Charging and double-frequency Aharonov-Bohm effects in an open system

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We have investigated experimentally an open semiconductor system in which electron confinement around an obstacle is obtained using a magnetic field. The magnetic field gives rise to Landau levels, and each associated edge state circulates around the obstacle, forming a set of quantized states. Tunable constrictions are fabricated by using a technique which enables us to control transport in and out of these states, producing Aharonov-Bohm oscillations as the magnetic field is swept. Surprisingly, a strong extra oscillation with the same h/e frequency develops, phase shifted by π so that the frequency appears to have doubled. We explain these results in terms of charging of isolated circulating edge states.

In a metal or semiconductor, the isolation of electrons in a potential well leads to charging effects because electrons are indivisible. For example, at low applied bias, transport through a cavity via tunnel barriers on either side is blocked when extra energy is required to add an electron, a phenomenon known as Coulomb blockade (CB). For a semiconductor in a magnetic field, it is possible to confine the highest Landau levels (LL's) within the cavity while allowing the lowest ones to extend into the leads, by reducing the height of the tunnel barriers below the Fermi energy E_F. The associated edge states can form closed paths and thus give rise to Aharonov-Bohm (AB) oscillations.[1,2] Recently, combined AB and CB oscillations were found, indicating that the confined LL's can charge even when there are also extended states in the cavity.[3–6]

The confinement is usually produced by a physical barrier, such as the edge of a metal sample or the depletion region in a semiconductor structure. In contrast, we have made a completely open (two-dimensional) system, in which confinement *around* a microscopic obstacle is provided solely by a perpendicular magnetic field B. All LL's extend into the bulk, but some edge states (shown schematically as solid lines in the insets, Fig. 1) form closed paths around the obstacle. If the path length is small and the temperature low, these paths are phase coherent. The accumulated phase depends on the circumference, wavelength, and the AB effect which causes a change of 2π for each increase of h/e in the flux enclosed. Thus, a ladder of allowed single-particle (SP) states forms. The states are also confined to an LL, which rises in energy as it approaches the edge, so states enclosing less area have higher energy and shorter wavelength. Changing B sweeps the states, each containing one electron, through E_F, causing the net charge near the obstacle to oscillate. In contrast, in electrostatically-confined systems the charge is independent of B. It might be expected that such excess charge would not occur, as electrons within the same, *unconfined,* LL would move to compensate. However, we observe a phenomenon which provides evidence for such charging, showing that edge states encircling the obstacle are unable to move sufficiently to screen the charge because they consist of a series of *quantized* orbits. Two sets of h/e oscillations are interleaved, exactly out of phase. This can only be explained in terms of a Coulomb interaction between two edge states.

Using three Schottky gates we have fabricated such an obstacle ("dot gate") in between two narrow side gates (shaded regions in insets, Fig. 1) on the surface of a high-mobility GaAs-$Al_xGa_{1-x}As$ heterostructure. Applying negative voltages to the gates depletes out electrons in the two-dimensional electron gas below leaving narrow channels under the gaps, the widths of which depend on the voltages. If there is no tunneling between states at the edges of the gaps, there is no backscattering and hence the SP states cannot be detected. However, as the width of each constriction is reduced, electrons from a given edge state may tunnel from one edge of the sample to the other via the SP states around the dot. This produces backscattering which depends on whether or not there is an SP state at E_F through which to tunnel.

We have made separate contact to the dot gate using a technique with a second level of metalization on top of an insulating layer.[7,8] This allows us unprecedented control of the number of edge states and the tunneling in each

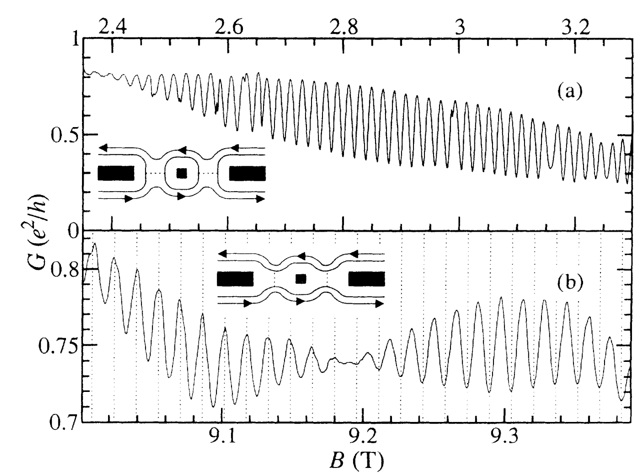

FIG. 1. AB oscillations due to partial reflection of the final edge state (with one additional reflected state in the bulk). (a) Constrictions tuned close to pinch off (sample A). (b) A π phase change in the AB oscillations. Inset: an edge state mostly transmitted (top) or reflected (bottom) (sample B).

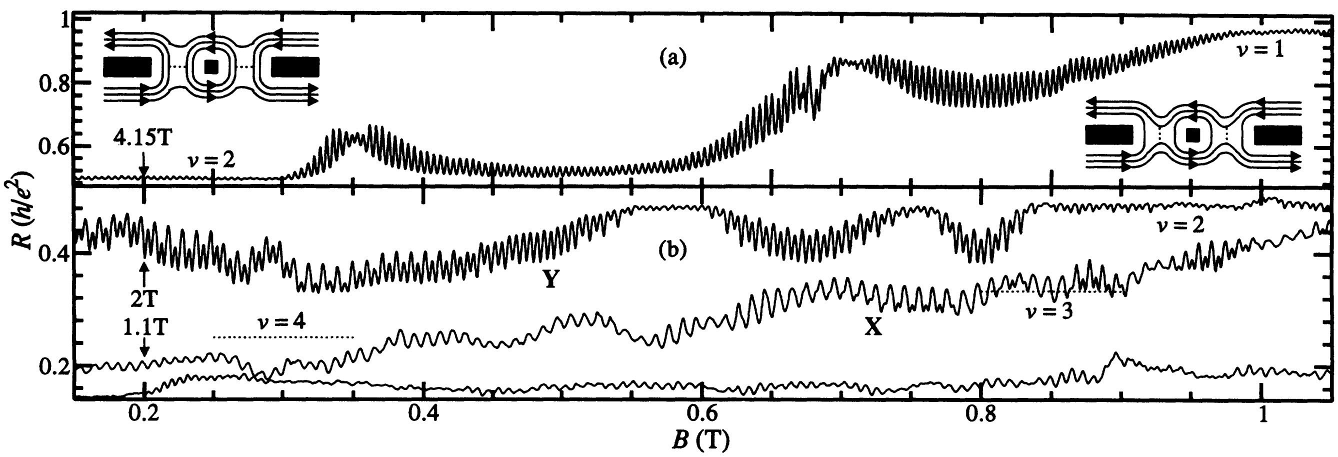

FIG. 2. AB oscillations from $B = 0.15$–4.95 T, for sample C. The data have been divided into four traces and overlaid, with the same scale along the x axis; the figure at the left of each trace is the field there. The plateau region between 2.85 and 4.1 T, where there were no AB oscillations, has been omitted. The number of transmitted edge states, ν, is labeled. The gate voltages for (b) were slightly different to those for (a). The insets show the edge states schematically near $\nu=2$ and 1.

constriction. Previous systems of this geometry did not have a tunable dot[9,10] and so gave very few AB oscillations. Measurements were made at $T<100$ mK using standard low-bias ac techniques. The two-terminal resistance R through the constrictions was calculated from the sum of the four-terminal and Hall resistances.[11] Alternatively the two-terminal conductance G was measured and corrected for a small field-dependent series resistance. The maximum number of spin-polarized edge states ν in either constriction is then just given by $G=1/R$ in units of e^2/h.[11] Results are presented from four samples A–D with carrier concentrations of $(2-4)\times 10^{15}$ m^{-2}, mobilities of 100–200 m^2/Vs and dot gates 0.1–0.3 μm on a side.

We can obtain extremely large AB conductance oscillations as a function of B [Fig. 1(a)] when $\nu \leq 1$. We come close to the maximum possible amplitude e^2/h corresponding to symmetric constrictions and no phase breaking around the loop. The 20 mT period corresponds to adding h/e of flux to a loop of radius 0.25 μm, consistent with a 0.1 μm depletion width and a 0.30 μm lithographic width of the dot gate.

If an edge state is almost fully transmitted, electrons may tunnel *across* the gaps, in and out of the state around the dot, producing some backscattering [dashed lines in inset, Fig. 1(a)]. However, for narrower constrictions, the edge state is mainly reflected, and electrons tunnel *through* the constrictions [see inset, Fig. 1(b)]. In both cases, tunneling is at a maximum when E_F coincides with the energy of an SP state, but in the former (latter), backscattering is enhanced (suppressed).[2] We often observe π phase changes [Fig. 1(b)], spread over just a few oscillations,[12] which one might expect to be due to a rapid changeover between the two types of tunneling. However, very recent calculations show that phase changes cannot be due to such a changeover but may occur when there is tunneling between different edge states or an adjacent impurity.[13]

Figure 2 shows what happens when there are more edge states in the constrictions. At low B [lowest trace, Fig. 2(b)] the usual h/e oscillations are seen. However, extra peaks start to appear between each of the others, for example, near $B=1.6$ T (middle trace, labeled **X**). These peaks become more pronounced as B increases, until the two have almost the same amplitude (labeled **Y**). The superposition of the two nonsinusoidal h/e frequencies out of phase causes the oscillations to look like double-frequency, $h/2e$ oscillations, with a modulation at the h/e frequency. For some gate voltages and fields, the h/e modulation disappears altogether, leaving an apparently pure double frequency. Figure 2(a) shows such

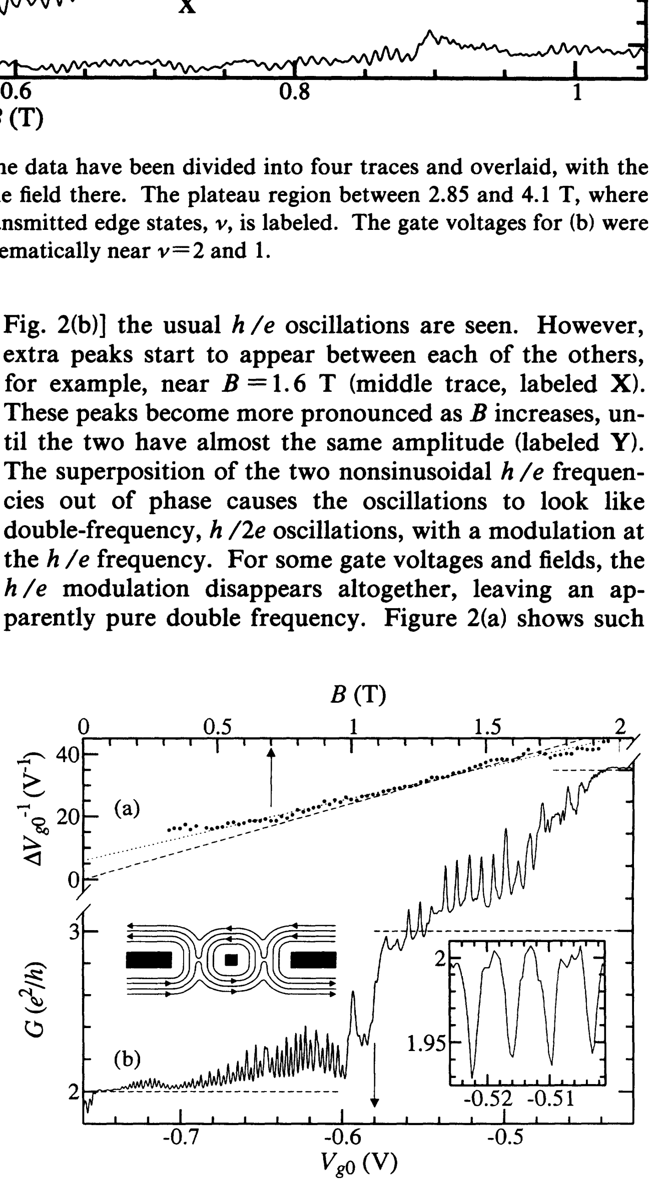

FIG. 3. (a) The frequency of the h/e oscillations in dot gate voltage V_{g0} versus B, near $V_{g0}=-0.7$ V, for sample B. The straight lines are guides to the eye: the dashed line passes through the origin, the other is a good fit to the points. (b) G versus V_{g0} for sample D at $B=1.4$ T (there are six edge states in the bulk). $h/2e$ oscillations occur for $V_{g0}<-0.6$ V (two edge states fully transmitted, see diagram inset); spiked h/e oscillations are seen near $V_{g0}=-0.5$ V, on reflection of the third edge state. The graph inset shows downward $h/2e$ conductance spikes (sample B).

behavior with fewer than two edge states in the constrictions.

The frequency doubling is also observed if the dot gate voltage V_{g0} is swept at fixed B [Fig. 3(b)]. The h/e frequency ΔV_{g0}^{-1} increases linearly with B [Fig. 3(a)], extrapolating almost to the origin, although it levels off at low B. This is consistent with V_{g0} just changing the area A enclosed by an edge state of radius r, since $\Delta V_{g0} \propto \Delta r \propto \Delta A = h/eB$ for small ΔV_{g0}.

Near plateaux the oscillations can become very spiked, as in Fig. 3(b) and its inset. Each corresponds to tunneling through a very well defined state, in which electrons can orbit the dot many times. The dips in G in the inset to Fig. 3(b) imply tunneling across the constrictions into such states. Surprisingly, spiked *peaks* are usually observed [Fig. 3(b)], implying resonances in the *transmission* of a mostly reflected, and hence extended, edge state. Transmission may instead be via an inner, well isolated, edge state, since the spikes do not seem to occur when $\nu \leq 1$. Fourier transforms show up to six harmonics, corresponding to multiple orbits. Successive harmonics are attenuated by the probabilities of leaving the orbit via either constriction and by phase-randomizing collisions. Thus the phase-coherence length in these samples must be many times the circumference (~ 2 μm). However, frequency doubling cannot simply be explained as the second harmonic $h/2e$ as there is no reason why this harmonic should dominate over a wide field range. Nor does it arise from tuning the constrictions precisely, since it is insensitive to the degree of asymmetry. Furthermore, we have seen the effect in at least ten samples, for $1 < \nu < 4$. Similar extra peaks were seen in a ballistic constriction with an impurity[14] but phase locking was not apparent and the result was attributed to spin splitting. In our case, spin splitting is often complete and so cannot explain our results.

At intermediate fields [upper curve, Fig. 2(b)], the two sets of peaks are nearly always π out of phase. *No* model of noninteracting electrons can explain this locking. One could only expect to obtain either a single frequency, or, if two or more edge states contribute to the oscillatory conductance, interspersed sets of peaks with slightly different periods and unrelated amplitudes. However, AB oscillations are accompanied by charging, since as B decreases, the area of each SP state increases in order to enclose the same flux, and a net positive charge builds up between the highest occupied and the lowest unoccupied orbits in a given edge state, until it reaches $e/2$. At this point an electron may enter the next SP state with zero energy cost, so the edge-state charge becomes $-e/2$. This resembles CB in a cavity.[15] Each circulating edge state has its own ladder of SP states and may, therefore, be able to charge if it is sufficiently isolated.

We have calculated the effect of this charging in a simple model where two isolated edge states are likened to two Coulomb-blockade cavities in parallel.[16] The cavities are capacitively coupled to each other and to the rest of the system. If the mutual capacitance C_{12} between the two is quite large, the B dependence of G is essentially the same as for noninteracting electrons (two sets of peaks with slightly different periods and no phase relation between them). However, if the interaction between the two "cavities" is increased (giving a smaller C_{12}), the peaks start to repel each other, and eventually peaks in G have equal spacing. Alternate peaks correspond to tunneling through different edge states. They will in general have different amplitudes since the tunneling probabilities are likely to be different. A *pure* double frequency, as in Fig. 2(a), might occur if the edge states hybridize, which would require that electrons circulate for long enough for spin-flip scattering to occur. This could sometimes be the case as adjacent edge states are close together since only the outermost one is fully decoupled;[17] also, the spiked peaks we have observed imply that the electrons can circulate many times.

This model of two charging edge states assumes that the charging energy is much larger than the SP energy spacing. h/e and $h/2e$ oscillations both die out at around 400 mK, presumably due to thermal smearing of the SP states which are, therefore, of order 40 μeV apart. On application of a dc bias the peaks split, yielding a level spacing (charging energy) of 200–500 μeV,[12] so the assumption is reasonable. If not, the locking may come from the edge states adjusting their positions due to the charging, because they are in an open system. This may lead to a feedback mechanism which ensures that the two edge states charge up alternately, exactly out of step.[18] Tunneling into an inner edge state is also assumed, but if this cannot occur easily, then charging of an inner state may just affect the outer state electrostatically, changing its overlap with states at the other edges.

Thus, with two isolated edge states, the phase locking that we observe can be understood. However, locking occurs even when the outer of two edge states is mainly backscattered and hence cannot charge because it is extended. We have no explanation for this at present; a full calculation taking into account self-consistently the charging and screening of each edge state and its effect on

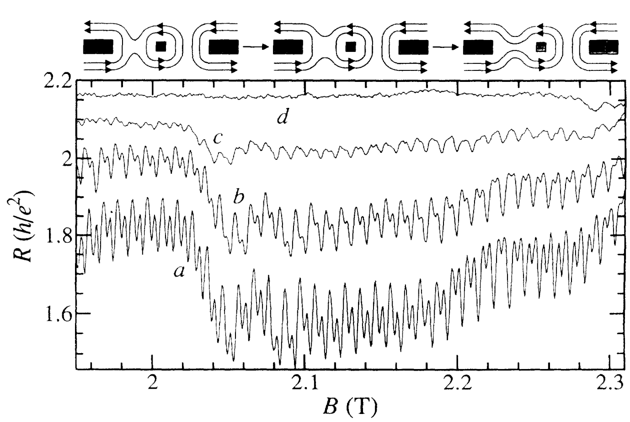

FIG. 4. $h/2e$ oscillations versus B for sample C. In trace a the two constrictions have similar widths. By incrementing one of the side-gate voltages, one constriction is successively narrowed for traces b–d (shown schematically in the diagrams at the top). In c the intermediate peaks are attenuated but still visible. However, careful measurements in the region between c and d show both oscillations disappearing completely at the same gate bias. For clarity, traces b, c, and d are vertically offset by 0.12, 0.16, and 0.2 h/e^2, respectively.

the other edge state's position and charge is required.

There is no sign of a corresponding $h/3e$ frequency when there are two isolated inner edge states. Tunneling into the innermost state is likely to be insignificant. Recent calculations find that edge states must be broad compressible strips in order to screen the sloping potential at the edge, with narrow incompressible regions between them.[17,19] Thus, adjacent states may interact strongly, whereas other states are much further away, and may also be screened, so they are not detected.

If one constriction alone is squeezed by changing a side-gate voltage, the edge states should progress through the configurations shown at the top of Fig. 4. The outer edge state ceases to surround the dot. The inner, formerly isolated, edge state then becomes connected to that edge and hence should gradually cease to charge. Thus, one might expect oscillations due to charging of the inner edge state to persist for a while after those due to the outer edge state have disappeared, affecting the backscattering of the outer edge state across the open constriction. However, we find that both oscillations disappear together (Fig. 4), although one of each pair becomes much smaller than the other. In our charging model, this implies that the inner state rapidly ceases either to be isolated or to affect the tunneling of the outer state.

Similar charging effects have been invoked to explain combinations of CB and AB oscillations in cavities.[3–6] However, around an obstacle, it is the confinement by the magnetic field and the discreteness of the SP states that cause the charge oscillations, rather than the change in size of a region using a gate. Note, also, that the edge state is surrounded by a sea of electrons, and is isolated not by an electrostatic potential barrier, but by the incompressible states between LL's.[17]

In conclusion, our fabrication technique provides unprecedented control over an isolated dot gate in an open system. Extra peaks appear exactly midway between the usual AB oscillations. These are attributed to charging of single-particle states in edge states isolated by the magnetic field. This system was originally proposed for the study of the interference of quasiparticles in the factional quantum Hall effect (QHE) regime,[20] but we have shown that even in the integer QHE regime, tunneling is strongly influenced by charging effects, suggesting that charging will also be important for quasiparticles. It may also prove a useful tool for investigating the more general QHE problem of transmission of Landau eigenstates through a smooth fluctuating potential in a high magnetic field, where the extended states result from percolation paths among an array of such obstacles.

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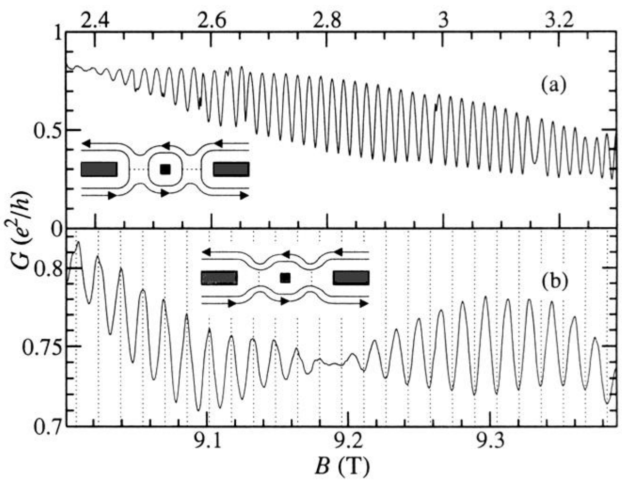

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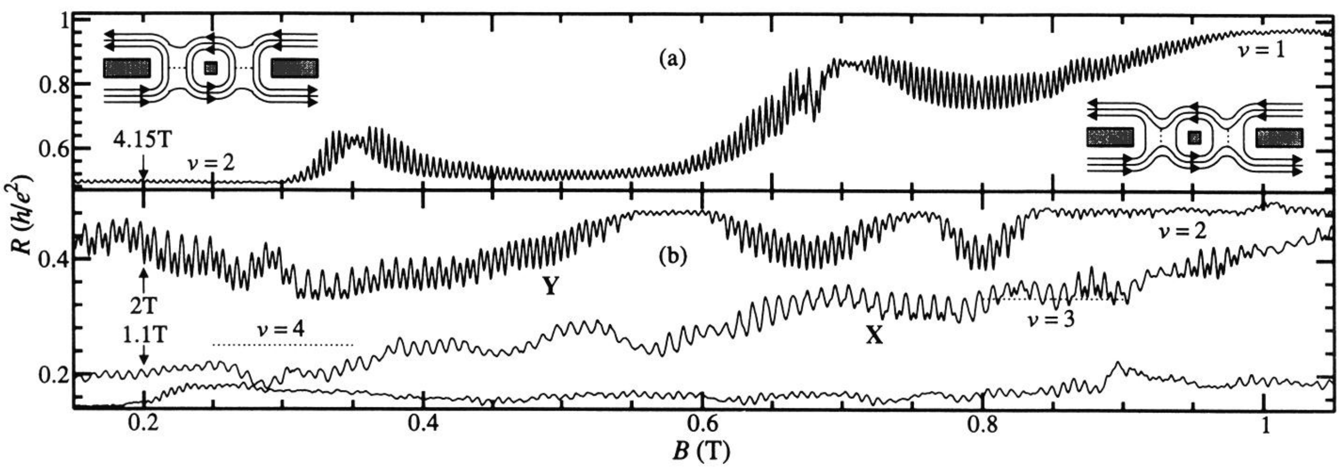

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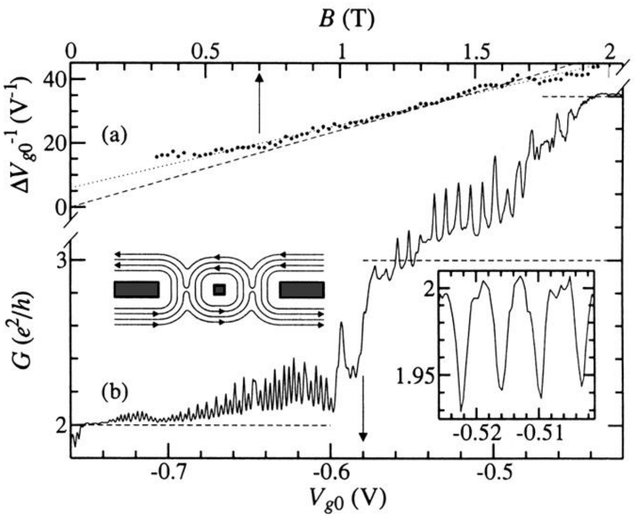

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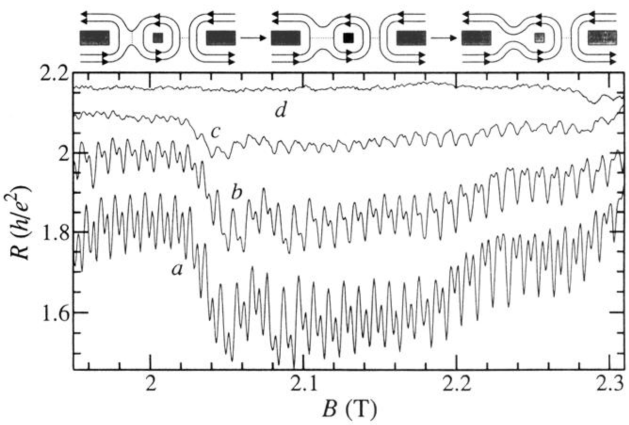

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