Self-consistent calculation of traversal time in a double-barrier resonant-tunneling structure in the presence of a transverse magnetic field

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A self-consistent calculation of the traversal time in a resonant tunneling device has been presented. The model is then used to study the effect of a transverse magnetic field on the traversal time. The calculation shows the presence of bistability in the traversal time. It is also observed that the traversal time becomes progressively longer in the presence of increasing magnetic field. A quantity called the *effective traversal time* has been defined which can be compared to an important time scale, namely, the carrier thermalization time, and may be experimentally measurable.

INTRODUCTION

The behavior of double-barrier resonant-tunneling structures (DBRTS) in the presence of a magnetic field has been studied extensively in recent years. A number of works,[1-5] both theoretical and experimental, have been reported so far. In almost all the theoretical and experimental works, the main emphasis was on the quenching of tunneling current and the fluctuation of conductance with increasing magnetic field. However, another important aspect, that is, the behavior of the traversal (or, tunneling) time in the presence of an applied magnetic field, has not been addressed much.

The study of tunneling time, in general, has attracted much attention. Despite great efforts, the issue of tunneling time still remains a highly debated topic. This is partly due to the nonavailability of any quantum-mechanical operator that gives *time* as an expectation value. Thus, indirect methods capable of providing access to tunneling time have been solicited rather than the standard quantum-mechanical prescription of tunneling time. Depending on the prescriptions, three different time scales can be identified:[6,7] the transmission time τ_T, the reflection time τ_R, and the dwell time τ_D. τ_T is often called the traversal (tunneling) time if the incident energy of the particle is less (greater) than the barrier potential. The early work on tunneling time dates back to the late 1940s and 1950s when Wigner[8] used the phase method to calculate it. Phase time is an asymptotic quantity and refers to the event when transient phenomenon has died out. Moreover, phase time cannot be defined locally as it only refers to asymptotic free motion. Smith[9] introduced the concept of dwell time, which does not suffer from this difficulty and can be computed locally. However, dwell-time formalism does not provide a mechanism to separate the time between different scattering channels. Baz'[10] introduced the Larmor precision to calculate particle lifetime in nuclear reactions. The method was extended by Rybachenko[11] and later by Büttiker[12] to calculate traversal time. Guéret, Baratoff, and Marclay[1] used the concept of the larmor clock to measure the tunneling time of a heterostructure barrier experimentally. They have shown that tunneling time can be determined from the reduction of current in a magnetic field applied perpendicular to the current. Büttiker and Landauer[13] calculated tunneling time using the WKB approximation by considering tunneling through a time-modulated barrier. Numerical studies[14] using wave packets are also reported for a lifetime calculation. Although many different approaches for calculating tunneling time have been reported, there exists no general consensus among these methods. As pointed out by Leavens,[15] different approaches give different results simply because they are concerned with different characteristic times.

In this paper, a self-consistent calculation of tunneling time (i.e., the time it takes for the particle to traverse the structure) will be presented for a DBRTS. The treatment is then extended to the case when the electric and magnetic fields are crossed. For DBRTS, the time that a particle takes to traverse the structure is particularly important in the understanding of different scattering processes. It is observed that the tunneling time in the present study can be compared to an important time scale, namely, the carrier thermalization time.

THEORY

A self-consistent calculation of tunneling (traversal) time requires the knowledge of the amount of space charge formed in the quantum structure. The computation of the developed space charge demands a self-consistent solution of Schrödinger and Poisson's equations. The Schrödinger equation is solved by using the logarithmic derivative of the wave function. The one electron Schrödinger equation in the effective mass approximation is written as

$$\frac{d\Xi(x,E_x)}{dx} = -j\left[\frac{m^*}{2\hbar}\Xi(x,E_x)^2+\frac{4}{\hbar}[V(x)+V_m(x)-E_x]\right]. \quad (1)$$

Here, $\Xi(x,E_x)=2\hbar/jm^*[\psi'(x)/\psi(x)]$ is the logarithmic derivative[16] and $\psi(x)$ is the electronic wave function. $V(x)$ accounts for the algebraic summation of the potential energies due to the applied bias $V_a(x)$, the space charge $V_{sc}(x)$, the conduction band discontinuity

$\Delta E_c(x)$, and magnetic field $V_m(x)$. $V_m(x)$ can be expressed as

$$V_m = \frac{m^*}{2}\omega_c^2(x-x_0)^2 . \tag{2}$$

Here, $x_0 = \hbar k_y/qB, \omega_c = qB/m^*$ is the cyclotron frequency, B is the applied magnetic field, q is the electronic charge, m^* is the electronic effective mass, and $\hbar$ is the modified Planck's constant. Equation (1) may be solved numerically and for a piecewise constant potential profile, the recurrence relation may be written as

$$\Xi_n = \left[\frac{8A}{m^*}\right]^{1/2} \tanh\left\{-j\left[\frac{2m^*A}{\hbar^2}\right]^{1/2}\Delta x + \operatorname{arctanh}\left[\Xi_{n+1}\left[\frac{m^*}{8A}\right]\right]^{1/2}\right\}, \tag{3}$$

where $A = -[V(x_n) - E_x + 0.5m^*\omega_c^2(d_1+0.5d_2)^2 - 2x_n\Delta x]$, $\Delta x = x_{n+1} - x_n$, and d_1 and d_2 are the width of the barriers and the quantum well, respectively.

The solution of Poisson's equation requires the space-charge distribution $n(x)$ inside the quantum well and is given as

$$n(x) = \int_0^\infty dE_x \sum_{\pm} n^{\pm}(x,E_x) , \tag{4}$$

where

$$n^{\pm}(x,E_x) = \frac{J^{\pm}(E_x)}{qv_g^{\pm}(x,E_x)} , \tag{5}$$

and $J(E_x)$ is the total current density $[J^+(E_x)+J^-(E_x)]$, the $+(-)$ sign implies that the electron is moving from the emitter (collector) to the collector (emitter) electrode,[17] and v_g is the electron group velocity and is related to the real part of the logarithmic derivative as[18]

$$v_g^{\pm}(x,E_x) = \tfrac{1}{2}\mathrm{Re}[\Xi^{\pm}(x,E_x)] . \tag{6}$$

The calculation of $J(E_x)$ follows the method presented by Alam and Khondker;[17] however, $V_m(x)$ has to be incorporated in the $V(x)$ calculation. The updated potential profile is obtained by solving Poisson's equation and is given by

$$\phi(x) = \phi(0) - \frac{q}{\epsilon}\int_0^x dx' \int_0^{x'} n(x'')dx'' \tag{7}$$

and $V_{sc}(x) = -q\phi(x)$. Equations (1)–(7) are solved till a consistent current is obtained.

As soon as the self-consistent potential profile is achieved, tunneling time is calculated next. Tunneling time for a DBRTS of length $L\,(=2d_1+d_2)$ is written as[18]

$$\tau(E_x) = \int_0^L \frac{dx}{v_g^+(x,E_x)} = 2\int_0^L \frac{dx}{\mathrm{Re}[\Xi(x,E_x)]} . \tag{8}$$

In the absence of scattering, $\tau^+(E_x) = \tau^-(E_x) = \tau(E_x)$ is implied.

Instead of depicting tunneling time as a function of energy, an effective tunneling time $\langle\tau\rangle$ may be defined as

$$\langle\tau(V,B)\rangle = \frac{\int_{QW}\tau(E_x)n(E_x)dE_x}{\int_{QW}n(E_x)dE_x} , \tag{9}$$

where

$$n(E_x) = \sum_{\pm}\int_{QW}\frac{J^{\pm}(E_x)dx}{qv_g^{\pm}(x,E_x)} . \tag{10}$$

$\langle\tau\rangle$ depends on the applied bias and magnetic field and provides an important tool to investigate phenomena like carrier thermalization. $\langle\tau\rangle$ will provide a more realistic measure of tunneling time in the context of device modeling and in the understanding of different scattering phenomena.

RESULTS AND DISCUSSION

A double-barrier resonant-tunneling structure consisting of a 50-Å-wide quantum well and 50-Å-wide barriers is used. The effective mass of the electron inside the well and barriers are $0.067m_0$ and $0.096m_0$, respectively, where $m_0 = 9.1\times10^{-31}$ kg. The barrier height is 0.275 eV. The Fermi level E_F is assumed to be 0.03 eV above the bottom of the conduction band in the emitter contact.

In Fig. 1, tunneling times are plotted as a function of energy (E_x) with and without invoking self-consistency. It is observed that the self-consistently obtained tunneling time is longer than the corresponding non-self-consistent case. In the figure, curve 1 corresponds to bias voltage 0.01 V and curve 2 corresponds to the bias voltage 0.11 V. It is interesting to note that the difference between self-consistent and non-self-consistent tunneling time becomes greater at 0.11 V as compared to 0.01 V. This can be attributed to the accumulated space charge[19] in the quantum well which is greater at 0.11 V than that at 0.01

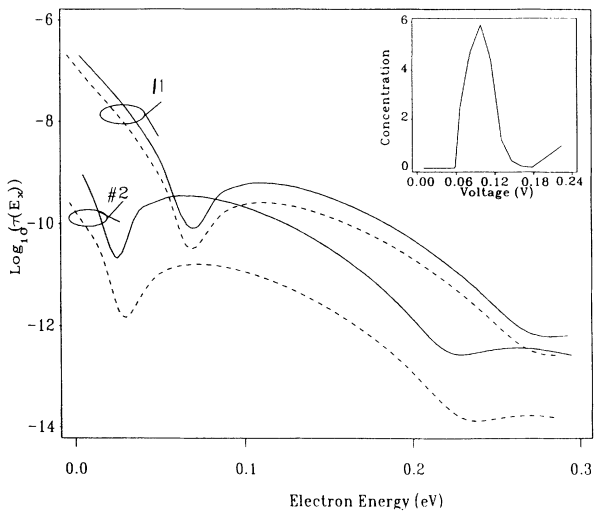

FIG. 1. Tunneling time $\tau(E_x)$ is plotted as a function of incident electron energy with (solid line) and without (dashed line) invoking self-consistency. Sets 1 and 2 correspond to the bias of 0.01 and 0.11 V, respectively. The inset shows the space-charge concentration in the quantum well ($10^{22}/m^3$) as a function of bias voltage.

V. This indeed is observed in the inset of Fig. 1 which shows the space-charge distribution in the quantum well. Depending on the magnitude of stored charge, the potential profile will be modified according to Eq. (7) and the tunneling time will also be affected correspondingly.

In Fig. 2, tunneling time $\tau(E_x)$ is plotted as a function of the applied voltage (V) and incident electron energy (E_x). It is noted that with increasing voltage the minima of the tunneling time moves to lower energy. At sufficiently high bias voltage (e.g., $V > 0.16$ V), the first dip disappears completely. This can be attributed to the shift of eigenenergies to lower values with increasing bias. The dip in tunneling time corresponds approximately to the value of the eigenenergy. Thus, tunneling time for an electron having the resonant energy will be relatively smaller.

In Fig. 3, $\tau(E_x)$ is plotted as a function of applied magnetic field and incident electron energy for a particular applied bias. It is observed that due to diamagnetic shift[3] the dip in tunneling time moves to higher electron energy. Moreover, for a given electron energy, tunneling time is a gradually increasing function of magnetic field. This may be attributed, in the classical analogy, to an increasing traversal path with increasing magnetic field.[4]

The positive and negative bias voltage sweep tunneling times, $\tau^+(E_x)$ and $\tau^-(E_x)$, respectively, are not the same and show bistability. In Fig. 4, tunneling time $\tau^+(E_x)(\tau^-(E_x))$ is plotted as a function of energy for positive (negative) voltage sweep. These plots show that bistability exists in $\tau^+(E_x)$ in a DBRTS. However, with the application of a transverse magnetic field, the width and the extent of the bistable region change along with the position of minima of $\tau(E_x)$. For example, at 0 T, $\tau^+(E_x)$ and $\tau^-(E_x)$ differ by 0.1 nS for $E_x = 100$ meV and 1 fS for $E_x = 220$ meV as compared to 0.7 nS and 0.1 pS, respectively, at 10 T. The difference in tunneling times $[\tau^+(E_x) \sim \tau^-(E_x)]$ increases with increasing magnetic field and may be in the range of nano second for certain electron incident energies.

In Fig. 5, the effective tunneling time $\langle \tau \rangle$ is plotted as a function of the applied bias and magnetic field. As observed, $\langle \tau \rangle$ increases with magnetic field but decreases with applied voltage. However, the decrease in tunneling time is rather sluggish in a certain voltage range (ΔV_τ). The extent of ΔV_τ again decreases with the increase of magnetic field. This decrease in ΔV_τ may be related to the height of the bistable region observed in the current-voltage characteristics, whereas the width may be related to $\Delta\tau_E[=\tau^+(E_x)-\tau^-(E_x)]$ in the previous diagram.

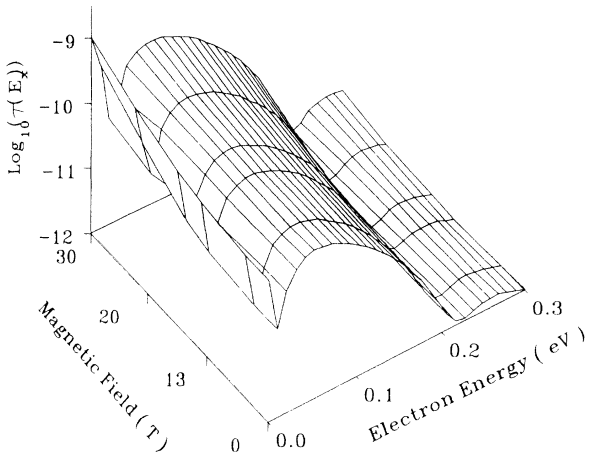

FIG. 3. Tunneling time $\tau(E_x)$ is plotted as a function of applied magnetic field and incident electron energy for an applied bias of 0.13 V. Note that due to a diamagnetic shift the dip in tunneling time moves to higher electron energy.

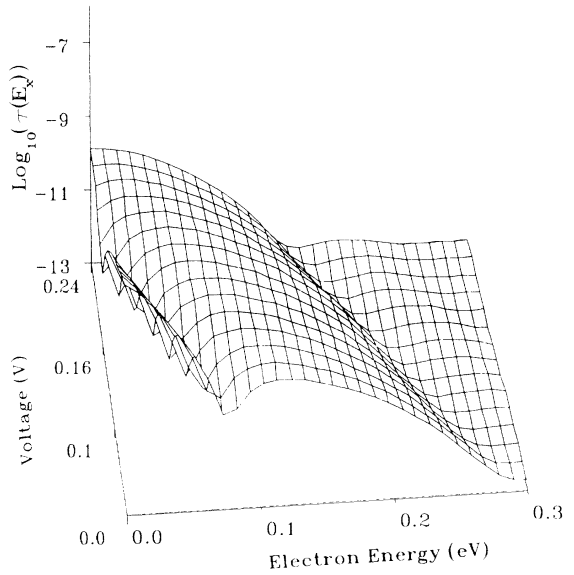

FIG. 2. Tunneling time $\tau(E_x)$ is plotted as a function of the applied voltage and incident electron energy. It is to be observed that with increasing voltage the minima of tunneling time moves to lower energy.

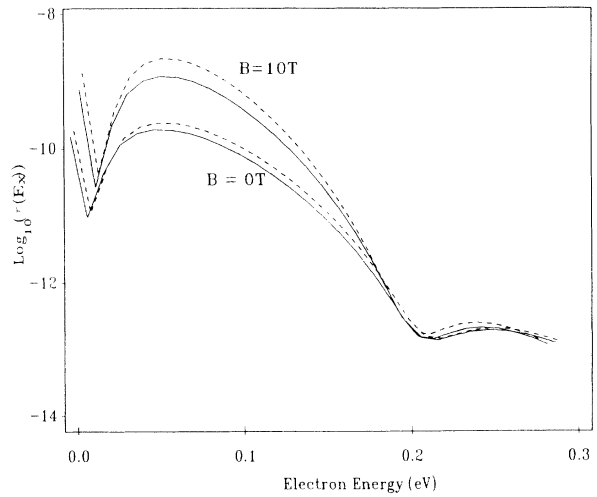

FIG. 4. Tunneling time is plotted as a function of energy for both the positive and negative voltage sweep with magnetic field as a parameter. The solid (dashed) curve corresponds to the positive (negative) voltage sweep. The plots show bistability in tunneling time in DBRTS.

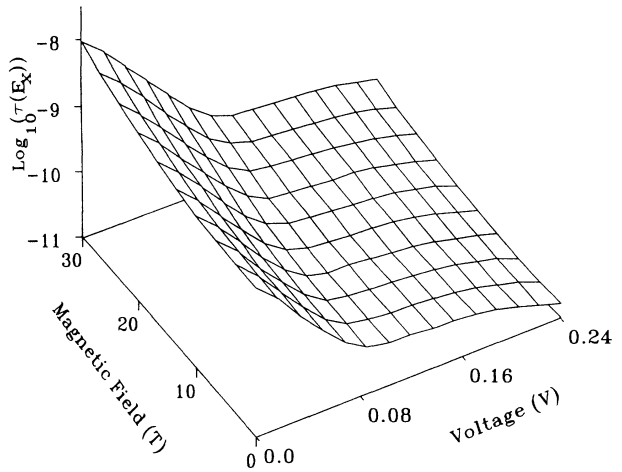

FIG. 5. The effective tunneling time $\langle \tau(V,B) \rangle$ is plotted as a function of the applied bias and magnetic field. Note $\langle \tau(V,B) \rangle$ increases with magnetic field but decreases with applied bias.

Figure 5 can also be used to study carrier thermalization in DBRTS. At low-temperature, energy relaxation is via emission of acoustic phonons and the characteristics time, τ_{ph}, is approximately 10^{-9} sec.[5] At very low bias, irrespective of the magnitude of the magnetic field, $\langle \tau \rangle > \tau_{ph}$ making the thermalization possible. As observed, for a given magnetic field $\langle \tau \rangle$ becomes smaller than τ_{ph} with increasing bias. However, with increasing magnetic field, the effective tunneling time characteristic moves up, making $\langle \tau \rangle > \tau_{ph}$ and allowing thermalization to be possible again. The last observation can be explained in terms of the space charge stored in the quantum well.

CONCLUSION

In this paper, a self-consistent calculation of the tunneling time has been presented. The study of the tunneling time is useful in the context of the carrier thermalization inside the DBRTS. It has been observed that the application of a transverse magnetic field includes enhancement in the relaxation rate and thereby makes thermalization possible.

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