, |\epsilon|)$ are eliminated. During each step of rescaling the cutoff, transmission amplitude t is renormalized according to (19) with t_0 being substituted by the modified t from the previous step. The accumulated effect of these small renormalizations may be found as a solution of the differential equation

$$\frac{dt}{d\ln(D_0/D)} = -\alpha t(1 - |t|^2). \tag{20}$$

The renormalization procedure should be stopped at the scale $D \simeq |\epsilon|$, because at lower energy scales the perturbative correction is no longer logarithmically large. Thus we integrate this equation from $D = D_0$ to $D = |\epsilon|$ and use the boundary condition $t(\epsilon)|_{D=D_0} = t_0$. We obtain the renormalized t identical to (17). The transmission coefficient $\mathcal{T} = |t|^2$ is then

$$\mathcal{T}(\epsilon) = \frac{\mathcal{T}_0|\epsilon/D_0|^{2\alpha}}{\mathcal{R}_0 + \mathcal{T}_0|\epsilon/D_0|^{2\alpha}}, \tag{21}$$

where $\mathcal{T}_0 = 1 - \mathcal{R}_0 = |t_0|^2$ is the bare transmission coefficient.

B. Electrons with spin

In Sec. II we have considered a system of spinless electrons; such a system can be realized by applying a large magnetic field to lift the spin degeneracy. For systems with spin degeneracy the above theory must be revised to take into account the difference between forward and

backward scattering processes. In the spinless case, since all electrons are identical, there is no difference between the final states that result from the two processes. However, final states become distinguishable for real spin-$\frac{1}{2}$ electrons. This difference leads to the renormalizations of the interaction constants in the spin-$\frac{1}{2}$ case; see, e.g., Ref. 17. The related phenomena are studied within the framework of the following model Hamiltonian:

$$H_{\rm int} = \frac{1}{2\pi}\int dk dp dq \sum_{\sigma\sigma'}\Big[g_1 a^\dagger_{k\sigma} b^\dagger_{p\sigma'} a_{p+2k_F+q,\sigma'} b_{k-2k_F-q,\sigma} + g_2 a^\dagger_{k\sigma} b^\dagger_{p\sigma'} b_{p+q,\sigma'} a_{k-q,\sigma} + \frac{1}{2} g_4 \left(a^\dagger_{k\sigma} a^\dagger_{p\sigma'} a_{p+q,\sigma'} a_{k-q,\sigma} + b^\dagger_{k\sigma} b^\dagger_{p\sigma'} b_{p+q,\sigma'} b_{k-q,\sigma}\right)\Big]. \tag{22}$$

Here $a^\dagger_{k\sigma}$ and $b^\dagger_{k\sigma}$ are the operators creating, respectively, right- and left-moving electrons with momentum k and spin σ. The dispersion relation is taken to be linear within a band of half-width D_0. Interaction constant g_1 describes the backscattering, while g_2 and g_4 characterize the density-density interaction between the electrons moving in the opposite directions and in the same direction, respectively.[18] Unrenormalized constants g_1, g_2, and g_4 are identified with the Fourier components of the interaction potential,

$$g_1 = V(2k_F), \qquad g_2 = g_4 = V(0). \tag{23}$$

The renormalizations caused by the interaction may be found by the RG method. The bandwidth cutoff D serves as the scaling parameter in the problem. It is reduced to $D-\delta D$ at each step of the renormalization. By requiring the invariance of the scattering matrix for the states in the narrow band of half-width $D-\delta D$ under this RG transformation, we obtain the Hamiltonian

$$H'_{\rm int}(\omega) = PH_{\rm int}P + PH_{\rm int}(1-P)\frac{1}{\omega - H_0}H_{\rm int}P + \cdots, \tag{24}$$

which has the same scattering properties as the original one[16,17] in the narrow band. Here P is a projection operator into the subspace of the eigenstates of the quadratic Hamiltonian H_0 without any excitations in the high-energy range $(D-\delta D, D)$. As we will see, the first term in Eq. (24) generates additional terms quadratic in fermion operators in the transformed Hamiltonian, while the second term gives rise to quartic terms leading to renormalizations of the interaction constants.

We now calculate the renormalizations of the transmission amplitudes using the above model interaction (22). The procedure is as follows. We first transform (22) to the basis of the wave functions (1) and (2) by a unitary transformation. In this basis the Hamiltonian H_0 consisting of the kinetic energy and the barrier potential is diagonal. The effect of scattering on the barrier is incorporated exactly in the basis. This is the main reason for using the scattered wave basis. The RG transformation is then performed for the interaction Hamiltonian in this representation. We reduce the bandwidth cutoff step by step, each step accompanied by renormalizations of the transmission amplitudes and interaction constants. We will find the differential equations for these stepwise renormalizations. The renormalized transmission amplitudes are then calculated by integrating these equations.

To transform the Hamiltonian to this basis, we introduce operators $c^\dagger_{k\sigma}$ and $d^\dagger_{k\sigma}$ that create electrons in states (1) and (2), respectively. Operators (a,b) and (c,d) are related by unitary transformation:

$$a_{k\sigma} = \int dq\,[\mathcal{A}^{q\sigma}_k c_{q\sigma} + \mathcal{B}^{q\sigma}_k d_{q\sigma}], \tag{25}$$

$$b_{k\sigma} = \int dq\,[\mathcal{C}^{q\sigma}_k d_{q\sigma} + \mathcal{D}^{q\sigma}_k c_{q\sigma}], \tag{26}$$

where

$$\mathcal{A}^{q\sigma}_k = \frac{i}{2\pi}\left[\frac{1}{k-q+i0^+} - \frac{t_{q\sigma}}{k-q-i0^+}\right], \tag{27}$$

$$\mathcal{B}^{q\sigma}_k = \frac{-i}{2\pi}\frac{r'_{q\sigma}}{k-q-i0^+}, \tag{28}$$

$$\mathcal{C}^{q\sigma}_k = \frac{i}{2\pi}\left[\frac{1}{k-q+i0^+} - \frac{t'_{q\sigma}}{k-q-i0^+}\right], \tag{29}$$

$$\mathcal{D}^{q\sigma}_k = \frac{-i}{2\pi}\frac{r_{q\sigma}}{k-q-i0^+}. \tag{30}$$

Here r, t, r', and t' are reflection and transmission amplitudes defined in the way similar to the bare amplitudes in Eqs. (1) and (2).

Under transformations (25) and (26), the interaction Hamiltonian (22) becomes

$$\begin{aligned} H_{\rm int} = \frac{i}{(2\pi)^2}\sum_{\sigma\sigma'}\int dq_1 dq_2 dq_3 dq_4 \Big\{ & g_1\Big[\frac{c^\dagger_{q_1\sigma}\left(t'^*_{q_2\sigma'} d^\dagger_{q_2\sigma'} + r^*_{q_2\sigma'} c^\dagger_{q_2\sigma'}\right) c_{q_3\sigma'}\left(t'_{q_4\sigma} d_{q_4\sigma} + r_{q_4\sigma} c_{q_4\sigma}\right)}{q_1+q_4-q_2-q_3+i0^+} \\ & - \frac{\left(t^*_{q_1\sigma} c^\dagger_{q_1\sigma} + r'^*_{q_1\sigma} d^\dagger_{q_1\sigma}\right) d^\dagger_{q_2\sigma'}\left(t_{q_3\sigma'} c_{q_3\sigma'} + r'_{q_3\sigma'} d_{q_3\sigma'}\right) d_{q_4\sigma}}{q_1+q_4-q_2-q_3-i0^+}\Big] \\ & + g_2\Big[\frac{c^\dagger_{q_1\sigma}\left(t'^*_{q_2\sigma'} d^\dagger_{q_2\sigma'} + r^*_{q_2\sigma'} c^\dagger_{q_2\sigma'}\right)\left(t'_{q_3\sigma'} d_{q_3\sigma'} + r_{q_3\sigma'} c_{q_3\sigma'}\right) c_{q_4\sigma}}{q_1+q_3-q_2-q_4+i0^+} \\ & - \frac{\left(t^*_{q_1\sigma} c^\dagger_{q_1\sigma} + r'^*_{q_1\sigma} d^\dagger_{q_1\sigma}\right) d^\dagger_{q_2\sigma'} d_{q_3\sigma'}\left(t_{q_4\sigma} c_{q_4\sigma} + r'_{q_4\sigma} d_{q_4\sigma}\right)}{q_1+q_3-q_2-q_4-i0^+}\Big] + g_4 \times [\cdots]\Big\}. \end{aligned} \tag{31}$$

The g_4 term is not written explicitly, since it only contributes to the renormalization of the Fermi velocity and does not affect the interaction constants nor the reflection and transmission amplitudes. Thus the renormalizations of the transmission amplitudes are caused by the density-density interaction g_2 and backscattering g_1 between the left- and right-movers.

We now reduce the bandwidth to $D_0 - \delta D_0$. The RG transformation of the Hamiltonian is given by Eq. (24). The new Hamiltonian defined in the narrow band is obtained by eliminating the degrees of freedom in the energy range $(D_0 - \delta D_0, D_0)$. This is done by replacing any product of two fermion operators in $H'_{\rm int}$ that do not belong to the narrow band by its average value. The first term in $H'_{\rm int}$ generates additional quadratic terms in the new Hamiltonian. They correspond to the Hartree and exchange potentials created by the electrons in the eliminated band. In the absence of a barrier these terms are diagonal and yield only a renormalization of the electron dispersion relation. In the present case the quadratic part of the Hamiltonian contains also nondiagonal terms:

$$H'_0 = \sum_\sigma \int dk \tilde{\epsilon}(k) \left(c^\dagger_{k\sigma} c_{k\sigma} + d^\dagger_{k\sigma} d_{k\sigma}\right) - \frac{i}{8\pi^2} \sum_{\sigma\sigma'} (g_1 - g_2 \delta_{\sigma\sigma'}) \int dk dp \left[\left(t^*_{k\sigma} r'_{q\sigma'} - t'_{k\sigma} r^*_{q\sigma'}\right) c^\dagger_{k\sigma} d_{p\sigma} - \text{H.c} \right] \frac{\delta D_0}{D_0 + \epsilon(k) + \epsilon(p)}, \tag{32}$$

where $\tilde{\epsilon}(k)$ is the renormalized dispersion relation; q is the wave vector of an electron with energy $-D_0$.

This part of the Hamitonian can be diagonalized by slight changes of r, t, r', and t' in transformation (25)–(30). These changes correspond to the renormalizations of the reflection and transmission amplitudes under the RG transformation. In the case of a symmetric barrier ($r = r'$), all these renormalizations can be expressed in terms of the corrections to $t_\sigma(\epsilon)$,

$$\delta t_\sigma(\epsilon) = \left[\frac{g_2 - g_1}{2\pi\hbar v_F} t_\sigma |r_\sigma|^2 - \frac{g_1}{2\pi\hbar v_F} t_\sigma r_\sigma r^*_{-\sigma} \right] \frac{\delta D_0}{D_0 + \epsilon}. \tag{33}$$

The second term in Eq. (24) generates the renormalizations of the interaction constants. A straightforward calculation gives the differential RG-equations for these constants, and yields[17] the cutoff dependences:

$$g_1(\xi) = \frac{V(2k_F)}{1 + \frac{V(2k_F)}{\pi\hbar v_F}\xi}, \tag{34}$$

$$g_2(\xi) = V(0) - \frac{1}{2} V(2k_F) + \frac{1}{2} \frac{V(2k_F)}{1 + \frac{V(2k_F)}{\pi\hbar v_F}\xi}, \tag{35}$$

where $\xi = \ln(D_0/D)$. In the presence of a single scatterer the above results are still valid and do not depend on t and r. For a finite system of length L, it is easy to show that the correction to the interaction constants due to the barrier is small as $1/L$ and vanishes in the limit of long channel.

In the leading logarithm approximation we take into account only the corrections linear in $g_{1,2}$. Thus we only consider the first two terms in the right-hand side of Eq. (24).

The next steps of the RG procedure are described by the differential equations for the transmission amplitudes t_σ. In the spin-degenerate case, one finds from Eq. (33) the RG equation:

$$\frac{dt}{d\xi} = -\frac{g_2(\xi) - 2g_1(\xi)}{2\pi\hbar v_F} t(1 - |t|^2). \tag{36}$$

Due to the spin degeneracy, the g_1 term emerging from the Hartree potential has an extra factor of 2. Taking into account the ξ dependence (35) of g_1 and g_2 and integrating Eq. (36) from $\xi = 0$ to $\xi = \ln(D_0/|\epsilon|)$, we find

$$t(\epsilon) = \frac{t_0 \left[1 + 2\alpha_1 \ln \frac{D_0}{|\epsilon|}\right]^{3/4} \left|\frac{\epsilon}{D_0}\right|^{\alpha_2 - \alpha_1/2}}{\sqrt{\mathcal{R}_0 + \mathcal{T}_0 \left[1 + 2\alpha_1 \ln \frac{D_0}{|\epsilon|}\right]^{3/2} \left|\frac{\epsilon}{D_0}\right|^{2\alpha_2 - \alpha_1}}}, \tag{37}$$

where the parameters α_1 and α_2 are defined by Eq. (12). The above formula is different from the spinless case due to the renormalization of interaction constants caused by the backscattering.

IV. LINEAR CONDUCTANCE AND NONLINEAR *I-V* CHARACTERISTICS

A. Spinless case

The renormalized transmission coefficient (21) allows us to find the temperature dependence of the linear conductance of a 1D spinless interacting electron system with a single barrier. At high temperatures $k_B T > D_0$ the conductance is given by the Landauer formula for an ideal Fermi gas, $G_0 = (e^2/2\pi\hbar)\mathcal{T}_0$. At smaller temperatures the transmission coefficient is renormalized. Because of the smearing of the Fermi surface, ϵ in Eq. (21) should be replaced by $k_B T$, and the following temperature dependence of the linear conductance is found:

$$G(T) = \frac{e^2}{2\pi\hbar} \frac{\mathcal{T}_0 (k_B T/D_0)^{2\alpha}}{\mathcal{R}_0 + \mathcal{T}_0 (k_B T/D_0)^{2\alpha}}. \tag{38}$$

Formula (38) gives the power-law dependence of $G(T)$ at small T. This asymptotics coincides with the result of the Luttinger-liquid theory.[10] In contrast with this theory, our result (38) describes the behavior of linear conductance at all temperatures. Besides, Eqs. (11) and (12) give the microscopic definition of the exponent α in terms of the interaction potential. In the case of a smooth potential Eq. (11) coincides with the exponent found in Ref. 13.

The differential conductance $G(V)$ at a high voltage $eV > k_BT$ may be obtained by substitution $T \to (e/k_B)V$.

For a channel of finite length L, renormalization in (20) should be stopped at the level spacing, i.e., at $D \sim \hbar v_F/L$. The conductance is temperature and voltage independent at $k_BT, eV \ll \hbar v_F/L$.

B. Spin-$\frac{1}{2}$ electrons

Using the transmission amplitude (37), we can now find the conductance for spin-$\frac{1}{2}$ electrons using the Landauer formula,

$$G(T) = \frac{e^2}{\pi\hbar} \frac{\mathcal{T}_0 \left[1 + 2\alpha_1 \ln \frac{D_0}{k_BT}\right]^{3/2} \left(\frac{k_BT}{D_0}\right)^{2\alpha_2-\alpha_1}}{\mathcal{R}_0 + \mathcal{T}_0 \left[1 + 2\alpha_1 \ln \frac{D_0}{k_BT}\right]^{3/2} \left(\frac{k_BT}{D_0}\right)^{2\alpha_2-\alpha_1}}. \tag{39}$$

The above expression indicates a qualitative difference from the Luttinger-liquid behavior. In the absence of backscattering, $\alpha_1 = 0$, the existence of the spin degrees of freedom leads only to a trivial factor of 2, as compared to the conductance (38) in the spinless case. At $\alpha_1 > 0$ the backscattering gives rise to the logarithmic factors in Eq. (39) that determine the deviation from the power-law asymptotics of conductance at $T \to 0$,

$$G(T)|_{T\to 0} = \frac{e^2}{\pi\hbar} \frac{\mathcal{T}_0}{\mathcal{R}_0} \left[2\alpha_1 \ln \frac{D_0}{k_BT}\right]^{3/2} \left(\frac{k_BT}{D_0}\right)^{2\alpha_2-\alpha_1}. \tag{40}$$

At sufficiently strong backscattering, $\alpha_1 > \frac{1}{2}\alpha_2$, the temperature dependence of conductance becomes nonmonotonic. As the temperature is lowered from D_0/k_B, the conductance (39) first grows, reaches a maximum value at

$$T \sim \frac{D_0}{k_B} \exp\left(-\frac{2\alpha_1 - \alpha_2}{\alpha_1(2\alpha_2 - \alpha_1)}\right), \tag{41}$$

and finally drops to zero at $T \to 0$; see Fig. 2. This can be seen also from Eqs. (34), (35), and (36). At relatively high temperature, renormalizations of g_1 and g_2 are small, the left-hand side of Eq. (36) is positive, and $\mathcal{T}$ grows. However, at lower temperature, g_1 finally renormalizes towards zero, while g_2 flows to a constant value. At temperature (41), the maximum is reached, since $2g_1 = g_2$. Below this temperature we have $2g_1 < g_2$, and the conductance vanishes at $T = 0$.

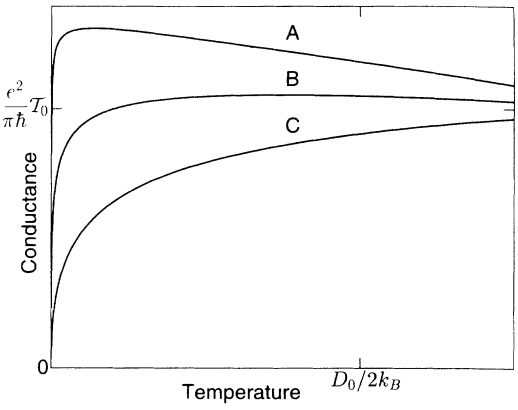

FIG. 2. Temperature dependence of linear conductance for spin-$\frac{1}{2}$ electrons. Plot of Eq. (39) with $\mathcal{T}_0 = 1/2$ for (a) $\alpha_1 = 3/8, \alpha_2 = 3/8$; (b) $\alpha_1 = 1/4, \alpha_2 = 3/8$; (c) $\alpha_1 = 1/8, \alpha_2 = 3/8$. The conductance has a maximum when $2\alpha_1 > \alpha_2$.

C. Linear conductance in the presence of a magnetic field

The deviation of $G(T)$ from the Luttinger-liquid behavior is due to the renormalizations induced by the backscattering of electrons with opposite spins. The effect of electrons of opposite spin is twofold: first, they produce one more Friedel oscillation that causes extra renormalization of the transmission amplitudes; second, the backscattering renormalizes the interaction constants. The interaction between electrons of opposite spin is responsible for the deviations from the Luttinger-liquid behavior at all energies. In the presence of magnetic field B, virtual states in higher order scattering processes have energy at least $2\mu_BB$. So the backscattering is important only at energy scales exceeding $2\mu_BB$. At these energies the transmission amplitude $t(|\epsilon| \geq 2\mu_BB)$ is adequately described by Eq. (37). At energies below the Zeeman splitting $2\mu_BB$ the renormalization due to the backscattering is suppressed, and the Luttinger-liquid behavior restores. The renormalizations of the transmission amplitude for each spin direction follow Eq. (20) for spinless electrons. The only difference occurs in the initial conditions that result from the renormalizations at higher energies according to (37),

$$\tilde{t}_0 = t|_{\epsilon=\tilde{D}_0}, \quad \tilde{D}_0 = 2\mu_BB. \tag{42}$$

Thus the conductance at low temperatures $k_BT \ll \mu_BB$ is described by Eq. (38) with initial conditions (42) and with the extra factor of 2 accounting for the two possible spin directions:

$$G(T) = \frac{e^2}{\pi\hbar} \frac{\tilde{\mathcal{T}}_0 (k_BT/\tilde{D}_0)^{2\alpha}}{\tilde{\mathcal{R}}_0 + \tilde{\mathcal{T}}_0 (k_BT/\tilde{D}_0)^{2\alpha}}, \quad \tilde{\mathcal{T}}_0 \equiv (1 - \tilde{\mathcal{R}}_0) = |\tilde{t}_0|^2. \tag{43}$$

V. NONLINEAR *I-V* CHARACTERISTICS IN THE MAGNETIC FIELD

Magnetic field removes the degeneracy between the electron states with opposite spin directions. Zeeman splitting results in the appearance of two different values of Fermi wave vector, $k_{F\uparrow}$ and $k_{F\downarrow}$. These wave vectors

define two different periods of Friedel oscillations in the two spin subsystems. At low temperatures only the electrons with energies close to the Fermi level contribute to the linear conductance. The low-energy electrons are scattered effectively only by the same-spin Friedel oscillation, therefore the renormalizations of transmission coefficients occur in each spin subsystem independently. However, it is possible to make the electrons scatter effectively on Friedel oscillations produced by the opposite-spin subsystem by applying a finite bias $eV \approx 2\mu_B B$. From Sec. II A, the Hartree potential enhances transmission, therefore we expect a peak in the differential conductance at this bias.

At a finite bias V the current $I(V)$ can be expressed in terms of transmission coefficients $\mathcal{T}_\sigma(\epsilon, V)$ that in turn depend on voltage,

$$I(V) = \frac{e}{2\pi\hbar} \sum_\sigma \int_{-eV}^{0} \mathcal{T}_\sigma(\epsilon, V) d\epsilon. \tag{44}$$

Here $\sigma = \pm 1$ characterizes the spin direction of tunneling electron and ϵ is its energy counted from the Fermi level at the left lead. To calculate the transmission coefficients $\mathcal{T}_\sigma(\epsilon, V)$ in the presence of bias and Zeeman splitting, one has to generalize the RG equation (33). The renormalization of the transmission amplitude occurs only if the wave vector k of the scattering electron is close to at least one of the four Fermi wave vectors of the four Friedel oscillations (corresponding to two spin directions and two leads). Thus we will do the RG in momentum space, cutting the electron bands at $k \pm D/\hbar k_F$, with D being some cutoff parameter. The resulting RG equations have the form:

$$\frac{dt_\sigma}{d\ln(D_0/D)} = -\frac{g_2 - g_1}{4\pi\hbar v_F} t_\sigma |r_\sigma|^2 \left[\theta(D - |\epsilon|) + \theta(D - |\epsilon + eV|)\right] + \frac{g_1}{4\pi\hbar v_F} t_\sigma r_\sigma r^*_{-\sigma} \left[\theta(D - |\epsilon - 2\sigma\mu_B B|) + \theta(D - |\epsilon + eV - 2\sigma\mu_B B|)\right]. \tag{45}$$

Each of the four Friedel oscillations contributes to Eq. (45) only if the corresponding Fermi wave vector belongs to the band of half-width $D/\hbar v_F$. These constraints are represented by the four step-functions in the right-hand side of Eq. (45). The terms proportional to $g_2 - g_1$ are due to the Friedel oscillations from the same-spin electrons which contribute to both exchange and Hartree potentials. The remaining terms are produced by interaction with the opposite-spin electrons.

Equation (45) can be easily integrated. In the limit of strong barrier, $\mathcal{T}_\sigma \ll 1$, the solution of RG equation (45) at $D \to 0$ results in the transmission coefficient

$$\mathcal{T}_\sigma(\epsilon) = \mathcal{T}_0 \left|\frac{\epsilon}{D_0}\right|^{\alpha_2 - \alpha_1} \left|\frac{\epsilon + eV - 2\sigma\mu_B B}{D_0}\right|^{-\alpha_1} \times \left|\frac{\epsilon + eV}{D_0}\right|^{\alpha_2 - \alpha_1} \left|\frac{\epsilon - 2\sigma\mu_B B}{D_0}\right|^{-\alpha_1}. \tag{46}$$

Here, the renormalizations of the interaction constants were neglected.[19] This result combined with Eq. (44) allows us to calculate the I-V characteristic. As it is clearly seen from Eq. (46), two singularities in $\mathcal{T}_\sigma(\epsilon)$ merge at $eV = 2\mu_B B$. As a result a singularity appears in the I-V characteristic. The latter manifests itself as a peak in differential conductance,

$$\delta G = \frac{e^2}{\pi\hbar} \widetilde{\mathcal{T}}_0 \frac{\alpha_1}{\alpha_2 - 2\alpha_1} \left(1 - \left|\frac{eV - 2\mu_B B}{2\mu_B B}\right|^{\alpha_2 - 2\alpha_1}\right), \tag{47}$$

schematically shown in Fig. 3. The last result was obtained in the linear $\mathcal{T}$ approximation that accounts for the processes in which an electron crosses the tunnel barrier only once. This approximation is equivalent to the standard method of calculation of tunnel current in terms of the transmission coefficient and tunnel densities of states in the leads. Indeed, the expression (46) may be interpreted as the product of the bare transmission coefficient $\mathcal{T}_0$ and the power-law factors corresponding to the singular energy dependence of the tunnel densities of states. Apparently this method fails if the differential conductance becomes of the order of $e^2/\hbar$. This does not happen for a sufficiently long-range interaction potential, $g_2 - 2g_1 > 0$. In the case of a short-range potential, $g_2 - 2g_1 < 0$, formula (47) is not applicable in a very narrow vicinity of the point $V = 2\mu_B B/e$.

Although transmission amplitudes can be calculated exactly from Eq. (45) for any barrier strength, it is difficult to find the conductance at arbitrary α_1 and α_2. A surprisingly simple expression for the differential conductance near the peak can be obtained, however, for the short-range interaction when $\alpha_1 = \alpha_2$. In this case the peak shape has the form

$$G = \frac{e^2}{\pi\hbar} \frac{\widetilde{\mathcal{T}}_0}{\widetilde{\mathcal{T}}_0 + \widetilde{\mathcal{R}}_0 \left|\frac{eV - 2\mu_B B}{2\mu_B B}\right|^{\alpha_1}}; \tag{48}$$

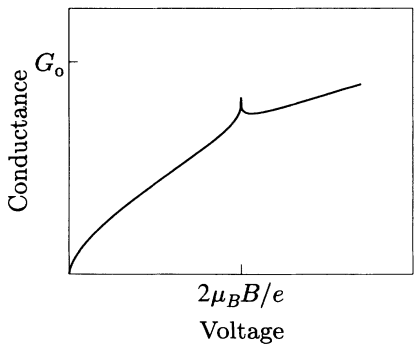

FIG. 3. The differential conductance $G(V) = dI/dV$ of a quantum wire with a strong barrier in the presence of magnetic field B. The power-law behavior at $V \to 0$ is consistent with the Luttinger-liquid theory (Ref. 10). The power-law peak (47) at $V = 2\mu_B B/e$ is due to the scattering of electrons on the Friedel oscillation produced by the opposite-spin electrons. The curve was calculated for $\alpha_1 = 1/16$, $\alpha_2 = 3/8$.

see the Appendix. The enhancement of tunneling due to the scattering on the opposite-spin Friedel oscillation gives rise to the perfect conductance $e^2/\pi\hbar$ at $eV = 2\mu_B B$. In the limit of small transmission coefficient, Eq. (48) agrees with the asymptotic formula (47).

VI. CONCLUSION

In this paper, we solved the problem of scattering on a single impurity for weakly interacting 1D electrons. A simple renormalization-group method allows us to derive the equations (20), (36), and (45) for transmission amplitudes. In these equations, scattering on the barrier is incorporated exactly. The electron-electron backscattering is also trivially included in Eq. (36). This enables us to find the transmission amplitudes (37) for a spin-$\frac{1}{2}$ electron system in the presence of a barrier of arbitrary strength. Conductance of the system is found at any temperature, see Eqs. (38), (39), and (43). For a real spin-$\frac{1}{2}$ system, the backscattering gives rise to deviations from the power-law temperature dependence of linear conductance at $T \to 0$; see Eq. (40). In the presence of a magnetic field, the backscattering creates a peak in the differential conductance at bias $V = 2\mu_B B$, as described by Eqs. (47) and (48). The found transmission amplitudes can be used for investigating other properties of interacting 1D electron systems.[20]

ACKNOWLEDGMENTS

The authors are grateful to H. U. Baranger, C. L. Kane, A. I. Larkin, and B. Z. Spivak for helpful discussions. This work was supported by NSF Grant No. DMR-9117341.

APPENDIX A: DERIVATION OF CONDUCTANCE PEAK SHAPE FOR A SHORT-RANGE INTERACTION

In this appendix we outline the intermediate steps in obtaining formula (48). Let us consider tunneling of a spin-up electron incoming from the left with energy close to the Fermi level. Since we are interested in the vicinity of the singularity at $eV = 2\mu_B B$, the most important terms in Eq. (45) are the two terms corresponding to

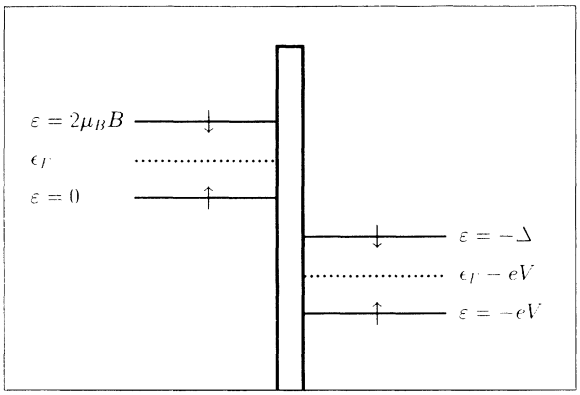

FIG. 4. Energy diagram of a biased junction in a magnetic field. Horizontal solid lines correspond to the mechanical energies of Fermi electrons with different spins shown by arrows. Dotted lines represent the Fermi levels of electrons on the two sides of the barrier. By definition, $\Delta = eV - 2\mu_B B$.

Friedel oscillations formed by spin-up electrons on the left and spin-down electrons on the right, the remaining two terms can be neglected. Similar arguments can be applied to the tunneling of spin-down electrons. For short-range interaction ($\alpha_1 = \alpha_2$) the contribution of the Friedel oscillation formed by the same-spin electrons vanishes. Since renormalizations depend on the difference between the wave vector k of a tunneling electron and the closest Fermi wave vector characterizing a Friedel oscillation, it is natural to write a closed system of equations for $t_\uparrow$ and $t_\downarrow$ in terms of the mechanical energy ε rather than the total energy ϵ. It is convenient to count ε from the "mechanical" Fermi level $\epsilon_F - \mu_B B$ for the spin-up electrons; see Fig. 4. Then RG equations (45) take the form

$$\frac{dt_\uparrow}{d\ln(D_0/D)} = \frac{1}{2}\alpha_1 t_\uparrow r_\uparrow r_\downarrow^* \theta(D - |\varepsilon + \Delta|), \tag{A1}$$

$$\frac{dt_\downarrow}{d\ln(D_0/D)} = \frac{1}{2}\alpha_1 t_\downarrow r_\downarrow r_\uparrow^* \theta(D - |\varepsilon|). \tag{A2}$$

Integrating over D, we can obtain the transmission coefficients. At $|\varepsilon + \Delta| > |\varepsilon|$ we have

$$\mathcal{T}_\uparrow(\varepsilon) = \frac{\mathcal{T}_0}{\mathcal{T}_0 + \mathcal{R}_0 \left|\frac{\varepsilon+\Delta}{D_0}\right|^{\alpha_1}}, \tag{A3}$$

$$\mathcal{T}_\downarrow(\varepsilon) = 1 - \left\{ \frac{[1+\rho_\uparrow(\varepsilon)]\left|\frac{\varepsilon}{\varepsilon+\Delta}\right|^{\alpha_1\rho_\uparrow(\varepsilon)} - [1-\rho_\uparrow(\varepsilon)]}{[1+\rho_\uparrow(\varepsilon)]\left|\frac{\varepsilon}{\varepsilon+\Delta}\right|^{\alpha_1\rho_\uparrow(\varepsilon)} + [1-\rho_\uparrow(\varepsilon)]} \right\}^2, \quad \rho_\uparrow(\varepsilon) \equiv \sqrt{1-\mathcal{T}_\uparrow(\varepsilon)}. \tag{A4}$$

Similarly, at $|\varepsilon + \Delta| < |\varepsilon|$ one finds

$$\mathcal{T}_\downarrow(\varepsilon) = \frac{\mathcal{T}_0}{\mathcal{T}_0 + \mathcal{R}_0 \left|\frac{\varepsilon}{D_0}\right|^{\alpha_1}}, \tag{A5}$$

$$\mathcal{T}_{\uparrow}(\varepsilon) = 1 - \left\{ \frac{[1+\rho_{\downarrow}(\varepsilon)]\left|\frac{\varepsilon+\Delta}{\varepsilon}\right|^{\alpha_1\rho_{\downarrow}(\varepsilon)} - [1-\rho_{\downarrow}(\varepsilon)]}{[1+\rho_{\downarrow}(\varepsilon)]\left|\frac{\varepsilon+\Delta}{\varepsilon}\right|^{\alpha_1\rho_{\downarrow}(\varepsilon)} + [1-\rho_{\downarrow}(\varepsilon)]} \right\}^2, \quad \rho_{\downarrow}(\varepsilon) \equiv \sqrt{1-\mathcal{T}_{\downarrow}(\varepsilon)}. \tag{A6}$$

Notice that $\mathcal{T}_{\uparrow,\downarrow}(\varepsilon)$ are also functions of $\Delta = eV - 2\mu_B B$. To calculate conductance, one differentiates the current (44) over the voltage:

$$\begin{aligned} G(V) &= \frac{e}{2\pi\hbar}\frac{\partial}{\partial V}\left(\int_{-eV}^{0}\mathcal{T}_{\uparrow}(\varepsilon)d\varepsilon + \int_{-eV+2\mu_B B}^{2\mu_B B}\mathcal{T}_{\downarrow}(\varepsilon)d\varepsilon\right) \\ &= \frac{e^2}{2\pi\hbar}\left(\mathcal{T}_{\uparrow}(-eV) + \frac{1}{e}\int_{-eV}^{0}\frac{\partial}{\partial V}\mathcal{T}_{\uparrow}(\varepsilon)d\varepsilon + \mathcal{T}_{\downarrow}(-eV+2\mu_B B) + \frac{1}{e}\int_{-eV+2\mu_B B}^{2\mu_B B}\frac{\partial}{\partial V}\mathcal{T}_{\downarrow}(\varepsilon)d\varepsilon\right). \end{aligned} \tag{A7}$$

The limits of integration for the spin-down part in (A7) are shifted by $2\mu_B B$ due to our choice of the reference level for mechanical energy ε. Substituting (A3)–(A6) into (A7) one finds the result (48).

---

* On leave from the Institute of Solid State Physics, Chernogolovka, Moscow distr., 142432, Russia.

[1] B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G Williamson, L.P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. **60**, 848 (1988).

[2] C. W. J. Beenakker and H. van Houten, in *Solid State Physics,* edited by H. Ehrenreich and D. Turnbull (Academic Press, San Diego, 1991), Vol. 44.

[3] L. I. Glazman, G. B. Lesovik, D. E. Khmel'nitskii, and R. I. Shekhter, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 239 (1988) [JETP Lett. **48**, 238 (1988)].

[4] G. Timp, in *Semiconductors and Semimetals,* edited by M. A. Reed (Academic Press, New York, 1990).

[5] L. I. Glazman and M. Jonson, Phys. Rev. B **44**, 3810 (1991).

[6] R. Landauer, IBM J. Res. Dev. **1**, 223 (1957).

[7] D. C. Mattis, Phys. Rev. Lett. **32**, 714 (1974).

[8] A. Luther and I. Peschel, Phys. Rev. Lett. **32**, 992 (1974).

[9] A. Luther and V. J. Emery, Phys. Rev. Lett. **33**, 589 (1974).

[10] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. **68**, 1220 (1992).

[11] A. Furusaki and N. Nagaosa, Phys. Rev. B **47**, 4631 (1993).

[12] L. I. Glazman, I. M. Ruzin, and B. I. Shklovskii, Phys. Rev. B **45**, 8454 (1992).

[13] K. A. Matveev and L. I. Glazman, Phys. Rev. Lett. **70**, 990 (1993); Physica B **189**, 266 (1993).

[14] M. H. Devoret, D. Esteve, H. Grabert, G.-L. Ingold, H. Pothier, and C. Urbina, Phys. Rev. Lett. **64**, 1824 (1990).

[15] S. M. Girvin, L. I. Glazman, M. Jonson, D. R. Penn, and M. D. Stiles, Phys. Rev. Lett. **64**, 3183 (1990).

[16] P. W. Anderson, J. Phys. C **3**, 2436 (1970).

[17] J. Solyom, Adv. Phys. **28**, 201 (1979).

[18] The umklapp processes conventionally described by g_3 are neglected, since in 1D semiconductor structures electron density is always far from half-filling.

[19] This approximation is valid for small g_1, when $g_1 \ln(D_0/\mu_B B) \ll 1$. If the latter condition is not satisfied, the result (47) should be corrected by the substitution of the renormalized values (34) and (35) of $g_{1,2}$ at $D = 2\mu_B B$ into the definition $\alpha_{1,2} = g_{1,2}/2\pi\hbar v_F$.

[20] C. L. Kane, K. A. Matveev, and L. I. Glazman, Phys. Rev. B (to be published).