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# Correlation functions for mesoscopic conductance at finite frequency

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We extend to finite frequency the calculation of the conductance correlation function for a mesoscopic device, using linear-response theory for noninteracting electrons. Because the conductance is complex, two functions are required to fully specify correlations in the real and imaginary parts. The two functions have different frequency dependences due to the placement of the diffusion propagators in the simplest diagram. We apply our results to the case of $h/e$ conductance oscillations in a ring.

## I. INTRODUCTION

Linear-response theory has been shown to account for quantum-transport effects on the dc conductance of mesoscopic devices.[1] Recently, some attention has been given to the problem of finite-frequency mesoscopic transport.[2,3] The characteristic frequencies for a device are the inverse electron-transit time through the device $\tau_L^{-1}$ and the thermal frequency $kT/\hbar$. The dephasing rate $\tau_\varphi^{-1}$ may also play a role if $\tau_\varphi^{-1}>\tau_L^{-1}$. For diffusive transport, $\tau_L=L^2/D$, where $L$ is the device size and $D$ the diffusion constant. When the measurement frequency becomes comparable to these characteristic frequencies, new complications appear in the calculation of the conductance. In particular, the replacement of the microscopic electric field within the device by the average or classical field no longer gives an exact result,[4] as it does in the dc limit.[5] This implies that screening and other interaction effects may become important. Also, nonlinear effects may occur if the ac field is sufficiently strong.[6] Still, the linear-response conductance of noninteracting electrons is a necessary starting point for the consideration of finite-frequency mesoscopic effects.

In this work, we extend the calculation of the conductance correlation function $F(\Delta B)=\langle \delta G(B)\delta G(B+\Delta B)\rangle$ to finite frequency, within the context of linear response and for noninteracting electrons, in the diffusive regime. The resulting functions,

$$\begin{aligned} F_1(\omega,\Delta B)&=\langle \delta G(\omega,B)\delta G(\omega,B+\Delta B)\rangle \ , \\ F_2(\omega,\Delta B)&=\langle \delta G(\omega,B)\delta G^*(\omega,B+\Delta B)\rangle \ , \end{aligned} \tag{1}$$

describe features of the magnetoconductance $\delta G(\omega,B)$ of a mesoscopic device at measurement frequency $\omega$. We apply the result to the case of an isolated ring made of quasi-one-dimensional wire and obtain the frequency dependence of the flux-periodic Aharonov-Bohm conductance oscillations. The case of periodic oscillations in a ring is especially interesting because the distribution of transit times for the relevant interfering paths is sharply peaked near $\tau_L$, so that any frequency dependence should become rapidly evident once $\omega\tau_L>1$. Also, the result can be compared directly with results of recent experiments.[3]

## II. DERIVATION OF FINITE-FREQUENCY CORRELATION FUNCTIONS

Following Altshuler *et al.*, we write the nonlocal conductivity tensor as[7]

$$\sigma_{\alpha\beta}(\mathbf{r},\mathbf{r}';\omega)=\frac{1}{\omega}[K_{\alpha\beta}(\mathbf{r},\mathbf{r}';\omega)-K_{\alpha\beta}(\mathbf{r},\mathbf{r}';0)] \ , \tag{2}$$

in which

$$K_{\alpha\beta}(\mathbf{r},\mathbf{r}';\omega)=\frac{e^2}{2\pi m^2}\int_{-\infty}^{\infty}d\varepsilon\{[f(\varepsilon)-f(\varepsilon-\omega)]\hat{p}_\alpha G_\varepsilon^+(\mathbf{r},\mathbf{r}')\hat{p}_\beta G_{\varepsilon-\omega}^-(\mathbf{r}',\mathbf{r}) + f(\varepsilon)[\hat{p}_\alpha G_\varepsilon^-(\mathbf{r},\mathbf{r}')\hat{p}_\beta G_{\varepsilon-\omega}^-(\mathbf{r}',\mathbf{r})-\hat{p}_\alpha G_{\varepsilon+\omega}^+(\mathbf{r},\mathbf{r}')\hat{p}_\beta G_\varepsilon^+(\mathbf{r}',\mathbf{r})]\} \ . \tag{3}$$

Here, $G^+$ and $G^-$ are, respectively, the retarded and advanced Green functions, $\hat{\mathbf{p}}$ is the momentum operator $-i\nabla$, and $f(\varepsilon)$ is the Fermi function. In the usual calculation of the zero-frequency limit of Eq. (2) the real part is taken, which results in additional terms. For our purposes, Eqs. (2) and (3) contain all the needed information. The quantity $K_{\alpha\beta}(\mathbf{r},\mathbf{r}';0)$ in Eq. (2) represents the diamagnetic part of the conductivity and, in the impurity average, cancels the last two terms in Eq. (3) except for quantities of order $\omega\tau$, where $\tau$ is the elastic-scattering time. For all frequencies we are considering, $\omega\tau \ll 1$, so these terms can be neglected and we obtain

$$\sigma_{\alpha\beta}(\mathbf{r},\mathbf{r}';\omega)=\frac{e^2}{2\pi m^2}\int_{-\infty}^{\infty}d\varepsilon\frac{f(\varepsilon)-f(\varepsilon-\omega)}{\omega} \times\hat{p}_\alpha G_\varepsilon^+(\mathbf{r},\mathbf{r}')\hat{p}_\beta G_{\varepsilon-\omega}^-(\mathbf{r}',\mathbf{r}) \ . \tag{4}$$

This expression contains contributions to the conductivi-

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ty only from electrons near the Fermi surface, as expected. Its impurity average has been shown to yield the classical conductivity[8] and the weak-localization correction.[9]

We now wish to calculate correlation functions of the diagonal part of the conductance tensor obtained from Eq. (4). We consider the functions $F_1(\omega,\Delta B)$ and $F_2(\omega,\Delta B)$ of Eq. (1); these together provide all information about correlations of the real and imaginary parts of the conductances. In the approximation of a uniform electric field within the sample, the conductance of a wire or between opposite points of a ring is given by $(1/L^2)\int d\mathbf{r}\,d\mathbf{r}'\,\sigma(\mathbf{r},\mathbf{r}';\omega)$, where $L$ is the length of the wire or half the circumference of the ring. The expression for the simplest diagram contributing to $F_1$ [see Fig. 1(a)] is

$$F_1(\omega,\Delta B)=2\left[\frac{e^2}{2\pi m^2L^2}\right]^2\int_{-\infty}^{\infty}d\varepsilon\int_{-\infty}^{\infty}d\varepsilon'\frac{f(\varepsilon)-f(\varepsilon-\omega)}{\omega}\frac{f(\varepsilon')-f(\varepsilon'-\omega)}{\omega}$$
$$\times\int d\mathbf{r}_3d\mathbf{r}_4d\mathbf{r}_5d\mathbf{r}_6 j(\mathbf{r}_3,\mathbf{r}_6)j(\mathbf{r}_4,\mathbf{r}_5)P_{\omega+\varepsilon-\varepsilon'}(\mathbf{r}_6,\mathbf{r}_5)P_{\omega-\varepsilon+\varepsilon'}(\mathbf{r}_4,\mathbf{r}_3)\ , \quad (5)$$

where

$$j(\mathbf{r}_3,\mathbf{r}_6)=\int d\mathbf{r}_1d\mathbf{r}_2\hat{p}_\alpha G_\varepsilon^+(\mathbf{r}_2,\mathbf{r}_3)G_{\varepsilon-\omega}^-(\mathbf{r}_6,\mathbf{r}_2)\times\hat{p}_\alpha G_{\varepsilon'}^+(\mathbf{r}_1,\mathbf{r}_6)G_{\varepsilon'-\omega}^-(\mathbf{r}_3,\mathbf{r}_1) \quad (6)$$

and $P_\Omega(\mathbf{r},\mathbf{r}')$ is the diffusion propagator, which satisfies the equation

$$\{-D[\nabla-ie(\mathbf{A}\pm\mathbf{A}')]^2-i\Omega+\tau_\varphi^{-1}\}P_\Omega(\mathbf{r},\mathbf{r}')=\frac{1}{2\pi N_0\tau^2}\delta(\mathbf{r}-\mathbf{r}')\ . \quad (7)$$

The Green functions in Eq. (6) are impurity averaged. In Eq. (7), $N_0$ is the density of states at the Fermi level, and $\mathbf{A}$ and $\mathbf{A}'$ are the vector potentials in the two conductances; the plus sign refers to the particle-particle or Cooper channel, while the minus sign refers to the particle-hole channel. In the following we shall consider the particle-hole channel, since the particle-particle contribution decays at moderately small fields because of the positive sign.

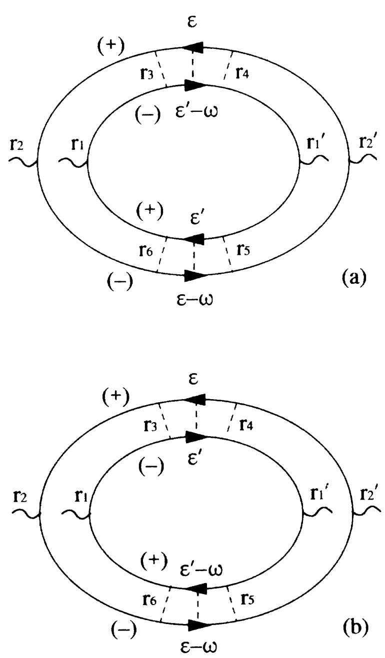

FIG. 1. Feynman diagrams in position space contributing to the correlation functions (a) $F_1(\omega,\Delta B)=\langle\delta G(\omega,B)\delta G(\omega,B+\Delta B)\rangle$, and (b) $F_2(\omega,\Delta B)=\langle\delta G(\omega,B)\delta G^*(\omega,B+\Delta B)\rangle$. Diagrams giving equal contributions are obtained by exchanging $\mathbf{r}_1$ and $\mathbf{r}_1'$, accounting for the overall factors of two in Eqs. (10) and (13).

Since the impurity-averaged Green functions decay exponentially over distances on the order of the mean free path, in the diffusive approximation $j(\mathbf{r}_3,\mathbf{r}_6)$ may be regarded as proportional to $\delta(\mathbf{r}_3-\mathbf{r}_6)$, and is approximately translationally invariant in the finite sample, so that it may be evaluated in momentum space:[10]

$$j(\mathbf{r}_3,\mathbf{r}_6)=\delta(\mathbf{r}_3-\mathbf{r}_6)\int\frac{d\mathbf{p}}{(2\pi)^d}p_\alpha^2G_\varepsilon^+(\mathbf{p})\times G_{\varepsilon-\omega}^-(\mathbf{p})G_{\varepsilon'}^+(\mathbf{p})G_{\varepsilon'-\omega}^-(\mathbf{p})\ . \quad (8)$$

Here $d$ is the dimensionality, and $G_\varepsilon^\pm(\mathbf{p})=[\varepsilon-\varepsilon(\mathbf{p})\pm i/2\tau]^{-1}$. The integral can be evaluated by the contour method; using $|\varepsilon-\varepsilon'|\tau\ll1$ [which is insured by the factors containing the Fermi functions in Eq. (6)], and $\omega\tau\ll1$, we have to sufficient accuracy

$$j(\mathbf{r}_3,\mathbf{r}_6)=\frac{p_F^2}{d}4\pi N_0\tau^3\delta(\mathbf{r}_3-\mathbf{r}_6)\ . \quad (9)$$

The expression for $j(\mathbf{r}_4,\mathbf{r}_5)$ is similar and gives the same result. $F_1$ can now be written as

$$F_1(\omega,\Delta B)=2\left[\frac{2e^2DN_0\tau^2}{L^2}\right]^2\int\int_{-\infty}^{\infty}d\varepsilon\,d\varepsilon'\,g(\varepsilon,\varepsilon';\omega)\times\int d\mathbf{r}\,d\mathbf{r}'P_{\omega+\varepsilon-\varepsilon'}(\mathbf{r},\mathbf{r}')P_{\omega-\varepsilon+\varepsilon'}(\mathbf{r}',\mathbf{r})\ , \quad (10)$$

where we have used $v_F^2\tau/d=D$, and defined

$$g(\varepsilon,\varepsilon';\omega)\equiv\frac{f(\varepsilon-\omega)-f(\varepsilon)}{\omega}\frac{f(\varepsilon'-\omega)-f(\varepsilon')}{\omega}\ . \quad (11)$$

To calculate $F_2$, we take the complex conjugate of one of the conductances by using the symmetry property of the Green functions,

$$[G_\varepsilon^{\pm}(\mathbf{r},\mathbf{r}')]^* = G_\varepsilon^{\mp}(\mathbf{r}',\mathbf{r}) \; . \tag{12}$$

Since the diffusion ladders must connect Green functions of opposite analyticity to give a singular correction, the simplest diagram for $F_2$ is that shown in Fig. 1(b). The current vertex functions $j(\mathbf{r}_3,\mathbf{r}_6)$ and $j(\mathbf{r}_4,\mathbf{r}_5)$ differ from those in $F_1$ only in the energy arguments of the Green functions and so are unchanged in our approximation. The result for $F_2$ is

$$F_2(\omega,\Delta B) = 2\left[\frac{2e^2 D N_0 \tau^2}{L^2}\right]^2 \int\int_{-\infty}^{\infty} d\varepsilon\, d\varepsilon' g(\varepsilon,\varepsilon';\omega) \times \int d\mathbf{r}\, d\mathbf{r}' P_{\varepsilon-\varepsilon'}(\mathbf{r},\mathbf{r}') P_{-\varepsilon+\varepsilon'}(\mathbf{r}',\mathbf{r}) \; . \tag{13}$$

Note that the frequency does not appear in the diffusion propagators in $F_2$; this is because the diffuson depends only on the difference of the energies of the Green-function lines.

### III. APPLICATION TO A MESOSCOPIC RING

We consider an isolated ring of one-dimensional conductor, with radius $a$. If we choose the gauge $\mathbf{A}-\mathbf{A}'=\Delta\mathbf{A}=(\Delta B r/2)\hat{\theta}$, where $\hat{\theta}$ is the azimuthal unit vector, then Eq. (7) for the diffuson can be written

$$\left[-\left[\frac{\partial}{\partial\theta} - i\frac{\Delta\Phi}{\Phi_0}\right]^2 + \frac{a^2}{D}(\tau_\varphi^{-1} - i\Omega)\right] P_\Omega(\theta,\theta') = \frac{a}{2\pi D N_0 \tau^2}\delta(\theta-\theta') \; . \tag{14}$$

Here, $\Delta\Phi$ is the difference in flux threading the ring, and $\Phi_0$ is the flux quantum $h/e$. The operator on the left-hand side has normalized eigenfunctions $\varphi_m(\theta) = (2\pi)^{-1/2}\exp(im\theta)$ on the ring; since $P_\Omega(\theta,\theta')$ must satisfy periodic boundary conditions on the ring, $m$ must be an integer. The solution of Eq. (14) is then

$$P_\Omega(\theta,\theta') = \frac{a}{2\pi D N_0 \tau^2} \sum_m \frac{\varphi_m(\theta)\varphi_m^*(\theta')}{\lambda_m(\Omega)} \; , \tag{15}$$

where $\lambda_m(\Omega)$ is the eigenvalue associated with $\varphi_m$,

$$\lambda_m(\Omega) = \left[m - \frac{\Delta\Phi}{\Phi_0}\right]^2 + \frac{a^2}{D}(\tau_\varphi^{-1} - i\Omega) \; . \tag{16}$$

Substituting into Eqs. (10) and (13) and evaluating the integrals over the ring coordinates, we obtain

$$F_1(\omega,\Delta B) = 2\left[\frac{e^2}{\pi^3}\right]^2 \int\int_{-\infty}^{\infty} d\varepsilon\, d\varepsilon' g(\varepsilon,\varepsilon';\omega) \times \sum_m \frac{1}{\lambda_m(\omega+\varepsilon-\varepsilon')\lambda_m(\omega-\varepsilon+\varepsilon')} \; ,$$

$$F_2(\omega,\Delta B) = 2\left[\frac{e^2}{\pi^3}\right]^2 \int\int_{-\infty}^{\infty} d\varepsilon\, d\varepsilon' g(\varepsilon,\varepsilon',\omega) \times \sum_m \frac{1}{\lambda_m(\varepsilon-\varepsilon')\lambda_m(-\varepsilon+\varepsilon')} \; , \tag{17}$$

where we have taken $L=\pi a$ for the ring.

The sums can be evaluated using the Poisson summation formula,

$$\sum_{m=-\infty}^{\infty} F(m) = \sum_{p=-\infty}^{\infty} \int_{-\infty}^{\infty} dx\, F(x) e^{2\pi i p x} \; , \tag{18}$$

and by making the variable change $x' = x + \Delta\Phi/\Phi_0$ the flux dependence can be taken out of the integral, leading directly to the Fourier expansion in the flux of the correlation functions. In this case, the expansion is a cosine series in the flux, and the coefficient of $\cos(2\pi p \Delta\Phi/\Phi_0)$ in $F_1$ is

$$F_{1p}(\omega,\Delta B) = 4\left[\frac{e^2}{\pi^3}\right]^2 \int\int_{-\infty}^{\infty} d\varepsilon\, d\varepsilon' g(\varepsilon,\varepsilon';\omega) \int_{-\infty}^{\infty} \frac{dx\, e^{2\pi i p x}}{[x^2 + (a/L_\varphi(\omega))^2]^2 + [a^2(\varepsilon-\varepsilon')/D]^2} \; , \tag{19}$$

in which $[L_\varphi(\omega)]^{-2} \equiv L_\varphi^{-2} - i\omega/D$, and $L_\varphi \equiv (D\tau_\varphi)^{1/2}$. The expression for the $p$th component of $F_2$ is identical to this with the exception that $L_\varphi(\omega)$ is replaced by $L_\varphi(0)$ in the integral over $x$.

The integrals over the energies can be approximated in the limits $\omega \ll kT$ and $\omega \gg kT$. In the former case $g(\varepsilon,\varepsilon';\omega) \approx (\partial f/\partial\varepsilon)\,(\partial f/\partial\varepsilon')$, and the rest of the integrand depends only on $\varepsilon-\varepsilon'$. Upon changing to average and difference variables and integrating over the average energy, $(\partial f/\partial\varepsilon)\,(\partial f/\partial\varepsilon')$ becomes a function strongly peaked about $\varepsilon-\varepsilon'=0$, with width of order $kT$ and unit integral. Therefore, for $\omega \ll kT$ we may approximate $F_{1p}$ as

$$F_{1p}(\omega,\Delta B) \approx \left[\frac{e^2}{\pi^3}\right]^2 \frac{2}{kT} \int_{-kT}^{kT} d(\varepsilon-\varepsilon') \int_{-\infty}^{\infty} \frac{dx\, e^{2\pi i p x}}{[x^2 + (a/L_\varphi(\omega))^2]^2 + [a^2(\varepsilon-\varepsilon')/D]^2} \; . \tag{20}$$

The result of the integral over $\varepsilon-\varepsilon'$ involves an arctangent which is of order unity for $x^2<a^2kT/D$, which is the range of the main contribution to the $x$ integral. Setting this factor equal to 1 and evaluating the $x$ integral by the contour method, we obtain

$$F_{1p}(\omega,\Delta B)\approx 2\left[\frac{2}{\pi}\right]^3\left[\frac{e^2}{h}\right]^2\frac{L_T^2L_\varphi(\omega)}{a^3}e^{-2\pi pa/L_\varphi(\omega)} . \tag{21}$$

We have introduced the temperature diffusion length $L_T\equiv(\hbar D/kT)^{1/2}$, and restored factors of $\hbar$. The corresponding expression for $F_{2p}$ is again identical except for the replacement of $L_\varphi(\omega)$ by $L_\varphi(0)$.

For $\omega\gg kT$, $g(\varepsilon,\varepsilon';\omega)$ is approximately a rectangular function of width $\omega$ and height $1/\omega$, so that on changing variables in Eq. (19) as before and integrating over the energy difference from $-\omega$ to $\omega$, we have

$$F_{1p}(\omega,\Delta B)\approx 2\left[\frac{2}{\pi}\right]^3\left[\frac{e^2}{h}\right]^2\frac{L_\omega^2L_\varphi(\omega)}{a^3}e^{-2\pi pa/L_\varphi(\omega)} , \tag{22}$$

where $L_\omega\equiv(D/\omega)^{1/2}$. Again, the corresponding expression for $F_{2p}$ is obtained by replacing $L_\varphi(\omega)$ by $L_\varphi(0)$.

The meaning of these functions can be seen by expanding the conductances in their real and imaginary parts. We then find the correlations of the real and the imaginary parts of the conductance,

$$\begin{aligned}\langle \mathrm{Re}\delta G(\omega,B)\mathrm{Re}\delta G(\omega,B+\Delta B)\rangle &= \mathrm{Re}(F_2+F_1)/2 ,\\ \langle \mathrm{Im}\delta G(\omega,B)\mathrm{Im}\delta G(\omega,B+\Delta B)\rangle &= \mathrm{Re}(F_2-F_1)/2 .\end{aligned} \tag{23}$$

One can also consider the cross correlation $\langle \mathrm{Re}\delta G\, \mathrm{Im}\delta G\rangle$, which is equal to half the imaginary part of $F_1$. The functions $F_1$ and $F_2$ are equal at zero frequency, but the presence of the frequency in the exponential factor in the oscillatory terms in $F_1$ makes the contribution of $F_1$ to the conductance oscillation decay rapidly for $\omega\tau_L>1$. By contrast, the contribution of $F_2$ is nearly independent of frequency until $\omega>kT/\hbar$, and then decays only as $(\omega\tau_L)^{-1}$ in the high-frequency limit because of the energy-averaging factor $g(\varepsilon,\varepsilon';\omega)$. Therefore, for $\omega<kT/\hbar$ the absolute magnitude of the $h/e$ oscillation ($p=1$ term) in the ring is nearly frequency independent, but when $\omega\approx\tau_L^{-1}$ some of the oscillation is transferred from the real part to the imaginary part of the conductance, so that the oscillation in the real part decreases in amplitude by $1/\sqrt{2}$, and an oscillation appears in the imaginary part with an amplitude approaching that in the real part. The predictions for the $h/e$ amplitude as a function of frequency are shown in Fig. 2. When $\omega\gg kT/\hbar$, the amplitudes in both parts decay as $(\omega\tau_L)^{-1/2}$; this result has also been obtained for the magnitude of the universal conductance fluctuations in a one-dimensional wire.[11]

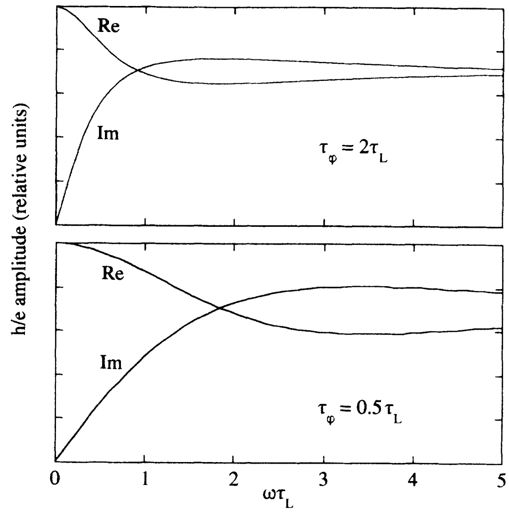


FIG. 2. The amplitude of the $h/e$-periodic conductance oscillations in a ring as a function of frequency, calculated from the diagrams in Fig. 1 for dephasing times $\tau_\varphi$ which are longer and shorter than the mean sample transit time $\tau_L$. These calculations assume that $\omega\ll kT/\hbar$ for all frequencies shown.

## IV. CONCLUSIONS

Because the frequency does not appear in the diffuson in $F_2$, the frequency dependence of the absolute magnitude of mesoscopic conductance fluctuations is only through energy averaging. On the other hand, the phase with which the fluctuations enter the complex conductance is contained in $F_1$ and does vary at frequencies of order $\tau_L^{-1}$. This reflects the phase delay of the interference between diffusive paths having traversal times of order $\tau_L$. The delay gives rise to a fluctuating imaginary part of the conductance at $\omega\sim\tau_L^{-1}$ which is comparable in size to the nonfluctuating imaginary part due to classical reactive effects at the same frequency in a typical device [for example, a narrow wire 1 $\mu$m long with an inductance of order 1 pH, a real resistance of 50 $\Omega$, and $(2\pi\tau_L)^{-1}$ of order 1 GHz]. Because the distribution of traversal times through a device is peaked near $\tau_L$, it might have been expected that mesoscopic effects should decay at frequencies greater than $\tau_L^{-1}$, independently of the energy averaging. This idea, however, is not supported by the calculation we have presented. It is possible that more complicated diagrams may become important at high frequency and modify the results, or that the microscopic electric potential in the device may play an essential role.

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## APPENDIX

In this appendix we consider diagrams of the type shown in Fig. 3, which describe the fluctuations in the density of states. A typical current vertex in these diagrams has the form

$$j(\mathbf{r}_3,\mathbf{r}_6)=\int d\mathbf{r}_2 d\mathbf{r}_2' \hat{p}_\alpha G_\varepsilon^+(\mathbf{r}_2,\mathbf{r}_3)\hat{p}_\alpha G_{\varepsilon-\omega}^-(\mathbf{r}_2',\mathbf{r}_2) \times G_{\varepsilon'}^+(\mathbf{r}_6,\mathbf{r}_2')G_{\varepsilon'-\omega}^-(\mathbf{r}_3,\mathbf{r}_6)\ , \tag{A1}$$

which can again be evaluated in momentum space to give the value of Eq. (9), with corrections for finite frequency only of order $\omega\tau$. These vertices can also be dressed with additional impurity lines; summing the dressed and undressed diagrams at dc gives a partial cancellation resulting in an overall factor[5] of $\frac{1}{4}$, but there are four equal contributions formed by exchanging $\mathbf{r}_1$ and $\mathbf{r}_1'$, or $\mathbf{r}_2$ and $\mathbf{r}_2'$, or both. Since the correction to the vertices at finite frequency is small, we assume that the numerical factor is unchanged for ac and obtain contributions to $F_1$ and $F_2$,

$$F_1(\omega,\Delta B)_N=\left[\frac{2e^2DN_0\tau^2}{L^2}\right]^2\int\int_{-\infty}^{\infty}d\varepsilon\, d\varepsilon' g(\varepsilon,\varepsilon';\omega) \times\int d\mathbf{r}\,d\mathbf{r}'[P_{\omega+\varepsilon-\varepsilon'}(\mathbf{r},\mathbf{r}')]^2\ ,$$

$$F_2(\omega,\Delta B)_N=\left[\frac{2e^2DN_0\tau^2}{L^2}\right]^2\int\int_{-\infty}^{\infty}d\varepsilon\, d\varepsilon' g(\varepsilon,\varepsilon';\omega) \times\int d\mathbf{r}\,d\mathbf{r}'[P_{\varepsilon-\varepsilon'}(\mathbf{r},\mathbf{r}')]^2\ . \tag{A2}$$

Again, the diffusion propagators in $F_2$ are not frequency dependent. For $\omega \ll kT$, the $F_2$ diagram is essentially equal to its dc value, which in one- and two-dimensional systems at finite temperature is small compared to the diagram of Fig. 1(b) at dc.[12] On the isolated ring the $F_2$ diagram can be written in terms of the eigenvalues of the ring as

$$F_2(\omega,\Delta B)_N=\left[\frac{e^2}{\pi^3}\right]^2\int\int_{-\infty}^{\infty}d\varepsilon\, d\varepsilon' g(\varepsilon,\varepsilon';\omega) \times\sum_m[\lambda_m(\varepsilon-\varepsilon')]^{-2}\ , \tag{A3}$$

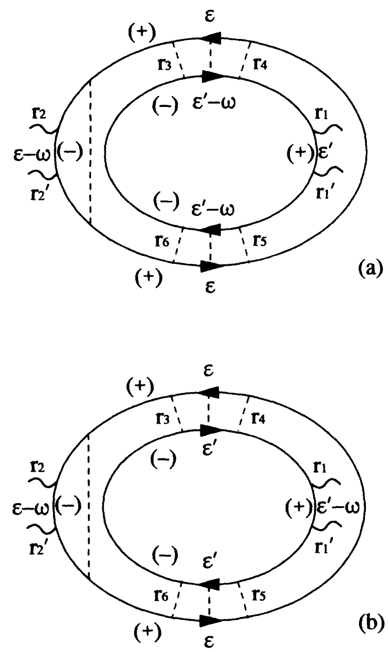


FIG. 3. Diagrams contributing to the correlation functions (a) $F_1$ and (b) $F_2$ through fluctuations in the density of states. An optional impurity dressing of a current vertex is shown.

and for $\omega \ll kT$, its $p$th component in the flux is

$$F_{2p}(\omega,\Delta B)_N\approx\left[\frac{e^2}{\pi^3}\right]^2\frac{1}{kT}\int_{-kT}^{kT}d(\varepsilon-\varepsilon') \times\int_{-\infty}^{\infty}\frac{dx\, e^{2\pi ipx}}{[x^2+(a/L_\varphi)^2-ia^2(\varepsilon-\varepsilon')/D]^2}\ , \tag{A4}$$

which is suppressed as $\exp(-2\pi pa/L_T)$ when $L_T \ll L_\varphi$, which is usually satisfied in experiments on metal devices. Since a finite frequency only increases the range of the energy averaging in $F_2$ for a fixed temperature, the absolute value of the contribution to the $h/e$ oscillation in the ring will be small compared to that of Fig. 1(b) at all frequencies.

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