

## Comments

*Comments are short papers which criticize or correct papers of other authors previously published in the Physical Review. Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

### Comment on “Electroconductance oscillations and quantum interference in ballistic nanostructures”

Zhen-Li Ji and Karl-Fredrik Berggren

*Department of Physics and Measurement Technology, Linköping University, S-581 83 Linköping, Sweden*

(Received 23 August 1993; revised manuscript received 5 October 1993)

We present quantum-mechanical calculations of conductance oscillations produced by modulating an applied-potential step  $V_3$  in one of the branches of a multiply connected system. We show that these oscillations arise from partial reflections of the wave function at the sharp steps defining the different potential regions. Quantum interference between the different branches is shown to be negligible in spite of a great deal of coherence present in ballistic multiply connected systems.

In a recent paper,<sup>1</sup> Joe and Ulloa have reported calculations of conductance oscillations in quantum ballistic narrow channels produced by modulating an applied transverse potential along one of the branches of a multiply connected region. They suggested, “These aperiodic oscillations arise as a quantum-interference effect between phase-shifted branches of the wave function in the structure, similar to the lag effects described by Boyer.”<sup>2</sup> However, we show in this Comment that conductance oscillations as a function of an applied transverse potential in one of the branches of a multiply connected system are due to partial reflections of the wave function at the sharp steps defining the different potential regions, and interference effects between branches of a multiply connected region are negligible. These resonances occur when an integral multiple of half the Fermi wavelength coincides with the length of the potential barrier.<sup>3</sup>

Figure 1 is a schematic drawing of the system which is similar to that described in Ref. 1. Two two-dimensional (2D) semi-infinite electron gases are connected by a con-

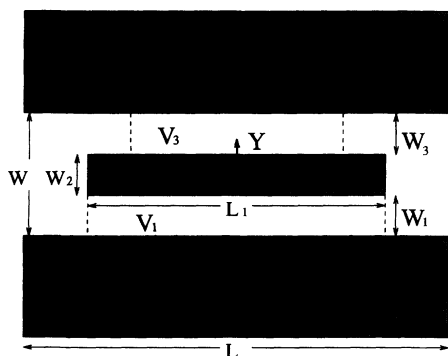


FIG. 1. Two 2D semi-infinite electron gases connected by a constriction. A rectangular infinite-repulsive potential obstacle is deposited in the center of the constriction. The potential  $V_i$  inside the branch of the structure can be varied independently, for  $i=1$  and 3. Shaded areas are forbidden for electrons.

striction. A rectangular infinite-repulsive potential obstacle is deposited in the center of the constriction, so that incident electron waves are forced to pass through two branches before they recombine at the other end of the channel. The potential inside each of the uniform sections of the constriction is constant. A potential barrier with uniform height  $V_3$  is placed in the top branch. In our model, the potential  $V_i$  inside the branch of the structure can be varied independently, for  $i=1$  and 3 as shown in Fig. 1, allowing us to study diverse physical situations. For simplicity, the potential in the shaded areas in Fig. 1 is infinite. We solve the Schrödinger equation in every uniform section shown in Fig. 1 by an eigenfunction expansion. Matching the wave function and its normal derivatives at the boundaries between the different sections, a system of linear equations for the expansion coefficients can be found. We solve this system numerically, and use the solution to calculate the conductance at zero temperature. Details of this procedure are found in Refs. 3 and 4.

Our numerical results are shown in Fig. 2 at a fixed Fermi energy  $E_F=9.4$  meV, as a function of potential barrier  $V_3$ . In Fig. 2(a) we have plotted conductance  $G$  versus potential barrier  $V_3$  along the top branch of the structure. Conductance oscillations induced by a changing potential  $V_3$  are clearly seen. Oscillations become sharper for larger  $L_T$  values, and the variation of the channel length  $L_1$  for a fixed  $L_T$  has no noticeable effect on these oscillations, even if  $L_1=L_T$  (not shown here). These results in Fig. 2(a) are in qualitative agreement with the work of Joe and Ulloa.<sup>1</sup> However, they suggested that these oscillations are produced by quantum interference between voltage-shifted states in the different branches. In the following, we will show in Figs. 2(b) and 2(c) that interference effects between the different branches of the structure are negligible, and the oscillatory structure is due to the reflection at the edges of the potential barrier  $V_3$  in the top branch of the constriction.

To show the interference effects on conductance  $G$  of the constriction, we have calculated  $G_1$  and  $G_3$ , where  $G_i$  is the conductance of the  $i$ th branch when the other is completely closed in the constriction. The differences between  $G$  and  $G_1 + G_3$  are shown in Fig. 2(b), where  $\Delta G = G - (G_1 + G_3)$ . Notice that the values of  $\Delta G$  are so small that interference effects between two branches of the constriction are negligible, i.e.,  $G \approx G_1 + G_3$ . These are in overall agreement with the results of the earlier studies.<sup>5,6</sup> Thus Fig. 2(b) demonstrates that conductance oscillations are not due to quantum interference between two branches of the constriction.

We argue that conductance oscillations produced by modulating a potential barrier  $V_3$  along the top branch of the structure in Fig. 2(a) are due to the longitudinal resonances arising from partial reflections of the wave function at the edges of the potential barrier  $V_3$ .<sup>3</sup> Resonant transmission occurs if the barrier length  $L_T$  is approximately an integer multiple of half the longitudinal wavelength  $\lambda = h[2m(E_F - E_n - V_3)]^{-1/2}$ , leading to oscillations on the conductance plateaus. Here,  $E_n + V_3$  is the subband energy in the top branch of the structure closest to  $E_F$  from below. The resonances are more numerous for a longer barrier length  $L_T$  as the resonance condition can be satisfied more times. The ratio between the barrier length  $L_T$  and the longitudinal wavelength  $\lambda$  of the top branch of the structure can then be written as

$$\frac{L_T}{\lambda} = L_T [2m(E_F - E_n - V_3)/h^2]^{1/2}. \quad (1)$$

In Fig. 2(c) we display the comparison between the predicted ratio changes as a function of  $V_3$  in Eq. (1) (solid line) and the calculated peak positions in Fig. 2(a) (dots), indicating the resonance condition in the top branch of the structure. Notice that the solid curve fits the dots very well in Fig. 2(c). This good agreement demonstrates that conductance oscillations produced by modulating a potential  $V_3$  along the top branch of the structure are due to longitudinal resonances arising from partial reflections of the wave function at the edges of the potential barrier  $V_3$ .

In conclusion, we have shown that quantum interference between two branches of the present model makes a negligible contribution to the conductance and that conductance oscillations produced by modulating a potential barrier  $V_3$  are due to reflections at the sharp steps defining the different potential regions in the top branch of the structure. Finite temperature broadening of the Fermi surface would tend to smooth out the fine structure shown in Fig. 2(a), making it disappear when  $kT$

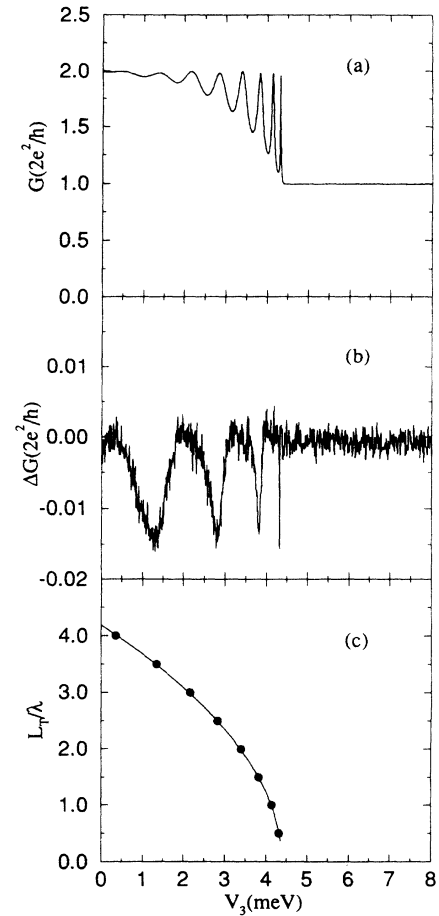


FIG. 2.  $E_F = 9.4$  meV,  $L_T = 300$  nm,  $L_1 = 1.2L_T$ ,  $L = 1.4L_T$ ,  $W = 100$  nm, and  $W_i = W/3$ ,  $i = 1, 2$ , and 3 for all curves. (a) Conductance  $G$  vs  $V_3$  for  $V_1 = 0$ . (b) The difference between  $G$  and  $G_1 + G_3$  as a function of  $V_3$ , where  $\Delta G = G - (G_1 + G_3)$ . (c) Predicted [solid line, Eq. (1)] and calculated [dots, (a)] ratio between the barrier length  $L_T$  and the longitudinal wavelength  $\lambda$  of the top branch of the structure vs  $V_3$ .

exceeds the separation of the resonances. Smooth barrier edges tends to reduce the reflections at the edges of the potential barrier, deteriorating the resonance condition.<sup>6</sup> A rounding of the corners has the same effect.<sup>7</sup> Therefore, observation of these oscillations in conductance is difficult, due to the potential barrier generated by gate bridges will not be a perfect step function.

We gratefully acknowledge the support of the Swedish Engineering Research Council and Swedish Natural Science Research Council.

<sup>1</sup>Y. S. Joe and S. E. Ulloa, Phys. Rev. B **47**, 9948 (1993).

<sup>2</sup>T. Boyer, Phys. Rev. Lett. **54**, 2469 (1985).

<sup>3</sup>G. Kirczenow, Phys. Rev. B **39**, 10452 (1989).

<sup>4</sup>K.-F. Berggren and Zhen-Li Ji, Phys. Rev. B **43**, 4760 (1991).

<sup>5</sup>Y. Avishai, M. Kaveh, S. Shatz, and Y. B. Band, J. Phys. Con-

dens. Matter **1**, 6907 (1989); Zhen-Li Ji and K.-F. Berggren, Semicond. Sci. Technol. **6**, 63 (1991).

<sup>6</sup>E. Castaño and G. Kirczenow, Phys. Rev. B **41**, 5055 (1990).

<sup>7</sup>Zhen-Li Ji, Semicond. Sci. Technol. **8**, 1561 (1993).

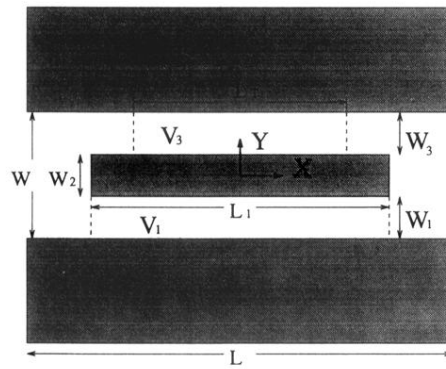


FIG. 1. Two 2D semi-infinite electron gases connected by a constriction. A rectangular infinite-repulsive potential obstacle is deposited in the center of the constriction. The potential  $V_i$  inside the branch of the structure can be varied independently, for  $i = 1$  and 3. Shaded areas are forbidden for electrons.