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Bosonization of current-current interactions

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We discuss a generalization of the conventional bosonization procedure to the case of current-current interactions which have a natural representation in terms of current instead of fermion-number-density operators. A consistent bosonization procedure requires a geometrical quantization of the Hamiltonian action of W_∞ on its coadjoint orbits. An integrable example of a nontrivial realization of this symmetry is presented by the Calogero-Sutherland model. For an illustrative nonintegrable example we consider transverse gauge interactions and calculate the fermion Green function.

I. GENERALIZED BOSONIZATION BASED ON W_∞

The method of bosonization[1] has proved to be a powerful tool for studying a great variety of one-dimensional systems of interacting fermions. However, the applicability of the conventional bosonization technique is restricted by such necessary requirements as a linear fermion dispersion and the locality of a four-fermion interaction which can be written solely in terms of density operators $\rho_\alpha(q)=\sum_{\alpha p>0}\Psi_\alpha^\dagger(p+q)\Psi_\alpha(p)$, where the subscript $\alpha=(R,L)$ labels chirality. Due to the commutation relations

$$[\rho_\alpha(q),\rho_{\alpha'}(q')]=\alpha q\delta_{\alpha\alpha'}\delta(q+q') \tag{1.1}$$

every Hamiltonian bilinear in $\rho_\alpha(q)$ can be represented as a quadratic form in terms of free bosons. Then the solution of the model can be easily achieved by means of the Bogoliubov transformation. It is supposed to yield an asymptotic long-wavelength description of a wide class of four-fermion interactions including long-ranged ones.

In a wider sense a bosonization procedure can be understood as a mapping of the fermion Hilbert space to a space of variables obeying commutation instead of anticommutation relations. In view of that one might ask whether there exists a formulation in terms of some variables of "bosonic" nature which would remain valid at all scales.

Apparently, to perform a consistent bosonization of more general Hamiltonians which would be correct away from the long-wavelength-scaling limit one has to enlarge the algebra of relevant operators. This stems naturally from the necessity to take into account higher-order spatial derivatives in Hamiltonians of the general form

$$H=-\tfrac{1}{2}\int dx\,\Psi^\dagger\partial_x^2\Psi+\tfrac{1}{2}\int dx_1dx_2dx_3dx_4\Psi^\dagger(x_1)\Psi(x_2)\times V(x_1,x_2;x_3,x_4)\Psi(x_3)^\dagger\Psi(x_4)\,. \tag{1.2}$$

Since the Hamiltonian (1.2) conserves the number of particles one can construct a required set of "bosonic" operators from various fermion bilinears $\Psi(x)^\dagger\Psi(x')$. According to Ref. 2 one can choose the following basis of operators:

$$W(x,q)=\int dr\,e^{iqr}\Psi^\dagger\left[r+\frac{x}{2}\right]\Psi\left[r-\frac{x}{2}\right]\,. \tag{1.3}$$

It can be readily seen that operators (1.3) obey the commutation relations

$$[W(x,q),W(x',q')]=2i\sin\tfrac{1}{2}(x'q-xq')W(x+x',q+q') \tag{1.4}$$

corresponding to the infinite dimensional algebra W_∞, which is a quantum deformation of the classical algebra w_∞ of area-preserving (symplectic) diffeomorphisms of the plane (x,q).

This algebra can also be understood as a particular limit of the trigonometric form of the SU(N) algebra at $N\to\infty$.[3] Notice that in the case of a finite chain of length N lattice counterparts of operators (1.3) do form the SU(N) algebra.

This algebraic structure finds numerous applications which include quantum mechanics of fermions on the first Landau level,[4] combinatorics of Laughlin wave functions,[5] topologically massive (2+1)-dimensional gauge theories,[6] two-dimensional turbulence of an incompressible fluid,[7,8] and two-dimensional gravity.[9]

In the context of one-dimensional classical (quantum) Hamiltonian dynamics the above-mentioned two-dimensional manifold is realized as a phase space of a single particle. accordingly, phase-space-volume-preserving diffeomorphisms correspond to canonical (unitary in the quantum case) transformations.

The physical meaning of W operators can be easily clarified by expanding $W(x,q)$ into a series $W(x,q)=\sum_{s=1}^{\infty}W^s(q)x^{s-1}/(s-1)!$. A simple analysis shows that the label s can be naturally identified with the conformal spin of the field $W^s(q)$.

In the case of a finite chain the mode index $q=2\pi n/N$ runs over N distinct values. In the case of a finite density of particles one may distinguish between left- and right-moving particles. The two ("left" and "right") algebras are isomorphic to each other. Then the algebra of, say, "right" fields $W_R^s(q)$ resulting from (1.4) gets a nontrivial central extension.

$$[W^s(q),W^{s'}(q')]=[(s-1)q'-(s'-1)q]W^{s+s'-2}(q+q') + \cdots + \delta(q+q')\delta_{ss'}c(q) , \tag{1.5}$$

where the dots denote contributions of field with spins ranging from $s+s'-4$ to zero. The c-number term providing a central extension to the algebra (1.5) appears in the same way as a conventional Schwinger anomaly in the right-hand side of (1.1). For a formal derivation of the c-number term in (1.5) one has to redefine the W generators: $W_R^s(q)\to W_R^s(q)-\sum_{p>0}(p+q/2)^{s-1}n(p)$ where $n(p)$ is the Fermi distribution function. A total anomaly obtained by a summation over all Fermi points cancels out.

Note that chiral W_∞ algebras appear naturally in the theory of edge states on boundaries of quantum Hall effect (QHE) droplets, the number of independent species being equal to the number of closed boundaries.[10]

Now we consider more concretely, the algebra of "right" current and fermion number densities. The spin 1 field $W^1(q)$ obeys the Abelian Kac-Moody algebra (1.1) and can be identified with the fermion density operator $W^1(q)=\rho_R(q)$ while the spin 2 field $W^2(q)$ has commutation relations

$$[W^2(q),W^1(q')]=-q'W^1(q+q') , \tag{1.6}$$

$$[W^2(q),W^2(q')]=(q-q')W^2(q+q') + \frac{c}{12}(k_F^2q-q^3)\delta(q+q') . \tag{1.7}$$

One can readily recognize the operators $W^2(q)$ obeying the Virasoro algebra (1.7) as current densities $\pi_R(q)=\sum_{p>0}(p+q/2)\Psi^\dagger(p+q)\Psi(p)$. In a proper normalization the central charge c corresponding to the case of free fermions is equal to 1.

To proceed with a consistent quantization of Hamiltonians expressed in terms of generators of the infinite algebra (1.5) one can apply the so-called method of geometrical quantization.[11] Recently it was shown that this method generalizing a coherent state representation provides a regular quantization procedure for the case of affine Lie algebras.[12]

In the framework of this approach one has to derive a covariant Lagrangian of the system in terms of variables taking their values on coadjoint orbits of the underlying algebra. An arbitrary element of the orbit Q can be represented by the projection operator into some coherent state $|\Psi\rangle$

$$Q=|\Psi\rangle\langle\Psi|=gPg^\dagger ,$$
$$g=\exp\left[i\int dq\,dx\,\phi(x,q)W(x,q)\right] , \tag{1.8}$$

where $P=|0\rangle\langle 0|$ is a projection operator into a reference state $|0\rangle$ belonging to the orbit. By construction one always has $Q^2=Q$ and the orbit is specified by the condition $\mathrm{tr}Q=1$ where the trace symbol stands for the integral over phase space coordinates, $\mathrm{tr}=\int dq\,dx$. The so-called symbol of an arbitrary operator $\hat{O}$ is given by its average over some coherent state, $\langle\Psi|\hat{O}|\Psi\rangle=\mathrm{tr}(QO)$.

A natural parametrization of the coadjoint orbit of W_∞ can be obtained in terms of the phase space density[2]

$$u(x,p)=\int dr\,dq\,e^{irp-ixq}\langle\Psi|W(r,q)|\Psi\rangle . \tag{1.9}$$

In terms of this variable the Lagrangian acquires the form

$$L=\langle\Psi|ig^\dagger d_t g-H|\Psi\rangle$$
$$=i\int_0^\infty d\sigma\,\mathrm{tr}(u\{\partial_\sigma u,\partial_t u\}_{\mathrm{MB}})-\mathrm{tr}(HQ) , \tag{1.10}$$

where $u(\sigma=0)=u(x,q)$ and $u(\sigma=\infty)=u_0=\mathrm{const}$. Note that the first term in (1.10) depends exclusively on the value of $u(\sigma;x,q;t)$ on the boundary $\sigma=0$. The use of the cocycle construction enables one to write down the Lagrangian (1.10) in a totally covariant form.

A definition of the so-called Moyal bracket $\{\ ,\ \}_{\mathrm{MB}}$ appearing in (1.10) reflects the complexity of commutation relations (1.4)

$$\{A,B\}_{\mathrm{MB}}=2\sin\tfrac{1}{2}(\partial_x\partial_{q'}-\partial_q\partial_{x'}) \times A(x,q)B(x',q')|_{x=x',q=q'} . \tag{1.11}$$

Despite numerous potential applications of the above formalism a geometry of W_∞ orbits remains quite unknown. Some attempt to get a first insight based on the finite N orbits of SU(N) was undertaken in Ref. 7.

However, one can easily obtain a local equation of motion for the phase space density,

$$\partial_t u+\{H,u\}_{\mathrm{MB}}=0 . \tag{1.12}$$

In the case of free fermions the equilibrium solution of (1.12) is merely $u_0(x,p)=\theta(k_F^2-p^2)$. To consider small deviations from the equilibrium state one can choose the parametrization $u(x,p)=\theta(k_F^2(x)-p^2)$ which leads to the approximation called the collective field theory.[13] Comparing this parameterization with the most general expansion

$$u(x,p)=\theta(k_F^2-p^2)+\sum_\alpha \delta(p-\alpha k_F)\rho_\alpha(x) + \partial_p\delta(p-\alpha k_F)[\pi_\alpha(x)-\alpha k_F\rho_\alpha(x)]+\cdots , \tag{1.13}$$

we conclude that in the framework of the collective field theory one can use an approximate identity

$$\pi_\alpha(x)\approx\left[\alpha k_F+\frac{1}{2i}\partial_x\right]\rho_\alpha(x) . \tag{1.14}$$

II. CALOGERO-SUTHERLAND MODEL

A remarkable example of a nontrivial realization of the W_∞ symmetry is provided by the famous Calogero-Sutherland model of one-dimensional fermions with long-range forces

$$H=\frac{1}{2}\int dx\, \Psi^\dagger(-\partial_x^2+\omega^2x^2)\Psi + \frac{1}{2}\int dx\, dx'\Psi^\dagger(x)\Psi(x)\frac{\lambda(\lambda-1)}{(x-x')^2}\Psi^\dagger(x')\Psi(x') \tag{2.1}$$

[or $V(x)\sim(L\sin x/L)^{-2}$ in the compact case]. This model manifests a surprisingly simple solution which allows one to consider the Hamiltonian (2.1) as some kind of a "statistical" interaction between fermions (compare the discussion of one-dimensional spin models with $1/x^2$ exchange in Ref. 14).

It was pointed out by Sutherland[15] that every state of the model (2.1) can be represented in the form

$$\Psi(x_1,\ldots,x_N)=\prod_{i<j}^{N}(x_i-x_j)^\lambda P(x_1,\ldots,x_N) \times\exp\left[-\frac{1}{4}\omega\sum_i^N x_i^2\right] , \tag{2.2}$$

where $P(x_1,\ldots,x_N)$ is a symmetric polynomial, the ground state corresponding to $P(\{x_i\})=1$.

It readily follows form the results of the previous studies[16,17] that the Hamiltonian (2.1) can be simply expressed in terms of the operators

$$W_n^s=\int dr\,\Psi^\dagger(r)\left[-i\partial_r-(\lambda-1)\int dr'\frac{\Psi^\dagger\Psi(r')}{r-r'}\right]^{s-1} \times r^n\Psi(r) , \tag{2.3}$$

where a proper normal ordering is assumed. It turns out that the operators (2.3) provide a nontrivial realization of W_∞.

The necessity to use the "covariant" derivative $-i\partial_r-(\lambda-1)\int dr'\Psi^\dagger\Psi(r')/(r-r')$ instead of the usual one can be easily deduced in the first quantized formalism. In the space of many-body wave functions the covariant derivative acts as $\prod_{i<j}^N(x_i-x_j)^{\lambda-1}\partial_{x_i}\prod_{i<j}^N(x_i-x_j)^{1-\lambda}$. Being applied to any of the functions (2.2) this operator leaves the result in the same set of functions, while an ordinary derivative has this property for $\lambda=1$ only.

One can also see that all modes with $0\le n<s-1$ annihilate the ground state $W_n^s|0\rangle=0$ and reveal its intrinsic symmetries. On the other hand, acting by the modes W_n^s with $n>s-1$ on the ground state, which appears to be the highest weight vector of the W_∞ representation, one creates all excited states of the form (2.2), $W_n^s|0\rangle=|\text{excitation}\rangle$.

Notice that the central charge c appearing in the commutator of $W^2(q)$ is still equal to 1 and in the scaling limit the only effect of the $1/x^2$ interaction is a renormalization of the Fermi velocity $v_F\to v_F\lambda$.[18] An apparent similarity of the functions (2.2) and the variational Laughlin wave functions proposed to describe ground states and low-lying excitations of the $\nu=\lambda^{-1}$ (for odd integer λ) yields a simple explanation of the reported high symmetries of Laughlin states.[5]

The nonlinear construction (2.3) demonstrates the fact that the model (2.1) describes an analog of a "statistical" interaction and can be understood as a one-dimensional counterpart of the two-dimensional anyon model. In fact, the Hamiltonian (2.1) does result from the anyon Hamiltonian with a statistical parameter equal to $\theta=\pi\lambda$ if all particles are placed on the same line.

It appears to be crucially important that, like the free fermion case, the Hamiltonian (2.1) can be written as a linear form in generators (2.3),

$$H=W_0^3+\omega^2W_2^1 . \tag{2.4}$$

Obviously, the set of mutually commuting operators $I_s=W_0^s$ commute with (2.4) and constitute a complete set of integrals of motion of the Calogero problem.[16]

III. CURRENT-CURRENT INTERACTIONS IN *D* DIMENSIONS

As an instructive (nonintegrable) example of an occurrence of W symmetries we consider fermion interactions bilinear in current operators,

$$H-\mu N=\sum_\alpha[W_\alpha^2(0)-k_F^2W_\alpha^1(0)] + \frac{1}{2}\sum_{\alpha\beta}\int dq\, W_\alpha^2(q)D_{\alpha\beta}(q)W_\beta^2(-q) . \tag{3.1}$$

The Hamiltonian (3.1) provides an example demonstrating a relevance of the subalgebra of W_∞ formed by $W_\alpha^1(q)$ and $W_\alpha^2(q)$. Indeed the only bosonic description of (3.1) which remains valid at all scales can be performed in terms of current and not fermion number densities. To treat the model (3.1) one has to carry out the geometrical quantization of the Virasoro subalgebra of W_∞ similar to that described in Ref. 19 in the context of the two-dimensional conformal field theory.

In general, the matrix $D_{\alpha\beta}(q)$ couples $N>1$ Fermi points all together. As another generalization allowed by the Lagrangian formalism one can also consider retarded interactions corresponding to a frequency-dependent vertex $D_{\alpha\beta}(\omega,q)$.

The equation of motion for the phase space density (1.13) reads as

$$\partial_t u_\alpha(x,p)=p\partial_x u_\alpha(x,p) +p(k_F^2\partial_x-\partial_x^3)\int dx' D_{\alpha\beta}(x-x')u_\beta(x',p) +\int dx'\int dp' pp' D_{\alpha\beta}(x-x') \times u_\beta(x',p')\partial_x u_\alpha(x,p) . \quad (3.2)$$

This nonlinear equation can only be treated perturbatively. In the long-wavelength limit one can neglect the last term in (3.2) as a higher-order gradient correction. The residual linear equation enables to determine a propagator of the field $u_\alpha(q,p;\omega)=\int dx\, dt\, e^{iqx-i\omega t}u_\alpha(x,p;t)$,

$$\langle u_\alpha(q,p;\omega)u_\alpha(-q,p;-\omega)\rangle =q\left[\omega-qp+\frac{p}{12}(k_F^2 q-q^3)\hat{D}(\omega,q)\right]^{-1}_{\alpha\alpha} . \quad (3.3)$$

In accordance with the previous discussion one obtains a long-wavelength limit of the expectation value $\langle\rho_\alpha(\omega,q)\rho_\alpha(-\omega,-q)\rangle$ simply by putting p equal to k_F in (3.3).

The formulas (3.2) and (3.3) can be used also for a perturbative computation of expectation values of any functionals $F\{u_\alpha\}$ representing various correlation functions $\langle W^n(q,t)\cdots W^m(q',t')\rangle$. Furthermore, in some circumstances the bosonization scheme can be generalized to the case of higher dimensions. Such a development of a conventional bosonization of density-density interactions was proposed in Ref. 20 and recently developed in Ref. 21 and also discussed in Refs. 22 and 23.

The basic assumption put forward in Ref. 21 is the existence of a Fermi surface obeying the Luttinger theorem. It can be considered as an extended object having infinite number of degrees of freedom corresponding to the Luttinger volume-preserving diffeomorphisms. Fluctuations of the Fermi surface are associated with the collective modes of the system (particle-hole excitations) which constitute the entire low energy physics. This conjecture is supposed to be essentially weaker than a statement about the applicability of the Landau-Fermi-liquid theory. Therefore it may facilitate an informative analysis of nontrivial non-Fermi-liquid states.

The key elements of the construction are commutation relations of the D-dimensional analogues of the W generators,

$$W_\alpha^s(\mathbf{q})=\sum_{\mathbf{p}\in\Lambda_\alpha}\left[\left(\mathbf{p}+\frac{\mathbf{q}}{2}\right)\mathbf{n}_\alpha\right]^{s-1} \times[\Psi^\dagger(\mathbf{p}+\mathbf{q})\Psi(\mathbf{p})-\delta(\mathbf{q})\langle n(\mathbf{p})\rangle] . \quad (3.4)$$

Here the unit vector $\mathbf{n}_\alpha$ is a normal to the Fermi surface "patch" Λ_α of the area $S_D\sim\Lambda^{D-1}$ ($\Lambda\ll k_F$) centered at the point $\mathbf{k}_\alpha$. Formally one should first construct a proper bosonization scheme for the case of $N\sim(k_F/\Lambda)^{D-1}$ coupled Fermi points and then tend N to infinity.

It was shown in Ref. 22 that a straightforward generalization of (1.1), (1.6), and (1.7) for $\rho_\alpha(\mathbf{q})=W_\alpha^1(\mathbf{q})$ and $\pi_\alpha(\mathbf{q})=W_\alpha^2(\mathbf{q})$,

$$[\rho_\alpha(\mathbf{q}),\rho_\beta(\mathbf{q}')]=S_D(\mathbf{n}_\alpha\mathbf{q})\delta_{\alpha\beta}\delta(\mathbf{q}+\mathbf{q}') , \quad (3.5)$$

$$[\pi_\alpha(\mathbf{q}),\rho_\beta(\mathbf{q}')]=(\mathbf{n}_\alpha\mathbf{q})\delta_{\alpha\beta}\rho_\alpha(\mathbf{q}+\mathbf{q}') , \quad (3.6)$$

$$[\pi_\alpha(\mathbf{q}),\pi_\beta(\mathbf{q}')]=\delta_{\alpha\beta}((\mathbf{q}-\mathbf{q}')\mathbf{n}_\alpha)\pi_\alpha(\mathbf{q}+\mathbf{q}') +\tfrac{1}{12}S_D\delta_{\alpha\beta}\delta(\mathbf{q}+\mathbf{q}') \times[(\mathbf{n}_\alpha\mathbf{q})k_F^2-(\mathbf{n}_\alpha\mathbf{q})^3] , \quad (3.7)$$

can only be derived if all moments lie inside a squat box with the size $\Lambda_\parallel$ along the normal to the Fermi surface being much less than the size $\Lambda_\perp$ in the tangent direction. Otherwise, one cannot neglect four-fermion terms in the right-hand side of (3.5)–(3.7) and the above algebra gets spoiled.

In the absence of any cutoff introduced by hand, one might think that this condition can be fulfilled dynamically if due to the specific features of the interaction vertex $D_{\alpha\beta}(\omega,\mathbf{q})$ the following relations among transferred energy and momentum hold:

$$\omega\lesssim q_\parallel\ll q_\perp . \quad (3.8)$$

Notice that these relations obviously fail in the case of the random-phase approximation (RPA) screened long-ranged density-density interaction $V(q)\sim 1/q^\alpha$ with $\alpha>0$. The vertex dressed by the RPA bubbles

$$D_{\rho\rho}(\omega,q)=\frac{V(q)}{1+\prod_{\rho\rho}(\omega,q)V(q)} , \quad (3.9)$$

where $\prod_{\rho\rho}(\omega,q)\approx 1-q^2/\omega^2$ (at $q\ll\omega$) is a scalar polarization operator, develops a pole characterized by the dispersion $\omega\sim q^{1-\alpha/2}$ at small q. Moreover, in the case of Coulomb interaction ($\alpha=2$) recently considered in Ref. 24, the collective mode acquires a finite plasmon gap and then is no longer relevant in the low-wavelength limit. Thus the applicability of the method to the case of long-range density-density interactions remains questionable.

However the conditions (3.8) certainly hold in the case of the RPA-summed effective current-current interaction governed by the transverse vector polarization $\prod_{\pi\pi}(\omega,q)=\chi(\omega,q)q^2+i\sigma(\omega,q)\omega$. In the gapless metallic state [$\chi(\omega,q)\approx$const;$\sigma(\omega,q)\sim 1/q$] the effective vertex

$$D_{\pi\pi}^{\mu\nu}(\omega,q)=\frac{g^2\left[\delta^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right]}{\chi q^2+i\gamma\frac{\omega}{q}} \quad (3.10)$$

demonstrates an overdamped pole at $\omega\sim iq^3$. It was suspected for a long time that the interaction (3.10) changes the behavior of fermions drastically with respect to the free case in both three[25] and two dimensions.[26]

In the long-wavelength approximation corresponding to the equality (1.14) a fermion operator can be represented solely in terms of $W_\alpha^1(\mathbf{q})$,

$$\Psi(\mathbf{r})\sim\sum_{\alpha=1}^{N}\exp[i\mathbf{k}_F(\alpha)\cdot\mathbf{r}+i\Phi_\alpha(r_\parallel)]O_\alpha(\mathbf{r}_\perp) ,$$

$$\Phi_\alpha(\mathbf{r})=\int\frac{d\mathbf{q}}{(2\pi)^D}e^{i\mathbf{q}\cdot\mathbf{r}}\frac{W_\alpha^1(\mathbf{q})}{\mathbf{n}_\alpha\mathbf{q}} . \quad (3.11)$$

The explicit form of the ordering operator $O_\alpha(\mathbf{r}_\perp)$ necessary to maintain anticommutativity of operators at equal $r_\parallel=\mathbf{r}\cdot\mathbf{n}$ is strongly dependent on dimension. In particular, in two dimensions one can use the operator $O_\alpha(\mathbf{r})=\exp[i\int d\mathbf{r}'\arg(\mathbf{r}-\mathbf{r}')W^1_\alpha(\mathbf{r}')]$, which is a counterpart of the one-dimensional Jordan-Wigner factor $O_\alpha(r)=\exp[i\pi\alpha\int dr'\Psi^\dagger(r')\Psi(r')]$.

IV. ONE-PARTICLE GREEN FUNCTION

Proceeding along the lines proposed in Refs. 21 and 22 and assuming for simplicity a spherical shape of the Fermi surface, one arrives at the integral representation of the one-particle Green function,

$$G(\mathbf{r},t)=\langle\Psi(\mathbf{r},t)\Psi^\dagger(0,)\rangle$$
$$\sim\sum_{\alpha=1}^{N}\exp\left[i\mathbf{k}_F(\alpha)\mathbf{r}-N\int_{q_\perp<\Lambda_\alpha}\frac{d\omega\, d\mathbf{q}}{(2\pi)^{D+1}q_\parallel}\frac{(1-e^{i\mathbf{q}\cdot\mathbf{r}-i\omega t})}{[\omega-q_\parallel+S_D\frac{1}{12}(k_F^2q_\parallel-q_\parallel^3)\hat{D}(\omega,\mathbf{q})+i\delta]_{\alpha\alpha}}\right]. \tag{4.1}$$

Taking the limit of $N\to\infty$ and keeping the first nonzero term in the $1/N$ expansion one obtains the formula

$$G(\mathbf{r},t)\sim\int\frac{d\mathbf{n}}{(2\pi)^{D-1}}\frac{e^{ik_F\mathbf{n}\cdot\mathbf{r}}}{\mathbf{n}\cdot\mathbf{r}-t+i\delta}\exp\left[-\int\frac{d\omega\, d\mathbf{q}}{(2\pi)^{D+1}}\frac{(1-e^{i\mathbf{q}\cdot\mathbf{r}-i\omega t})D(\omega,\mathbf{q})}{(\omega-\mathbf{n}\cdot\mathbf{q}+i\delta)^2}\right], \tag{4.2}$$

where $D(\omega,\mathbf{q})$ denotes a diagonal matrix element of the operator $D_{\alpha\beta}(\omega,\mathbf{q})$.

Notice that the formula (4.2) generalizes the result obtained in Ref. 27 for the case of $1<D<2$ by the method of "asymptotic Ward identities" based on the conjecture of a dominant rule of forward-scattering processes. The consideration in Ref. 27 was restricted to the case of local interactions. On the basis of the above observation we expect that their analysis could be extended to the case of long-range interactions in $D<2+\lambda$, the upper critical dimension being determined by the exponent λ which governs the asymptotics $D(\omega,\mathbf{q})\sim\max(\omega,q)^{-\lambda}$ at small ω,q.

We shall concentrate on the $D=2$ case first. Then the integral over the transverse transferred momentum $q_\perp$ yields

$$\int d^{D-1}q_\perp D(\omega,\mathbf{q})\sim\frac{g^2}{\omega^{1/3}}. \tag{4.3}$$

Other integrations in the exponent give the factor $\sim\exp[-g^2r_\parallel/\max(|t-r_\parallel|^{2/3},r_\perp^2)]$ which shows that the integrand in the residual integral over the Fermi surface is strongly peaked at $\mathbf{n}$ parallel (or antiparallel) to $\mathbf{r}$.

Calculating the Fourier transform of (4.2) at energy and momentum close to the (Luttinger) Fermi surface ($\varepsilon=0,p=k_F$) one obtains the expression

$$G(\epsilon,\mathbf{p})\sim\int_0^\infty r^2dr\,\sin k_Fr\times\int_0^\infty dt\frac{J_0(pr)}{t^2-r^2+i\delta}e^{i\epsilon t-g^2r/|t-r|^{2/3}}. \tag{4.4}$$

In particular, at $p=k_F$ and $\epsilon\to0$ we recover the asymptotic

$$G(\epsilon)\sim\frac{g^{3/2}}{\epsilon^{5/4}}\exp\left[-\frac{g^3}{\epsilon^{1/2}}\right], \tag{4.5}$$

which coincides with the result of the eikonal approximation[28] and exhibits a behavior drastically different from the Fermi-liquid one.

In the three-dimensional case the integral (4.3) behaves as $\ln\omega$ which means that $D=3$ is a critical dimension for the interaction (3.10). Our simplified consideration leads to the conclusion that the one-particle Green function has Luttinger-type features, $G(\epsilon)\sim\epsilon^{-1+\eta}$ where $\eta\sim g^2$. However we stress that in contrast to the $D=2$ result (4.5) the latter estimate can be strongly affected by neglected terms. A more precise analysis is needed to establish the $D=3$ behavior reliably. One can also obtain correlation functions of more than two fermion operators[28] using the same technique.

V. CONCLUSIONS

We observe that a heuristic attempt to accomplish a consistent bosonization of one-dimensional fermions with nonlinear dispersion and nonlocal interactions encounters such an algebraic structure as a central extension of W_∞ realized in terms of fermion bilinears. To proceed with a Lagrangian description one has to find a proper parametrization of coadjoint orbits of W_∞. The orbit parametrization in terms of the phase space density leads to a generalization of the collective field theory.[13] It also provides a regular way to derive corrections to results of the conventional bosonization due to higher gradient terms which become important away from the scaling limit. The Calogero-Sutherland model, presenting an example of a nonlinear realization of W_∞, allows a simple construction of integrals of motion in terms of generators of this algebra.

The bosonization procedure based on W_∞ can also be extended to higher dimensions by generalizing the approach of Ref. 21. We discuss current-current interactions mediated by a transverse gauge field as a physically relevant example where the Virasoro-type subalgebra of W_∞ occurs. On the basis of this consideration one can establish the status of the eikonal approximation earlier applied to this problem.[28] It corresponds to the neglect of higher order gradient terms and gives essentially the same results as a conventional bosonization as well as the method of "asymptotic Ward identities."[27]

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[1] S. Tomonaga, Prog. Theor. Phys. **5**, 544 (1950); D. C. Mattis and E. Lieb, J. Math. Phys. **6**, 304 (1964); A. Luther and V. J. Emery, Phys. Rev. Lett. **33**, 589 (1974); V. J. Emery, A. Luther, and I. Pechel, Phys. Rev. B **13**, 1272 (1976).

[2] S. R. Das, A. Dhar, G. Mandal, and S. R. Wadia, Mod. Phys. Lett. A **7**, 71 (1992); A. Dhar, G. Mandal, and S. R. Wadia, *ibid.* **7**, 3129 (1992).

[3] E. G. Floratos, Phys. Lett. B **232**, 467 (1989).

[4] S. Iso, D. Karabali, and B. Sakita, Phys. Lett. B **296**, 143 (1992); A. Capelli, C. A. Trugenberger, and G. R. Zemba, Nucl. Phys. B **396**, 465 (1993).

[5] D. Karabali (unpublished); M. Flohr and R. Varnhagen (unpublished).

[6] I. I. Kogan, Mod. Phys. Lett. A **7**, 3717 (1992).

[7] V. Zeitlin, Physica D **49**, 353 (1991).

[8] K. G. Selivanov, Mod. Phys. Lett. A **8**, 1139 (1993).

[9] I. R. Klebanov and A. M. Polyakov, Mod. Phys. Lett. A **6**, 3273 (1991); J. Avan and A. Jevicki, Phys. Lett. B **266**, 35 (1991).

[10] J. Martinez and M. Stone (unpublished).

[11] A. A. Kirillov, *Elements of the Theory of Representations* (Springer-Verlag, Berlin, 1978).

[12] P. B. Wiegmann, Nucl. Phys. **B323**, 311 (1989).

[13] A. Jevicki and B. Sakita, Nucl. Phys. **B165**, 511 (1980).

[14] F. D. M. Haldane, Phys. Rev. Lett. **67**, 937 (1991).

[15] B. Sutherland, J. Math. Phys. **12**, 246 (1971); **12**, 251 (1971); Phys. Rev. A **4**, 2019 (1971); **5**, 1372 (1972).

[16] A. P. Polychronakos, Phys. Rev. Lett. **69**, 703 (1992).

[17] B. S. Shastry, Phys. Rev. Lett. **69**, 164 (1992).

[18] N. Kawakami and S.-K. Yang, Phys. Rev. Lett. **67**, 2493 (1991).

[19] A. Alekseev and S. Shatashvili, Nucl. Phys. **B323**, 719 (1989).

[20] A. Luther, Phys. Rev. B **19**, 320 (1979).

[21] F. D. M. Haldane (unpublished).

[22] A. Houghton and J. B. Marston, Phys. Rev. B **48**, 7790 (1993).

[23] A. H. Castro Neto and E. Fradkin, Phys. Rev. Lett. **72**, 1393 (1994).

[24] P. Bares and X. G. Wen, Phys. Rev. B **48**, 8636 (1993).

[25] T. Holstein, R. E. Norton, and P. Pincus, Phys. Rev. B **8**, 2649 (1973); M. Y. Reiser, *ibid.* **39**, 1609 (1989); **40**, 11 571 (1989).

[26] P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989).

[27] C. Di Castro, C. Castellani, and W. Metzner, Phys. Rev. Lett. **72**, 316 (1994).

[28] D. V. Khveshchenko and P. C. E. Stamp, Phys. Rev. Lett. **71**, 2118 (1993); Phys. Rev. B **49**, 5227 (1994).