## Exciton polaritons in double versus single quantum wells: Mechanism for increased luminescence linewidths in double quantum wells

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Exciton radiative decay and polaritons in single and double quantum wells are compared and contrasted. Two types of eigenmodes exist in double quantum wells, with the dipole moment in the quantum wells parallel or antiparallel. Energy splittings  $\Delta(\mathbf{k}_{\parallel})$  between the two types of modes at small  $\mathbf{k}_{\parallel}$  are on the order of 0.1 meV for GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As-based structures of well width ~100 Å, which accounts for part of the increased luminescence linewidth in double compared with single quantum wells. Thus the polariton effect gives rise to an intrinsic source of inhomogeneous broadening through  $\Delta(\mathbf{k}_{\parallel})$  as well as homogeneous broadening through the radiative width, and both effects are of the same magnitude.

In single quantum wells (SQW's) the interaction of the vacuum electromagnetic (em) field with excitons gives rise to a nonzero radiative width for the coupled exciton—em-field states (polaritons)<sup>1</sup> with in-plane center-of-mass wave vector  $k_{\parallel} < \kappa_{ex} = E_{ex}(\mathbf{k}_{\parallel})/(\hbar c) [\approx \frac{2\pi}{\lambda}]$ where  $\lambda$  is the optical wavelength of the emitted photon,  $c = c_0/\sqrt{\epsilon_{\infty}}$  is the speed of light in the dielectric medium excluding the exciton resonance, and  $E_{ex}(\mathbf{k}_{\parallel})$  is the  $\mathbf{k}_{\parallel}\text{-dependent}$  exciton energy] and a radiative shift for  $k_{\parallel} > \kappa_{\text{ex}}$ .<sup>2-5</sup> Recent time-resolved photoluminescence measurements show that the radiative lifetime of the lowest-lying 1s heavy-hole exciton with  $\mathbf{k}_{\parallel}\sim\mathbf{0}$  in highquality GaAs/AlAs SQW's is  $\sim 10 \text{ ps},^6$  in agreement with theoretical predictions,<sup>3</sup> compared with  $\sim 1$  ns in bulk GaAs samples.<sup>7</sup> The large decay rates for the small- $\mathbf{k}_{\parallel}$  SQW excitons provide an example of super-radiance<sup>8</sup> while the vanishing rates associated with large- $\mathbf{k}_{\parallel}$  states is termed subradiance.

The general concept of the polariton has also proven fruitful in describing excitonic radiative decay in low-dimensional semiconductor structures other than SQW's.<sup>4,9-12</sup> We have recently traced the radiative behavior of excitons in multiple QW's as the number N of wells goes from 1 to  $\infty$ .<sup>9</sup> This treatment fully includes the effects of retardation in the framework of polaritons. The eigenmodes of an exciton interacting with the em field in such a structure can be characterized by the relative magnitude and phase of the dipole moment associated with the excitation in successive QW's. In addition, each mode is associated with an upper and a lower polariton branch, and are split one from another on an energy scale of 0.1 meV in  $GaAs/Al_xGa_{1-x}As$  structures.<sup>9</sup> As N becomes large, these different modes form a "miniband" of polariton states which are characterized by a wave vector  $k_z$  in the growth direction, as well as  $\mathbf{k}_{\parallel}$ . It was also shown that the radiative decay rates (radiative widths) of excitons not only depend strongly upon  $\mathbf{k}_{\parallel}$ , but also on N, the relative phase of the dipole moment in successive QW's, and the center-to-center QW spacing L. For  $N = \infty$ , the resulting system possesses

full translational invariance and the polariton dispersion displays a vanishing radiative width, as expected from the theory of bulk polaritons.<sup>1</sup> The theoretical treatment used is a Green-function approach by which the interacting modes composed of an exciton and the em field are obtained nonperturbatively from the poles of the dipole correlation function (CF).<sup>5,11</sup> The polariton dispersion then gives the radiative widths and shifts of the states. The exciton-polariton problem in the DQW is thus an exactly solvable model involving a finite number of discrete modes interacting with a continuum.

Apart from its pedagogic value, such an investigation is important in order to understand the fundamental excitonic optical properties of structures with  $N \neq 1$  due to their prevalent use in experimental studies and for applications. It has been known for quite some time that SQW's have narrower inhomogeneous linewidths than multiple QW's, in part because interwell fluctuations simply do not exist for a SQW.<sup>13</sup> Inhomogeneous linewidths in ultrahigh quality SQW's can be as small as a few tenths of a meV.<sup>14</sup> In very-high-quality samples, it is thought that well-width fluctuations show up primarily in terms of a spatially dependent exciton energy as the pump beam is moved between regions with different local thicknesses varying on the scale of a monolayer.<sup>6</sup> In this paper we report explicit theoretical results for the dispersion of exciton polaritons in the DQW, i.e., N = 2. We show that the  $\mathbf{k}_{\parallel}$ -dependent energy splitting  $\Delta(\mathbf{k}_{\parallel})$  between the two types of polariton modes in the DQW-dipole moment in the individual QW's parallel and antiparallel-can be on the order of 0.1 meV for  $GaAs/Al_xGa_{1-x}As$ -based structures with well widths ~100 Å.  $\Delta(\mathbf{k}_{\parallel})$  is of the same magnitude and moreover shares a physical origin, i.e., the polariton effect, as the radiative width which contributes to the homogeneous line. Thus,  $\Delta({f k}_{\parallel})$  contributes to the inhomogeneous line, and we suggest it may account for part of the increased luminescence linewidths in DQW's compared with SQW's. The splitting  $\Delta(\mathbf{k}_{\parallel})$  near  $\mathbf{k}_{\parallel} \sim \mathbf{0}$ represents a fundamental lower limit to the inhomoge-

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neous linewidth, even in a structure that has no inhomogeneities. The present study thus constitutes a fully retarded generalization of the work presented in Refs. 15 and 16 which considers the static dipole interaction in multiple QW's. In Ref. 15 the exchange-interaction induced splittings between modes in an infinite multiple QW stack were considered and splittings in the range of meV's were found for GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As-based structures. In Ref. 16 the splittings in a DQW were found to give rise to an energy minimum away from  $\mathbf{k}_{\parallel} = \mathbf{0}$  thus leading to long radiative decay times at very low temperature. Both these treatments, however, neglect the effects of retardation, which must be taken into consideration in order to obtain the radiative decay rates.

We consider a structure consisting of two identical QW's of width  $L_z$  separated a distance L between centers.<sup>17</sup> The barrier between the QW's is assumed suf-

ficiently high and wide so that the interwell electronic coupling can be neglected. Rather, the coupling between QW's is solely via the retarded em field.<sup>18</sup> We further assume that  $L_z \ll \lambda$  to avoid a proliferation of formulas. Using the method of Refs. 5 and 9, we find the polariton dispersion relation from the dipole CF. In order that the dispersion relation be tractable without extensive numerical computations, we assume a single exciton spin state s coupled to a unique polarization  $\epsilon$  of the em field. The directions  $\epsilon$  are chosen so that unit vectors  $\hat{\mathbf{n}}_{\epsilon}$ obey  $\hat{\mathbf{n}}_T \cdot \hat{\mathbf{k}}_{\parallel} = \hat{\mathbf{n}}_T \cdot \hat{\mathbf{z}} = \mathbf{0}, \ \hat{\mathbf{n}}_L = \hat{\mathbf{k}}_{\parallel}, \ \text{and} \ \hat{\mathbf{n}}_Z = \hat{\mathbf{z}}, \ \text{where}$  $\hat{\mathbf{z}}$  is the growth direction. The excitation in the DQW is expressed as a superposition of SQW exciton states in individual QW's l. We employ the tight-binding basis  $|l\rangle$ with  $l \in \{1, 2\}$ . The polariton dispersion relation is then given by the solutions of

$$0 = \begin{bmatrix} E_{\text{ex}}(\mathbf{k}_{\parallel})^2 + 2E_{\text{ex}}(\mathbf{k}_{\parallel})\hbar\Sigma_s(k) - E_{(i)}(k) & 2E_{\text{ex}}(\mathbf{k}_{\parallel})\hbar\Sigma_s^{(1)}(k)e^{-\alpha L} \\ 2E_{\text{ex}}(\mathbf{k}_{\parallel})\hbar\Sigma_s^{(1)}(k)e^{-\alpha L} & E_{\text{ex}}(\mathbf{k}_{\parallel})^2 + 2E_{\text{ex}}(\mathbf{k}_{\parallel})\hbar\Sigma_s(k) - E_{(i)}(k) \end{bmatrix} \underline{G}_{(i)}(k),$$

$$i\varepsilon = E_{(i)}(k), \qquad (1)$$

where  $\hbar \Sigma_s(k) = \hbar \Sigma_s^{(1)}(k) + \hbar \Sigma_s^{(2)}(\mathbf{k}_{\parallel})$  is the proper radiative self-energy (SE),  $\hbar \Sigma_s^{(1)}(k)$  is the regular part of the SE, and  $\hbar \Sigma_s^{(2)}(\mathbf{k}_{\parallel})$  is the singular part. The other parameters are defined as follows:  $k = (i\varepsilon, \mathbf{k}_{\parallel})$ ,  $i\varepsilon$  is the complex energy,  $\alpha = [k_{\parallel}^2 - (i\kappa)^2]^{1/2}$ , and  $i\kappa = i\varepsilon/(\hbar c)$ . The SE describes the effects on the exciton due to its interaction with the em field. The SE term  $\hbar \Sigma_s^{(1)}(k)$  is nonlocal and accounts for the retarded interaction, i.e., the coherent emission and reabsorption of photons. The term  $\hbar \Sigma_s^{(2)}(\mathbf{k}_{\parallel})$  is energy independent and purely local, and thus unrelated to retardation effects.  $\hbar \Sigma_s^{(2)}(\mathbf{k}_{\parallel})$  is nonzero only for the Z mode and at  $k_{\parallel} = 0$  is equal to the quasi-two-dimensional analog of the longitudinal-transverse splitting.<sup>11</sup> For the case of arbitrary N, Eq. (1) is replaced by an  $N \times N$  matrix equation where the diagonal components of the matrix are the same as in Eq. (1) and the *ij*th off-diagonal component is  $2E_{\text{ex}}(\mathbf{k}_{\parallel})\hbar \Sigma_s^{(1)}(k) \exp(-\alpha|L_{ij}|)$  where  $L_{ij}$  is the center-to-center distance between QW's *i* and *j*. The expressions for the SE terms  $\hbar \Sigma_s^{(1)}(k)$  and  $\hbar \Sigma_s^{(2)}(\mathbf{k}_{\parallel})$  for the different modes are

 $= \begin{cases} -\frac{2\pi}{\alpha} \frac{(i\varepsilon)^2}{E_{\text{ex}}(\mathbf{k}_{\parallel})^2} |\mathbf{C}_s \cdot \hat{\mathbf{n}}_T|^2 |S|^2, \\ \frac{2\pi}{\alpha} \frac{(i\varepsilon)^2}{E_s(\mathbf{k}_{\parallel})^2} \frac{\alpha^2}{(i\omega)^2} |\mathbf{C}_s \cdot \hat{\mathbf{n}}_L|^2 |S|^2, \end{cases}$ 

$$\begin{split} \hbar \Sigma_{s}^{(1)}(k) &= \begin{cases} \frac{1}{\alpha} \frac{1}{E_{\text{ex}}(\mathbf{k}_{\parallel})^{2}} \frac{(i\kappa)^{2}}{(i\kappa)^{2}} |\mathbf{C}_{s} \cdot \hat{\mathbf{n}}_{L}|^{2} |S|^{2}, \\ &- \frac{2\pi}{\alpha} \frac{(i\varepsilon)^{2}}{E_{\text{ex}}(\mathbf{k}_{\parallel})^{2}} \frac{k_{\parallel}^{2}}{(i\kappa)^{2}} |\mathbf{C}_{s} \cdot \hat{\mathbf{n}}_{Z}|^{2} |S|^{2}, \end{cases} \\ &\hbar \Sigma_{s\epsilon}^{(2)}(\mathbf{k}_{\parallel}) = \frac{4\pi}{\kappa_{\text{ex}}^{2}} \delta_{\epsilon Z} |\mathbf{C}_{s} \cdot \mathbf{n}_{Z}|^{2} \mathcal{I}, \end{split}$$

with  $\mathcal{I} = \int dz |f_c(z)f_v(z)|^2$  and  $S = \int dz |f_c(z)f_v(z)|$ . Here  $f_v(z)$  and  $f_c(v)$  are the valence- and conduction-band single-particle envelope functions, respectively.  $\mathbf{C}_s$  is the coupling constant and is proportional to the (vector) dipole moment produced by the transition from exciton state s to the crystal ground state. It is defined as  $\mathbf{C}_s = \kappa_{\mathbf{ex}} F_{\mathbf{ex}}(\mathbf{0}) \langle (cv)_s | e \mathbf{R} | \mathbf{0} \rangle$ , where  $F_{\mathbf{ex}}(\mathbf{r}_{\parallel})$  is the exciton envelope function,  $|(cv)_s\rangle$  is the electron-hole pair of definite spin s formed from a conduction and valence subband, and  $|\mathbf{0}\rangle$  is the dipole matrix element between the electron-hole pair and the crystal ground state; e is the electron charge and  $\mathbf{R}$  is the position vector.  $\mathbf{C}_s$  is

T mode L mode

 $Z \bmod e,$ 

(3)

(2)

related to the oscillator strength per unit area  $f_\epsilon$  via

$$f_{\epsilon} = \frac{2m_0 E_{\text{ex}}(\mathbf{0}) |F_{\text{ex}}(\mathbf{0})|^2}{\hbar^2} \sum_{s} |\langle (cv)_s | \mathbf{R} \cdot \hat{\mathbf{n}}_{\epsilon} | \mathbf{0} \rangle|^2,$$

where  $m_0$  is the free-electron mass. Detailed explanations of all the symbols are given in Refs. 5 and 11. Because  $\hbar \Sigma_s^{(2)}(\mathbf{k}_{\parallel})$  is energy-independent, its only role here is to renormalize the exciton energy. The *l* component of the *i*th eigenvector  $\underline{G}_{(i)}(k)$  gives the amplitude and phase of the excitation in QW *l* in eigenstate *i*. The exponential in the off-diagonal terms gives exponentially short-range coupling for  $k_{\parallel} > \kappa$  but infinite-range coupling for  $k_{\parallel} < \kappa$  where  $\kappa = E/(\hbar c) \in \Re^+$ . Its energy dependence accounts for the interwell retardation.

Note that the matrix in Eq. (1) is complex symmetric rather than Hermitian due to the nonstationarity of the problem at hand. The solution to the eigenvalue problem for N=2 is trivial. One finds the two roots

$$E_{\pm}(k)^{2} = E_{e\mathbf{x}}(\mathbf{k}_{\parallel})^{2} + 2E_{e\mathbf{x}}(\mathbf{k}_{\parallel})\hbar\Sigma_{s}^{(2)}(\mathbf{k}_{\parallel}) + 2E_{e\mathbf{x}}(\mathbf{k}_{\parallel})\hbar\Sigma_{s}^{(1)}(k)\left(1 \pm e^{-\alpha L}\right), E_{\pm}(k) = i\varepsilon.$$
(4)

The  $\pm$  eigenstates correspond to the dipole moments of the two wells being parallel and antiparallel, respectively. The set of coupled equations (4) is not susceptible to an analytic solution due to its transcendental nature; however, for our purposes we can employ the exciton-pole approximation (EPA) (Refs. 4 and 19) to obtain the approximate dispersion.<sup>20</sup> The EPA here amounts to replacing  $i\varepsilon$  by  $E_{\rm ex}(\mathbf{k}_{\parallel})+i0^+$  in the SE terms of the dispersion relation. The imaginary part  $i0^+$  of the energy ensures that retarded temporal boundary conditions are selected so that we obtain decaying solutions. We then have

$$E_{\pm}(\mathbf{k}_{\parallel}) = E_{\mathbf{ex}}(\mathbf{k}_{\parallel}) + \hbar \Sigma_{s}^{(2)}(\mathbf{k}_{\parallel}) + \hbar \Sigma_{s}^{(1)} [E_{\mathbf{ex}}(\mathbf{k}_{\parallel}), \mathbf{k}_{\parallel}] (1 \pm e^{-\chi L})$$
(5)

where real energy arguments in the SE are understood to contain the term  $i0^+$  and

$$\chi = \begin{cases} -i\sqrt{\kappa_{\rm ex}^2 - k_{\parallel}^2}, & k_{\parallel} < \kappa_{\rm ex} \\ \sqrt{k_{\parallel}^2 - \kappa_{\rm ex}^2}, & k_{\parallel} > \kappa_{\rm ex}. \end{cases}$$
(6)

The quantity  $i\chi$  is the **z** component of the wave vector of the em field associated with the polariton; for  $k_{\parallel} < \kappa_{ex}$ energy-momentum conservation dictates that it is real (propagating), while for  $k_{\parallel} < \kappa_{ex}$  it is imaginary (evanescent). Within the EPA, the dispersion for the SQW is<sup>21</sup>

$$E(\mathbf{k}_{\parallel}) = E_{\mathbf{ex}}(\mathbf{k}_{\parallel}) + \hbar \Sigma_{s}^{(2)}(\mathbf{k}_{\parallel}) + \hbar \Sigma_{s}^{(1)}[E_{\mathbf{ex}}(\mathbf{k}_{\parallel}), \mathbf{k}_{\parallel}].$$
(7)

Thus, the DQW dispersion differs from that of the SQW through the regular SE term  $\hbar \Sigma_s^{(1)}$ . From Refs. 5 and 11, the EPA SQW regular SE can be written as

$$\begin{split} \hbar \Sigma_{s}^{(1)}[E_{\mathrm{ex}}(\mathbf{k}_{\parallel}), \mathbf{k}_{\parallel}] &= -i\hbar \Gamma_{s}(\mathbf{k}_{\parallel})\theta(\kappa_{\mathrm{ex}} - k_{\parallel}) \\ &+ \hbar \Pi_{s}(\mathbf{k}_{\parallel})\theta(k_{\parallel} - \kappa_{\mathrm{ex}}), \end{split} \tag{8}$$

where

$$\begin{split} &\hbar\Gamma_s(\mathbf{k}_{\parallel}) = -\mathrm{Im}\hbar\Sigma_s[E_{\mathrm{ex}}(\mathbf{k}_{\parallel}),\mathbf{k}_{\parallel}],\\ &\hbar\Pi_s(\mathbf{k}_{\parallel}) = \mathrm{Re}\hbar\Sigma_s[E_{\mathrm{ex}}(\mathbf{k}_{\parallel}),\mathbf{k}_{\parallel}]. \end{split}$$

If  $|\hbar\Gamma_s(\mathbf{k}_{\parallel})| \ll |E_{\text{ex}}(\mathbf{k}_{\parallel}) + \hbar\Pi_s(\mathbf{k}_{\parallel})|$ , then  $\hbar\Gamma_s(\mathbf{k}_{\parallel})$  and  $\hbar\Pi_s(\mathbf{k}_{\parallel})$  are approximately the radiative width and shift, respectively. The radiative decay rate of the exciton is  $2\Gamma_s(\mathbf{k}_{\parallel})$ . Using Eq. (8), we can write Eq. (4) as

$$E_{\pm}(\mathbf{k}_{\parallel}) = E_{ex}(\mathbf{k}_{\parallel}) + \hbar \Sigma_{s}^{(2)}(\mathbf{k}_{\parallel}) \\ \pm \hbar \Gamma_{s}(\mathbf{k}_{\parallel}) \sin L \sqrt{\kappa_{ex}^{2} - k_{\parallel}^{2}} \\ -i\hbar \Gamma_{s}(\mathbf{k}_{\parallel}) \left(1 \pm \cos L \sqrt{\kappa_{ex}^{2} - k_{\parallel}^{2}}\right)$$
(9)

for  $k_{\parallel} < \kappa_{\rm ex}$  and

$$E_{\pm}(\mathbf{k}_{\parallel}) = E_{\mathrm{ex}}(\mathbf{k}_{\parallel}) + \hbar \Sigma_{s}^{(2)}(\mathbf{k}_{\parallel}) + \hbar \Pi_{s}(\mathbf{k}_{\parallel}) \left[ 1 \pm \exp\left(-L\sqrt{k_{\parallel}^{2} - \kappa_{\mathrm{ex}}^{2}}\right) \right]$$
(10)

(a)

for  $k_{\parallel} > \kappa_{ex}$ . Thus, as in the SQW case, states for which  $k_{\parallel} > \kappa_{ex}$  are nonradiative. The second line of Eq. (9) also shows that the upper branch develops a real dispersion for the DQW—a feature absent for the SQW in the EPA. The resulting energy splittings at  $\mathbf{k}_{\parallel} = \mathbf{0}$  between the *T*, *L*, and *Z* modes are a manifestation of the development of a bulklike longitudinal-transverse splitting,<sup>9</sup> while the  $\pm$  splitting  $\Delta(\mathbf{k}_{\parallel}) = \operatorname{Re}[E_{+}(\mathbf{k}_{\parallel}) - E_{-}(\mathbf{k}_{\parallel})]$  for fixed  $\epsilon$  indicates the formation of a polariton "miniband." These splittings oscillate as functions of *L*. From Eqs. (9) and (10) we have

$$\begin{split} \Delta(\mathbf{k}_{\parallel}) &= 2\hbar\Gamma_{s}(\mathbf{k}_{\parallel})\sin\left(L\sqrt{\kappa_{\mathrm{ex}}^{2}-k_{\parallel}^{2}}\right)\theta(\kappa_{\mathrm{ex}}-k_{\parallel}) \\ &+ 2\hbar\Pi_{s}(\mathbf{k}_{\parallel})\exp\left(-L\sqrt{k_{\parallel}^{2}-\kappa_{\mathrm{ex}}^{2}}\right)\theta(k_{\parallel}-\kappa_{\mathrm{ex}}). \end{split}$$
(11)

For GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As-based structures with  $L_z \approx 100$ Å,  $\kappa_{\rm ex}^{-1} \approx 400$  Å and  $\hbar \Gamma_s(\mathbf{0})$  is on the order of 0.1 meV. Thus for  $L = (2n+1)\frac{\pi}{2}\kappa_{\rm ex}^{-1}$ ,  $|\Delta(\mathbf{k}_{\parallel})|$  reaches its maximum value on the order of  $\pm 0.1$  meV with  $\Delta(\mathbf{k}_{\parallel})$  changing sign for even and odd integers n, respectively. These splittings are not negligible compared with the homogeneous and inhomogeneous linewidths of state-of-the art ultrahigh quality GaAs/AlAs SQW's and may contribute to the wider linewidths in multiple QW's.<sup>13</sup>  $\Delta(\mathbf{k}_{\parallel})$  thus provides a source of inhomogeneous broadening for fixed polarization  $\epsilon$  in perfect DQW's not present in SQW's. An important point to be made is that the same intrinsic radiative mechanism gives rise to both the homogeneous (radiative width) and inhomogeneous (splittings) contributions to the linewidth and that these contributions are of the same magnitude. Furthermore, note from Eq. (9) that the radiative width also oscillates with L. We see that the maxima in  ${\rm Im} E_{\pm}({\bf k}_{\parallel})$  occur when there is an integer n such that

$$\kappa_{\rm ex}^2 = \begin{cases} k_{\parallel}^2 + (2n)^2 \left(\frac{2\pi}{L}\right)^2, & + \text{ mode} \\ k_{\parallel}^2 + (2n+1)^2 \left(\frac{2\pi}{L}\right)^2, & - \text{ mode} \end{cases}$$
(12)

is satisfied. This means that radiative decay is favored if the excitation wave vector

$$egin{pmatrix} \left( \mathbf{k}_{\parallel}, (2n) rac{2\pi}{L} 
ight), &+ \mathrm{mode} \ \left( \mathbf{k}_{\parallel}, (2n+1) rac{2\pi}{L} 
ight), &- \mathrm{mode} \ \end{split}$$

nearly matches the wave vector of an energy-conserving photon  $(\mathbf{k}_{\parallel}, \sqrt{\kappa_{\mathrm{ex}}^2 - k_{\parallel}^2})$ . This is indicative of the incipient stage of conservation of the **z** component of momentum for multiple QW's with N > 2.

Insight into Eqs. (9) and (10) can be gained by considering the long-wavelength limit defined as  $L \ll \kappa_{ex}^{-1}, k_{\parallel}^{-1}$ . In this case, we can make the replacements  $\sin L(\kappa_{ex}^2 - \kappa_{ex}^2)$   $k_{\parallel}^2)^{1/2} \ 
ightarrow \ L(\kappa_{
m ex}^2-k_{\parallel}^2)^{1/2} \ \ {
m and} \ \ {
m cos} L(\kappa_{
m ex}^2-k_{\parallel}^2)^{1/2} \ 
ightarrow \ 1 \frac{1}{2}L^2(\kappa_{\rm ex}^2 - k_{\parallel}^2)$  in Eq. (9) and  $\exp[-L(k_{\parallel}^2 - \kappa_{\rm ex}^2)^{1/2}] \rightarrow$  $1 - L(k_{\parallel}^2 - \kappa_{ex}^2)^{1/2}$  in Eq. (10).<sup>22</sup> For the + solutions, the dipole moments in the two QW's are in phase. The radiative shift in  $k_{\parallel} < \kappa_{ex}$  is essentially zero while the decay rate is double the SQW value. This enhancement of the decay rate, or cooperativity, is the super-radiance effect;<sup>8</sup> for a collection of N dipole oscillators in phase within a region of dimensions small compared with the  $\lambda, ext{ the decay rate scales as } N. ext{ In } k_{||} > \kappa_{ ext{ex}}, ext{ the polari-}$ ton dispersion is twice what it is in the SQW. Thus, the system behaves approximately as a SQW with an effective oscillator strength two times the SQW value. Now consider the - solution where the dipole moments in the two QW's are  $\pi$  out of phase. In this case, we have  $E_{-}(\mathbf{k}_{\parallel}) \approx E_{\mathrm{ex}}(\mathbf{k}_{\parallel}) + \hbar \Sigma_{s}^{(2)}(\mathbf{k}_{\parallel})$ . The em fields associated with the individual QW's cancel out. The singular SE contribution  $\hbar \Sigma_s^{(2)}(\mathbf{k}_{\parallel})$  remains for any  $L \neq 0$  since this term is due to the interaction of the exciton with the local field.

To conclude, we have compared exciton polaritons in DQW's and SQW's and have given an exact dispersion relation for the modes [cf. Eq. (4)]. It was shown that for the DQW there are two types of modes; in the  $\pm$  mode the dipole moments associated with the two individual QW's are parallel or antiparallel, respectively. Depending upon the values of L and  $\mathbf{k}_{\parallel}$ , the energy splitting  $\Delta(\mathbf{k}_{\parallel})$  between the  $\pm$  modes is on the order of 0.1 meV for the DQW. The theoretical treatment<sup>9</sup> also applies for N > 2, although simple closed-form expressions for the dispersion and radiative widths do not exist. The DQW captures the salient features for multiple QW samples of total thickness less than  $\lambda$  and thus provides a paradigmatic example for this class of structures.  $\Delta(\mathbf{k}_{\parallel})$  is of the same order of magnitude as the observed luminescence

linewidths in ultrahigh quality SQW samples and therefore may account for part of the increased linewidths observed in the best multiple QW's with respect to SQW's. This contribution to the inhomogeneous line is shown to have the same underlying physical origin as that which gives rise to the radiative width, which contributes to the homogeneous line, namely the polariton effect. We have also shown that at fixed  $\mathbf{k}_{\parallel}$ ,  $\Delta(\mathbf{k}_{\parallel})$  is periodic in L. For example, at  $\mathbf{k}_{\parallel} = \mathbf{0}$  the period is  $\lambda$ . Thus we predict oscillations in the inhomogeneous linewidth with a period in L of  $\frac{\lambda}{2}$  for ultrahigh quality DQW's for normalincidence photoluminescence spectroscopy at low temperature where the radiative width is a substantial part of the homogeneous linewidth.  $\Delta(\mathbf{k}_{\parallel})$  is fundamentally difficult to observe directly since it intrinsically has the same magnitude as the radiative width. If imperfections in the structure giving rise to inhomogeneous linewidths greater than  $\Delta(\mathbf{k}_{\parallel})$  exist, then the effects discussed here may be obscured. Moreover, even if each well is morphologically perfect but with the exciton energy in the two wells differing by energy  $\Delta_{in}$ , then if  $\Delta_{in}$  is larger than typical values of  $\Delta(\mathbf{k}_{\parallel})$ , the polariton effects associated with the interwell coupling will be small. Using Eq. (1)adapted to this case, it is easy to show that the interwell coupling introduces additional radiative shifts and widths only on the order of  $\Delta(\mathbf{k}_{\parallel})^2/\Delta_{\rm in}$ . Thus in order to minimize these problems, ultrahigh quality DQW's rather than multiple QW's are the best candidates for a controlled study of the effects discussed here.

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- <sup>19</sup>J. Knoester, Phys. Rev. Lett. **68**, 654 (1992).
- <sup>20</sup>The dispersion relation for the SQW is in fact analytically solvable; however, the EPA will allow us to compare the SQW and DQW dispersion relations more directly. The EPA is adequate in the energy range near  $E_{\text{ex}}(\mathbf{k}_{\parallel})$  except close to  $k_{\parallel} = E_{\text{ex}}(\mathbf{k}_{\parallel})/(\hbar c)$ .
- <sup>21</sup>An exact analytical solution to the dispersion relation exists for N = 1, though for the purpose of comparison with the DQW we give the EPA expression.
- <sup>22</sup>Note that Eq. (4) is exactly solvable in the long-wavelength limit  $L < |i\kappa|^{-1}, k_{\parallel}^{-1}$  without the EPA since then Eq. (4) is a polynomial equation of degree no higher than 3 in  $(i\epsilon)^2$ .