Edge electrostatics of a mesa-etched sample and edge-state-to-bulk scattering rate in the fractional quantum Hall regime

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To study the effects of Coulomb interactions on the properties of fractional quantum Hall edge states, we introduce a realistic model of a two-dimensional electron gas at a mesa-etched sample edge and solve it within electrostatic and Hartree-Fock approximations. We discuss the physics of the fractional quantum Hall strips separating the conducting edge channels and ways of estimating the widths of these strips. We relate our results to the measurements of nonlocal resistance by estimating the quasiparticle scattering rate and the corresponding equilibration length between the edge states and the conducting bulk.

I. INTRODUCTION

In recent years considerable effort has been devoted to extending the treatment of quantum Hall edge states[1,2] to include the effects of electron-electron interactions, in the context of integer edge states[3-5] and fractional edge states.[3,6,7] In particular, it is important for any quantitative treatment of edge states to include the Coulomb interactions self-consistently. In the present work, we introduce a realistic model of sample edge, appropriate for mesa-etched samples, and solve for the self-consistent two-dimensional electron gas (2DEG) density profile analytically, within classical electrostatics. This model is useful for studying the spin-polarizing transition in integer edge states.[4] Here we give another application: a quantitative treatment of fractional edge states in the regime of slowly varying electron density. In this regime the edge-state structure may be described as a sequence of conducting channels sandwiched between strips of fractional quantum Hall (FQHE) liquid.[6] The positions and the widths of these alternating regions depend on the 2DEG density profile at the edge. To determine to what extent the effects of kinetic energy and exchange modify the electrostatic density profile we perform a numerical Hartree-Fock calculation in the lowest Landau level. We consider several approaches to estimating the widths of the FQHE strips that separate the edge channels. We discuss the incompressible strip approximation used by Chklovskii, Shklovskii, and Glazman[3] and introduce an alternative approximation, appropriate in the presence of disorder, that takes into account nonzero compressibility of the FQHE strips due to localized quasiparticle states. The conducting channels are equilibrated through quasiparticle scattering across the separating FQHE strips, with the scattering rates determined by the strip widths. A particularly interesting experiment that is relevant in this context is the measurement of nonlocal resistance in the FQHE regime by Wang and Goldman.[8,9] In the last part of our paper we discuss this experiment and compare our estimates of quasiparticle scattering rate with the experimental results.

II. DENSITY PROFILE OF 2DEG AT THE EDGE

A. Classical electrostatics

Our model of the charge distribution at an edge of a mesa-etched sample is shown in Fig. 1. The 2DEG is confined at the heterojunction and approaches sheet density n_0 away from the mesa wall. We take $\hat{z}$ to be in the growth direction and take the heterojunction to be in the x-y plane, with y parallel to the edge. The neutralizing donor layer is spaced at distance S from the heterojunction, and has constant sheet density n_0. On the mesa wall there is a certain surface-charge density $n_s(z)$, due to the occupation of the electronic surface states. This surface charge will deplete the 2DEG up to some distance W_D from the mesa wall. Weak localization measurements in wide wires give $W_D \approx 3000-5000$ Å.[10] Note that the depletion width is much larger than the magnetic length l_0. Within classical electrostatics the 2DEG density $n(x)$ is zero in the depletion region ($x<0$), and past the depletion region ($x>0$) the 2DEG screens the external po-

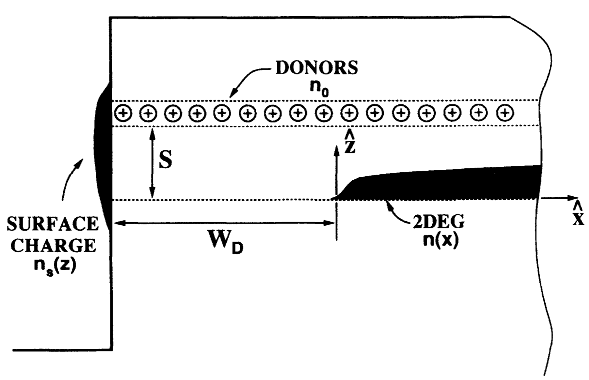

FIG. 1. Our model of a mesa-etched sample edge, with a thin donor layer spaced at distance S from the 2DEG and the surface charge $n_s(z)$ on the mesa wall, depleting the 2DEG out to distance W_D from the wall.

tential perfectly, i.e., it acts like a semi-infinite metal plane.

The electrostatic problem of a line charge parallel to a metal half-plane can be solved by conformal mapping. The complex Green's function is[11]

$$G(\zeta,\zeta_0)=-\frac{2}{\epsilon}\ln\left[\frac{\sqrt{\zeta}-\sqrt{R}e^{i\phi/2}}{\sqrt{\zeta}-\sqrt{R}e^{-i\phi/2}}\right], \quad (1)$$

where ζ is the complex coordinate $\zeta=x+iz$, the line charge is located at $\zeta_0=Re^{i\phi}$, and ϵ is the dielectric constant of the bulk semiconductor. The corresponding charge density induced on the metal plane is given by the discontinuity of the electric field at $z=0$:

$$n_{\rm ind}(x,\zeta_0)=\frac{\epsilon}{4\pi}{\rm Im}\left[\left.\frac{dG}{d\zeta}\right|_{\zeta=x+i0}-\left.\frac{dG}{d\zeta}\right|_{\zeta=x-i0}\right]. \quad (2)$$

Thus for an arbitrary external charge distribution $N(x,z)$ the charge density of the 2DEG is given by

$$n_{es}(x)=\int dX\,dZ\,N(X,Z)n_{\rm ind}(x,X+iZ)$$
$$=\frac{1}{\pi\sqrt{2x}}\int dX\,dZ\,N(X,Z)\frac{(x+R)\sqrt{R-X}}{(x-X)^2+Z^2}, \quad (3)$$

with $R=\sqrt{x^2+y^2}$.

In our model of mesa edge $N(x,z)=n_0\Theta(x+W_D)\delta(z-S)+n_s(z)\delta(x+W_D)$, where the first term corresponds to the sheet of donors (neglecting its thickness), the second term to the surface charge, and $\Theta(x)$ is the unit step function. We can write the 2DEG density as $n_{es}(x)=n_1(x)+n_2(x)$, where $n_1(x)$ is the contribution from the sheet of donors and $n_2(x)$ is the contribution from the surface charge. The charge density induced by the sheet of donors is

$$n_1(x)=\frac{n_0}{\pi}\left[\frac{2}{\sqrt{\xi}}+\tan^{-1}(\sqrt{\xi}-\beta)+\tan^{-1}(\sqrt{\xi}+\beta)\right], \quad (4)$$

where $W=\frac{1}{2}[W_D+(W_D^2+S^2)^{1/2}]$, the scaled distance is $\xi=x/W$, and $\beta=S/2W$ $(0<\beta<1)$. Given $n_s(z)$, we can similarly integrate (3) to obtain $n_2(x)$. The depletion width W_D is determined by the condition $n_{es}(0^+)=0$ that ensures equilibrium of the 2DEG boundary. Naturally, W_D and the 2DEG density profile will depend on the details of the surface-charge distribution.

For a localized external-charge distribution, such as $n_s(z)$, the integrand in (3) can be expanded in powers of $1/x$. The charge density (4) due to the donor sheet can be similarly expanded. Assuming that $n_s(z)$ is localized to a region $R<R_0$ (where we expect $R_0\sim W$) we get that for $x\gg R_0$,

$$n(x)\approx n_0\left[1-\eta\left[\frac{W}{x}\right]^{3/2}\right], \quad (5)$$

where the dimensionless constant η is given by

$$\eta=\frac{1}{\pi}\left[W^{-3/2}\int dz\frac{n_s(z)}{n_0}\{\tfrac{1}{2}[(W_D^2+z^2)^{1/2}+W_D]\}^{1/2}-\tfrac{2}{3}+2\beta^2\right]. \quad (6)$$

It follows that any external-charge distribution that is localized to a finite region near the edge induces a screening-charge density in the 2DEG that falls off as $x^{-3/2}$ at large distances. The slow power-law healing is characteristic of two-dimensional systems with three-dimensional Coulomb interactions. However, if the external charge is not localized then we may get a different exponent for the power law. For example, Chklovskii, Shklovskii, and Glazman[3] obtain $1/x$ power-law healing, instead of $1/x^{3/2}$. They use a model in which the 2DEG is confined by a negatively charged semi-infinite metal gate on top of the sample, instead of an etched mesa edge. In this case the net external charge seen by the 2DEG and the screening charge induced in the 2DEG both go as $1/|x|$ at large $|x|$.

To obtain the entire density profile of the 2DEG, we model the surface-charge distribution by a line charge in the same plane as the electron gas, $n_s(z)=\lambda_s\delta(z)$. This should be a good approximation if the actual surface-charge distribution is confined to $|z|\ll W_D$. The boundary condition on $n_{es}(x)$ gives us $\lambda_s=2n_0(WW_D)^{1/2}$. In particular, for $S=0$ we have $\lambda_s=2n_0W_D$, i.e., the depletion region screens out exactly half of the surface charge. The density profile of the 2DEG, for arbitrary S, is

$$n_{es}(x)=\frac{n_0}{\pi}\left[\frac{2\sqrt{\xi}}{\xi+1-\beta^2}+\tan^{-1}(\sqrt{\xi}-\beta)+\tan^{-1}(\sqrt{\xi}+\beta)\right]. \quad (7)$$

The long-distance behavior is given by (5), with $\eta=4/3\pi$ (Fig. 2). Note that, since $W\gg l_0$, the electron-density

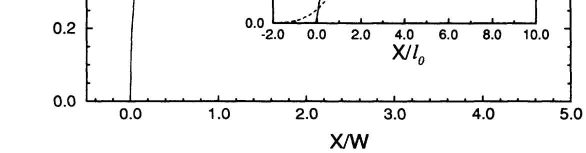

FIG. 2. Electrostatic density profile of the 2DEG given by Eq. (7) (solid line) and large distance approximation given by Eq. (5) (dot-dashed line). Inset: comparison of electrostatic density profile (solid) and Hartree-Fock calculation (dashed). Parameters used for Hartree-Fock are $\nu_{\rm bulk}=0.5$ and $k_bT=0.05e^2/\epsilon l_0$.

changes slowly on the scale of magnetic length everywhere, except near the edge ($x \sim l_0$), where $n_{es}(x) \sim x^{1/2}$ and the density gradient becomes large.

B. Hartree-Fock

Quantum-mechanical effects will change the screening properties of the 2DEG and will cause the electron-density profile at the edge to deviate from the electrostatic density $n_{es}(x)$. For $x \gg l_0$ the actual electron density should be close to $n_{es}(x)$, since at long wavelengths electrostatics will always dominate. However, for $x \sim l_0$ deviations from electrostatics could become appreciable, as the electron density no longer changes slowly on the scale of l_0.

To see to what extent kinetic energy and exchange effects modify the charge density near the edge, we perform a finite-temperature Hartree-Fock calculation restricted to the lowest, spin-polarized, Landau level. In Landau gauge ($\mathbf{A}=Bx\hat{y}$) the lowest-Landau-level eigenstates are

$$\psi_X(x,y)=(\pi^{1/2}Ll_0)^{-1/2}e^{-iXy/l_0^2}e^{-(x-X)^2/2l_0^2} . \tag{8}$$

The direct and the exchange terms in the Hartree-Fock Hamiltonian are diagonal in X. Hence the problem reduces to finding the Landau-level filling $\nu(X)$ and the corresponding single-particle Hartree-Fock energies $\varepsilon(X)$ that satisfy the self-consistency condition $\nu(X)=f(\varepsilon(X))$, where $f(\varepsilon)$ is the Fermi function. Up to a constant ($\frac{1}{2}\hbar\omega_c$), the single-particle energy is the sum of the corresponding Hartree and exchange contributions, $\varepsilon(X)=\varepsilon_H(X)+\varepsilon_{ex}(X)$. The Hartree contribution is given by

$$\varepsilon_H(X)=\frac{1}{\sqrt{\pi}l_0}\int dx\, e^{-(X-x)^2/l_0^2}\left\{V_{es}(x)-2\frac{e^2}{\epsilon}\int dx'[n(x')-n_{es}(x')]\ln|x-x'|\right\} , \tag{9}$$

where the electron density $n(x)$ is related to the Landau-level filling via

$$n(x)=\frac{1}{2\pi^{3/2}l_0^3}\int dX\, e^{-(x-X)^2/l_0^2}\nu(X) . \tag{10}$$

Also in Eq. (9), $n_{es}(x)$ is the charge density for the classical electrostatic problem given by Eq. (7), and $V_{es}(x)$ is the corresponding electrostatic potential. For slowly varying $\nu(X)$ the density is, of course, proportional to the filling, $n(x)=\nu(x)/2\pi l_0^2$. The exchange energy is given by

$$\varepsilon_{ex}(X)=-\frac{1}{4\pi}\frac{e^2}{\epsilon l_0^2}\int dX' e^{-(1/4)X'^2/l_0^2}K_0\left[\frac{X'^2}{4l_0^2}\right] \times \nu(X+X') , \tag{11}$$

where $K_0(x)$ is a modified Bessel function.[12] The calculation reduces to determining $\nu(X)$ self-consistently, which has to be done numerically. A homogeneous 2DEG in the lowest Landau level is unstable in Hartree-Fock against forming a charge-density wave at low temperature.[12] In our calculation we keep the temperature sufficiently high to avoid this instability. A comparison of a Hartree-Fock solution with the electrostatic solution is shown in Fig. 2 (inset). It can be seen that classical electrostatics gives a good approximation to the Hartree-Fock density profile down to x as small as $\approx 4l_0$.

III. SPATIAL STRUCTURE OF FQHE EDGE STATES

Two conducting edge channels will be separated by a strip of FQHE liquid when the Landau-level filling $\nu(x)$ reaches an appropriate FQHE value $\nu_f=p/q$, i.e., the position x_f of the strip is given by $\nu(x_f)=\nu_f$. If $x_f \gg l_0$, then we will be in the regime of slowly varying electron density, where the picture of edge states as conducting channels sandwiched between strips of FQHE liquid is indeed applicable and where electrostatics should give a good approximation for the Landau-level filling profile, $\nu_{es}(x)=2\pi l_0^2 n_{es}(x)$. However, to estimate the widths of the separating FQHE strips we need to consider more closely the response of a microscopically narrow band of FQHE liquid to external potential.

One possible approximation is to take the FQHE strip to be incompressible.[3,6] Then the electron density within the strip will be pinned at $2\pi l_0^2\nu_f$, deviating from $n_{es}(x)$ and thereby creating an electric field perpendicular to the edge. The resulting voltage drop across the FQHE strip is set equal to the discontinuity in chemical potential, given by $q\Delta_f$, where Δ_f is the FQHE quasiparticle energy gap. If the rest of the 2DEG is assumed to be perfectly screening then the problem reduces to one of classical electrostatics. Approximating $n_{es}(x)$ by a linear function about x_f, Chklovskii, Shklovskii, and Glazman[3] obtain

$$d_f^2=\frac{4\gamma_f q}{\pi^2 l_0(dn_{es}/dx)_{x=x_f}} , \tag{12}$$

where d_f is the width of the $\nu=\nu_f$ FQHE strip and $\gamma_f=\Delta_f/(e^2/\epsilon l_0)$.

However, the inevitable presence of disorder makes the situation somewhat more complicated. Disorder potential will give rise to some density of quasiparticle states in the gap. Due to the presence of these states, FQHE liquid will be compressible and the FQHE strips at the edge will now be able to partially screen the external potential. The resulting change in the filling factor across a $\nu=\nu_f$ FQHE strip ($\delta\nu_f$) will depend on the density of states in the gap. Since FQHE will not screen perfectly, the electron density will still deviate from electrostatics and there will still be a voltage drop across the FQHE strip. However, its compressibility may be sufficiently high (given enough disorder) that it would be sensible to make an alternative estimate of d_f by assuming perfect screening for all fillings, i.e., neglecting any deviations from $n_{es}(x)$ near x_f. This approach gives us

$$d_f = |x(\nu_f + \delta\nu_f) - x(\nu_f - \delta\nu_f)| , \tag{13}$$

where $x(\nu)$ is the inverse of $\nu_{es}(x)$ and $\nu_{\rm bulk} \geq \nu_f + \delta\nu_f$.

Of course, the appropriate $\delta\nu_f$ will depend on the nature and the strength of the disorder potential, as well as on the temperature (since localized states can conduct current at finite temperature). We can obtain a simple phenomenological estimate for $\delta\nu_f$ by introducing local diagonal conductivity $\sigma_{xx}(x)$, given by the bulk conductivity at corresponding fillings: $\sigma_{xx}(x,T) = \sigma_{xx}^{\rm bulk}(\nu(x),T)$. At low temperatures, $\sigma_{xx}^{\rm bulk}(\nu,T)$ is extremely small at $\nu=\nu_f$ and increases very rapidly if ν moves sufficiently far away from ν_f. Correspondingly, $\sigma_{xx}(x)$ will be very small at $x=x_f$ and will increase as x moves away from x_f. Thus there will be a band about $x=x_f$ in which tunneling rather than diffusive transport will be the dominant mechanism for interchannel charge flow. Hence in a disordered system at finite temperature it is sensible to define the transition between "conducting channel" and "FQHE strip" by the means of an appropriate conductivity cutoff $\sigma_{xx}^{\max}$. The condition $\sigma_{xx}^{\rm bulk}(\nu,T) < \sigma_{xx}^{\max}$ will be satisfied for fillings in some range $|\nu-\nu_f| < \delta\nu_f$, giving us an estimate of $\delta\nu_f$ that we can use in Eq. (13) to estimate d_f. We would like to caution here that bulk conductivity can probably give us only a fairly crude estimate of the desired edge-state properties. For example, long-range disorder that may be important in determining diagonal conductivity and other properties of the bulk 2DEG (Ref. 13) will have a rather different effect on a microscopically narrow FQHE strip at the edge than it does on the bulk.

We also note that even in the absence of disorder, the FQHE liquid will be compressible at *finite wave vector* due to collective excitations. Thus the static linear susceptibility in the FQHE state has a sharp peak at the magnetoroton minimum. For example, for the $\nu=\frac{1}{3}$ state the magnetoroton minimum is at $k_{mr} \approx 1.4 l_0^{-1}$, and the susceptibility maximum is $\chi(k_{mr}) \approx 10$ in units of $n_0/(e^2/\epsilon l_0)$.[14]

IV. EQUILIBRATION BETWEEN CONDUCTING CHANNELS

At low temperatures, two conducting channels separated by a FQHE strip will equilibrate mainly through the scattering of fractionally charged quasiparticles across the strip. The FQHE strip suppresses scattering by reducing the overlap between the initial- and final-state quasiparticle wave functions. In Born approximation (neglecting multiple scattering), at $T=0$, the scattering rate is reduced by a Gaussian factor, $G(d_f) = \exp(-d_f^2/2l_f^2)$, where $l_f = q^{1/2} l$ is the *quasiparticle* magnetic length. This is the same as the result for scattering between integer edge states,[15] except that the magnetic length is replaced by the quasiparticle magnetic length. The assumption of a Gaussian suppression of the scattering rate will break down at large values of d_f due to one or another competing process, e.g., multiple (virtual) scattering processes at $T=0$, or variable range hopping at $T \neq 0$. In either case, at large d_f the enhancement of the equilibration length will increase more slowly as a function of d_f rather than as an inverse Gaussian.

When bulk Landau-level filling lies between successive FQHE plateaus, $\nu_f < \nu_{\rm bulk} < \nu_f'$, then (in addition to edge states) the bulk itself forms a (dissipative) conducting channel, characterized by a local conductivity tensor. The filling profile $\nu_{es}(x)$ decreases monotonically towards the edge, hence as $\nu_{\rm bulk}$ approaches ν_f from above the $\nu=\nu_f$ FQHE strip moves into the bulk and its width d_f increases. The equilibration length between the edge states and the bulk ($L_{\rm eq}$) should become macroscopically large when $G^{-1} \sim 10^4$. Then current injected at a contact will flow through the bulk and through the edge states in parallel, with only small leakage between the two. This has been observed by Wang and Goldman[8,9] using four-terminal nonlocal resistance measurements of mesa-etched samples in the regime of a fractionally filled Landau level. Note that if particle-hole symmetry with respect to the lowest Landau level were present then the same would happen as $\nu_{\rm bulk}$ approached $1-\nu_f$ *from below*. This is not seen in the experiment[8] because at the edge the particle-hole symmetry is broken by the confining potential.

To illustrate these ideas, we have computed the Gaussian enhancement factor G^{-1} as a function of the bulk filling $\nu_{\rm bulk}$ for $\nu_f = \frac{1}{3}$ and $\frac{2}{3}$, for representative values of the parameters $\delta\nu_f$ and γ_f, using both Eq. (13) and the incompressible strip approximation (12) to compute the strip width. The results are plotted on a logarithmic scale in Fig. 3. Note that Eq. (13) gives larger d_f than the incompressible strip approximation [Eq. (12)]—in a real system we should be somewhere between the two limits. Also, deviations from a Gaussian suppression dis-

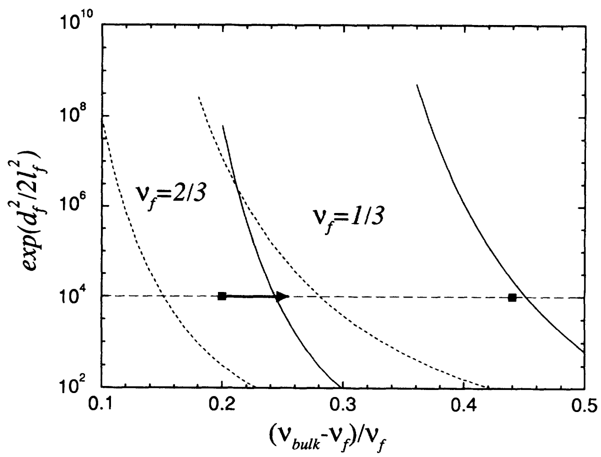

FIG. 3. Gaussian enhancement factor in the equilibration length as a function of bulk filling for $\nu_f = \frac{1}{3}$ and $\nu_f = \frac{2}{3}$ FQHE strips for parameters $W_D = 4000$ Å, $n_0 = 8.74 \times 10^{10}$ cm^{-2}, $\delta\nu_f = 0.017$, and $\gamma_f = 0.03$. Solid lines give the disordered sample estimates [Eq. (13)] and broken lines the incompressible strip estimates [Eq. (12)]. Above the long-dashed line the equilibration length $L_{\rm eq}$ is macroscopically large. Solid square indicates a transition to macroscopic $L_{\rm eq}$ inferred from the nonlocal resistance measurements of Ref. 8 ($\nu_f = \frac{1}{3}$), and the arrow indicates lower bound on similar transition for $\nu_f = \frac{2}{3}$.

cussed above will tend to decrease the steepness of the curves plotted in Fig. 3.

In Figs. 3 and 4 of Ref. 9, Wang and Goldman[8,9] have plotted the four-terminal nonlocal resistance $R_{\rm NL}$, as well as the ordinary resistance R_{xx}, for a single sample at low temperature ($T = 20$ mK), as a function of magnetic-field strength B, for the two opposite directions of the magnetic field (labeled $B+$ and $B-$). The edge currents are in the opposite directions for the two signs of B, and for a given set of current and voltage contacts, we find that $R_{\rm NL}$ is most sensitive to the equilibration length at only one of the two sample edges, depending on the direction of the applied magnetic field. (This conclusion is supported by a numerical calculation in which we have represented the system by an extended network model, similar to that employed by Richter, Wheeler, and Sacks.[16])

It is interesting to examine the data of Ref. 9 in light of the results of the present paper. As noted by Wang and Goldman, the necessary conditions for a measurable $R_{\rm NL}$ are that $\sigma_{xx}^{\rm bulk} > 0$ (i.e., that $\nu_{\rm bulk}$ lies in an interplateau region $\nu_f < \nu_{\rm bulk} < \nu_f'$), and that there exist a high conductivity edge state, well isolated from the bulk (i.e., $L_{\rm eq}$ must be very large). When $L_{\rm eq}$ is very large, according to the analysis of Sec. II of Ref. 9, the value of $R_{\rm NL}$ should qualitatively follow the value of $\sigma_{xx}^{\rm bulk} > 0$. Thus the observation of a magnetic-field interval where R_{xx} is appreciable but $R_{\rm NL}$ appears to be zero could indicate a region where $L_{\rm eq}$ is microscopic for the edge in question. The field regions where these effects of edge scattering are most likely to be observable, because of relatively large values of $\nu_{\rm bulk} - \nu_f$, are the regions $\frac{2}{3} < \nu_{\rm bulk} < 1$ and $\nu_{\rm bulk} \sim \frac{1}{2}$. If we examine the curve corresponding to $B-$ in Fig. 4(b) of Ref. 9, we see that for the region $\frac{2}{3} < \nu_{\rm bulk} < 1$, $R_{\rm NL}$ is nonzero (and hence $L_{\rm eq}$ is large) over the entire interplateau region, $4.2 < B < 5.2$ T. Hence the lower value of the magnetic field ($B \approx 4.2$) only gives us a lower bound on the value of $\nu_{\rm bulk}$ at which the equilibration length due to the $\nu = \frac{2}{3}$ strip should become macroscopically large (shown by a solid arrow in our Fig. 3). We note that, unlike the other nonlocal resistance peaks in the interplateau regions, the one between $4.2 < B < 5.2$ T is actually a double peak. In this field range $L_{\rm eq}$ decreases, while $\sigma_{xx}^{\rm bulk}$ increases sharply with decreasing B. Since $R_{\rm NL}$ increases with increasing $\sigma_{xx}^{\rm bulk}$ and decreases with decreasing $L_{\rm eq}$ the combination of the two effects may create a local minimum in $R_{\rm NL}$ and may be responsible for the double peak. Similarly, careful examination of $B-$ in Fig. 4(b) of Ref. 9 shows that in the $\nu_{\rm bulk} \sim \frac{1}{2}$ region $R_{\rm NL}$ becomes nonzero at $B \approx 7.5$ T. The corresponding value of $\nu_{\rm bulk} - \nu_f$ at which $L_{\rm eq}$ becomes macroscopically large is represented by a solid square in Fig. 3, assuming that the $\nu = \frac{1}{3}$ FQHE strip is more important than the $\nu = \frac{2}{5}$ strip.

By contrast, the curve corresponding to $B+$ in Fig. 4(b) of Ref. 9 (which also appears in Fig. 3 of Ref. 9) shows larger intervals of B, viz., $4.2 < B \lesssim 5$ T for $\frac{2}{3} < \nu_{\rm bulk} < 1$ and $B \lesssim 8.1$ T near $\nu_{\rm bulk} = \frac{1}{2}$, where $R_{NL} \approx 0$ but $R_{xx} \neq 0$. This suggests that for the edge that is important for this field direction $L_{\rm eq}$ may be considerably shorter than for the other edge and therefore considerably smaller than estimated from our theory. A possible interpretation is that the value of $L_{\rm eq}$ is reduced in the case of $B+$ because of one or more defective regions someplace along the corresponding edge.

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