RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 49, NUMBER 23 15 JUNE 1994-I

Conductivity of an interacting two-channel Tomonaga-Luttinger model

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(Received 6 April 1994)

Conductivity of a coupled two-channel dirty Tomonaga-Luttinger model is calculated from the Mori formalism. We find that the conductivity becomes enhanced as the interchannel interaction is turned on even when the interaction is repulsive. This effect is opposite to those from the $2k_F$ charge-fluctuation coupling in systems with the channel separation $< 1/k_F$, and is considered to come from the suppressed charge-density-wave fluctuations.

Recent studies of quantum transport in mesoscopic systems have brought to light many unusual features unexpected for classical systems.[1] The conductance quantization[2] in the ballistic regime and the universal conductance fluctuation[3] exemplify these in a two-dimensional (2D) system.

Quasi-one-dimensional (1D) systems, on the other hand, have been constructed with recent nanostructure technology and are called quantum wires. In one dimension, the electron-electron interaction can play a crucial role, which may be analyzed by what is called g-ology,[4–6] which is also extended to treat, e.g., the Coulomb blockade.[7] When only the interactions corresponding to the forward-scattering process are relevant, the system is reduced to the Tomonaga-Luttinger model. The conductivity of the dirty single-channel Tomonaga-Luttinger model has been discussed by Luther and Peschel for spinless fermions.[8] They find that the conductivity for $T \to 0$, which converges to a finite value for noninteracting electrons, diverges for attractive electron-electron interactions, and converges to zero for repulsive interactions due to the charge-density-wave (CDW) correlation. The suppression of the conductivity here can be attributed to the pinning of the CDW, which is enhanced (suppressed) by repulsive (attractive) interactions. Recently Fukuyama, Kohno, and Shirasaki have examined the effect of long-range Coulomb interaction on the conductivity by making the g parameters wave-number dependent.[9] The effect of the long-range interaction is found to appear as a weak temperature dependence of the critical exponent (γ in $\sigma \sim T^\gamma$ at low temperatures) for the conductivity. These studies have concentrated on the case of single-channel systems.

In this paper, we look into the conductivity of two-channel systems as a model for coupled quantum wires. We shall show that, surprisingly, the conductivity becomes enhanced by the interchannel (repulsive or attractive) interaction.

We consider the two-channel Tomonaga-Luttinger model with short-range intrachannel and interchannel electron-electron interactions with finite impurity densities. We take the two channels (α and β) to be equivalent, and we also assume that the interchannel electron tunneling is absent for simplicity. To concentrate on the many-body effect here, we assume that the system is short enough for the effect of the Anderson localization to be neglected as is done in previous studies,[8,9] although the problem can become crucial for long 1D systems.[3] We take $\hbar = k_B = 1$ in this paper.

The two-channel 1D system of interacting electrons with electron-impurity interaction is modeled by a phase Hamiltonian,

$$H = H_0 + H_{\rm imp}, \tag{1}$$

$$H_0 = H_{\rm charge} + H_{\rm spin}, \tag{2}$$

$$\begin{aligned} H_{\rm charge} = \int dx (A_\rho\{[\nabla\theta_\alpha(x)]^2 + [\nabla\theta_\beta(x)]^2\} \\ +C\{[P_\alpha(x)]^2 + [P_\beta(x)]^2\} \\ +2B\nabla\theta_\alpha(x)\nabla\theta_\beta(x)), \end{aligned} \tag{3}$$

$$\begin{aligned} H_{\rm spin} = \int dx (A_\sigma\{[\nabla\phi_\alpha(x)]^2 + [\nabla\phi_\beta(x)]^2\} \\ +C\{[\Pi_\alpha(x)]^2 + [\Pi_\beta(x)]^2\}), \end{aligned} \tag{4}$$

where $\theta_\nu(x)$ $[\phi_\nu(x)]$ is the charge (spin) phase of electrons in channel ν $(= \alpha, \beta)$, $P_\nu(x)$ $[\Pi_\nu(x)]$ is the momentum conjugate to $\theta_\nu(x)$ $[\phi_\nu(x)]$.

The coefficients are given by $A_\rho = (v_F/4\pi)(1 + 3g)$, $A_\sigma = (v_F/4\pi)(1 - g)$, $B = v_F g_{\alpha\beta}/\pi$, $C = \pi v_F(1 - g)$, where g $(g_{\alpha\beta})$ is the intrachannel (interchannel) coupling constant of the forward-scattering processes and v_F is the Fermi velocity. Here we have neglected the backward-scattering and umklapp-scattering processes, which have large momentum transfers.

The interchannel interaction enters as a charge-charge coupling $\propto v_F g_{\alpha\beta}\nabla\theta_\alpha(x)\nabla\theta_\beta(x)$. The charge-density operator in the Tomonaga-Luttinger liquid is originally expressed as

$$N_\nu(x) = \frac{\nabla\theta_\nu(x)}{\pi} + \frac{2}{\pi\Lambda}\cos[2k_F x + \theta_\nu(x)]\cos[\phi_\nu(x)]. \tag{5}$$

When the spatial separation of the two systems is of the order of less than $1/k_F$, the coupling ($\sim \cos[\theta_\alpha(x) - \theta_\beta(x')]$) of the $2k_F$ CDW fluctuations will couple the phases of the two channels.[6,10] By contrast, we assume here the opposite case of the separation larger than $1/k_F$, so that we are left with the $\nabla\theta_\alpha(x)\nabla\theta_\beta(x)$ coupling. In other words, we neglect the interchannel backward scattering. We shall return to this point later.

We can then diagonalize the charge part of the Hamiltonian $H_{\rm charge}$ as is done for the electron-hole system in a study of the excitonic phase by Nagaosa and Ogawa for a two-channel Tomonaga-Luttinger model,[10] by a linear transformation,

$$\theta_\pm = \frac{1}{\sqrt{2}}[\theta_\alpha(x) \pm \theta_\beta(x)]. \tag{6}$$

These charge modes are gapless with linear dispersions, in which the velocities are given by

$$v_\pm = v_F\sqrt{1+3g \pm 4g_{\alpha\beta}}. \tag{7}$$

Now we turn to the calculation of the conductivity. We assume that electrodes are attached to one of the two chains to measure the conductivity. The impurity-scattering part of the Hamiltonian $H_{\rm imp}$ is defined, for each of the two channels, by

$$H_{\rm imp} = \sum_{\nu=\alpha,\beta}\sum_l \int dx\, N_\nu(x)u(x-x_l^\nu), \tag{8}$$

where $u(x-x_l^\nu)$ is the potential of the impurity at x_l^ν in channel ν.

Following Götze and Wölfle for the Mori formalism for the conductivity,[11] we can calculate the relaxation time τ in the second order in $H_{\rm imp}$. Since we assume no interchannel hybridization we can calculate τ for each channel, where τ is the same for the two equivalent channels.

In the Tomonaga-Luttinger model, the conductivity $\sigma = n_e e^2\tau/m^*$, with m^* being the effective mass and n_i (n_e) the density of impurities (electrons), is given in terms of the density-density correlation function N as

$$\sigma = \sigma_0/F(T), \tag{9}$$

$$F(T) \equiv \frac{1}{4\pi v_F^2}\lim_{\omega\to 0}\sum_q \frac{{\rm Im}N(2k_F+q,\omega)}{\omega}. \tag{10}$$

Here $\sigma_0 = n_e e^2\tau_0/m^*$ is the conductivity of a noninteracting system with $\tau_0 = v_F/(n_i u^2)$ with $u \equiv u(zk_F)$.

The density-density correlation function is defined by

$$N(q,\omega) = i\int_0^\infty dt\, e^{i\omega t}\int dx\, e^{iqx}\langle[\rho(x,t),\rho(0,0)]\rangle, \tag{11}$$

where $\langle\,\rangle$ represents the thermal average, and the density for a two-channel system is expressed as

$$\begin{aligned}\rho(x) &= \rho_\alpha(x)+\rho_\beta(x),\\ \rho_\nu(x) &= \sum_\sigma[\psi^\dagger_{\nu 1\sigma}(x)\psi_{\nu 2\sigma}(x) + {\rm H.c.}].\end{aligned} \tag{12}$$

Here $\psi_{\nu i\sigma}$ is the annihilation operators of right-going ($i=1$) or left-going ($i=2$) electrons, with spin σ in channel ν. In general we obtain

$$\lim_{\omega\to 0}\sum_q \frac{{\rm Im}N(2k_F+q,\omega)}{\omega} = \frac{1}{2T}\int_{-\infty}^\infty dt\langle\rho(0,t)\rho(0,0)\rangle. \tag{13}$$

Although $\rho(x) = \rho_\alpha(x) + \rho_\beta(x)$, we can readily show that the cross term of the correlation function, $\langle\rho_\alpha(0,t)\rho_\beta(0,0)\rangle$, vanishes. This implies that, when we attach the electrodes to both channels, the total current will simply be twice the value for the case in which the electrode is only attached to one channel.

As for the diagonal terms,

$$\begin{aligned}\langle\rho_\alpha(0,t)\rho_\alpha(0,0)\rangle &\propto \langle e^{-i\theta_\alpha(t)}e^{i\theta_\alpha(0)}\rangle_{H_{\rm charge}}\\ &= e^{-\langle[\theta_+(t)-\theta_+(0)]^2\rangle/4}e^{-\langle[\theta_-(t)-\theta_-(0)]^2\rangle/4}\\ &\approx \exp\left\{-\frac{1}{2\pi}\left[\sqrt{C/(A_\rho+B)}+\sqrt{C/(A_\rho-B)}\right]\int_0^\infty d\omega\, e^{-\omega/\omega_F}\frac{1-\cos(\omega t)}{\omega}[n(\omega)+\tfrac{1}{2}]\right\},\end{aligned} \tag{14}$$

where $n(\omega)$ is the Bose distribution function and $\omega_F = v_F/\Lambda$ (Λ is the real space cutoff) is the cutoff frequency of the order of the Fermi energy. As evident from this, both θ_+ and θ_- are relevant.

Finally the quantity F (proportional to the resistivity) becomes

$$\begin{aligned}F(T) = \frac{\omega_F^2}{\pi\sqrt{\pi/3}T}\int_0^\infty dt\\ &\times\exp\left[-[2+G(g,|g_{\alpha\beta}|)]\int_0^\infty d\omega\frac{1-\cos(\omega t)}{\omega}\right.\\ &\left.\times[n(\omega)+\tfrac{1}{2}]e^{-\omega/\omega_F}\right]\\ &\times\cos\{\tfrac{1}{2}[2+G(g,|g_{\alpha\beta}|)]\tan^{-1}(\omega_F t)\},\end{aligned} \tag{15}$$

with the effective coupling

$$\begin{aligned}G(g,|g_{\alpha\beta}|) &= \sqrt{(1-g)/(1+3g+4g_{\alpha\beta})}\\ &\quad+\sqrt{(1-g)/(1+3g-4g_{\alpha\beta})}\end{aligned} \tag{16}$$

$$\approx 2\sqrt{(1-g)/(1+3g)}\left[1+2\left(\frac{g_{\alpha\beta}}{1+3g}\right)^2\right] \quad (|g_{\alpha\beta}|\ll 1). \tag{17}$$

The above equation is the key result of this paper. The conductivity is determined by $G(g,|g_{\alpha\beta}|) \propto (1/v_+ + 1/v_-)$, which is independent of the sign of $g_{\alpha\beta}$, with the dependence

$$\frac{\partial G(g,|g_{\alpha\beta}|)}{\partial g} < 0, \qquad (18)$$
$$\frac{\partial G(g,|g_{\alpha\beta}|)}{\partial |g_{\alpha\beta}|} > 0.$$

Surprisingly, the conductivity turns out to be more *enhanced* for increasing interchannel repulsion $g_{\alpha\beta}$, while the intrachannel interaction g reduces the conductivity as expected from the single-channel ($g_{\alpha\beta} = 0$) result for the spinless fermions.[8] The temperature dependence of σ/σ_0 numerically calculated in the absence of the interchannel interaction for various values of the intrachannel interaction g (Fig. 1) confirms that the conductivity for $T \to 0$ converges to zero for $g > 0$, converges to a finite constant for $g = 0$, and diverges for $g < 0$.

In the presence of the interchannel interaction $g_{\alpha\beta}$ with a fixed g in Fig. 2, σ/σ_0 is indeed seen to increase with $g_{\alpha\beta}$, in which the effect is more pronounced for lower temperatures. This result might naively seem to be an effect of the attraction due to the interchannel repulsion, i.e., in coupled channels, one can evoke an effective attraction, $-\delta g$, in the channel α as arising from a bubble diagram (in the lowest order) of channel β. However, we wish to stress the following point: if we look at Eq. (16), G cannot be given as a single-channel formula with a *shifted* value of $g \to g - \delta g$. This implies that the enhancement of the conductivity is intrinsically a *two-channel effect.*

In fact, the physics involved in the enhanced conductivity may be understood as a result of the quantum-mechanical suppression of CDW fluctuations: we can show by looking at the correlation functions in the two-band g-ology that the CDW fluctuation is suppressed when the interband interaction is turned on. This will weaken the CDW pinning due to impurities, resulting in the enhanced conductivity.

An interesting point to be noted is the following. There is an anticorrelation that superconductivity (SC) is enhanced when diagonal orders are suppressed, which is well known in single-channel systems and approximately holds in two-channel 1D systems as well.[10] In this sense, the enhanced conduction appears to occur concomitantly with an approach to an SC regime, even in two-channel systems. The SC regime in purely repulsive (two-band) systems might sound peculiar, but two of the present authors have shown that a parameter regime where SC dominates does indeed exist for repulsive intraband and interband interactions in the two-band system, although a different situation with inequivalent chains is considered there.[12] Thus the normal-state transport as unraveled here complements the study of superconductivity in such systems.

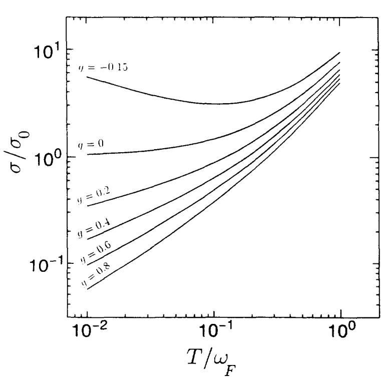

FIG. 1. The result for the conductivity σ/σ_0 for the single channel with intrachannel interaction $g = -0.15$, 0, 0.2, 0.4, 0.6, 0.8 with vanishing interchannel interaction $g_{\alpha\beta} = 0$.

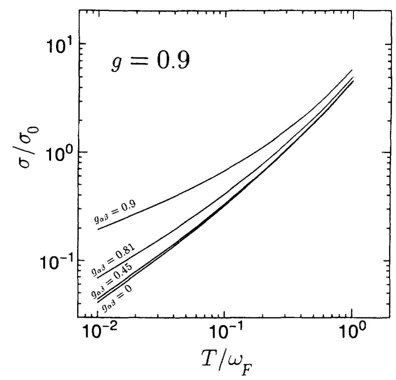

FIG. 2. The result of the conductivity σ/σ_0 for the two interacting channel systems for various interchannel interaction $g_{\alpha\beta} = 0.9$, 0.81, 0.45, 0 from the top for a fixed intrachannel interaction $g = 0.9$.

As mentioned in Ref. 9 the typical values of the parameters in the case of GaAs quantum wires are $n \sim 2 \times 10^{11}/\mathrm{cm}^2$, $m^* = 0.067 m_0$, and the single-channel corresponds to the value of the width of the wire $W \sim 400$ Å. These values correspond to the cutoff energy $\omega_F \sim 100$ K. Then the effect of the interchannel interaction on the conductivity may be observable around the usual experimental condition of T approximately a few K. A possible experimental situation to observe the effect would be two quantum wires, either laterally or vertically coupled with a short separation.

An interesting possibility, then, is to have a current-carrying wire on top of another wire, where the second wire is controlled by either doping or a back-gate electrode. Then the conductivity of the first wire should be modified when the electron density in the second wire is changed. To be more realistic the effect of hybridization in coupled quantum wires will have to be studied.

We can also envisage a completely different phenomenon when the spatial separation d of the two systems is varied across the value $1/k_F$ (~ 80 Å in quantum wires). The coupling of the $2k_F$ CDW fluctuations will lock the phases of the two channels (out of phase for the repulsive interchannel coupling) for $d < 1/k_F$, thereby *reducing* the conductivity due to the CDW pinning. In the coupling, $J \cos[\theta_\alpha(x) - \theta_\beta(x')]$, we can put $x \sim x'$ when $\nabla\theta/\theta < k_F$ and the coupling constant is $J \sim K_0(2k_F d)$ when the Coulomb coupling is integrated over $q_\perp$ with K_0, modified Bessel's function, being a rapidly decreasing function.[6] We can thus expect a crossover from the suppressed conduction for $d < 1/k_F$ down to the enhanced conduction for $d > 1/k_F$. We can alternatively make the k_F's in the two channels different by tuning the carrier densities, where the $2k_F$ coupling will vanish.

It is also an interesting future problem if the effect found here persists in two dimensions. The present formalism for the two-channel system can be extended to multichannel cases in a straightforward manner by introducing higher-dimensional matrices. To investigate the way in which the system crosses over to two dimensions, we will again have to incorporate the effect of hybridization.

We are much indebted to Professor H. Fukuyama and Dr. H. Kohno for illuminating discussions. Valuable comments from Professor A. Shimizu and Professor F. Komori are also appreciated.

[1] See, e.g., *Mesoscopic Phenomena in Solid*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (North-Holland, Amsterdam, 1991); *Transport Phenomena in Mesoscopic Systems*, edited by H. Fukuyama and T. Ando (Springer Verlag, Berlin, 1991).

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