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Localization of charged quantum particles in a static random magnetic field

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We consider a charged quantum particle in a random magnetic field with Gaussian, δ-correlated statistics. We show that although the single-particle properties are peculiar, two-particle quantities such as the diffusion constant can be calculated in perturbation theory. We map the problem onto a nonlinear σ model for Q matrices of unitary symmetry with renormalized diffusion coefficient for which all states are known to be localized in $d = 2$ dimensions. Our results compare well with recent numerical data.

The problem of a charged quantum particle moving in a static random magnetic field in two dimensions has received renewed interest recently.[1–5] For one, the problem may be considered as a limiting case of a system of particles interacting via a gauge field. Models of this type have been proposed to describe a state with charge-spin separation of the conduction electrons in high-T_c superconductors.[6,8] Second, an experimental realization of a random magnetic field due to the pinned vortex lines of a superconducting layer on top of a semiconductor heterostructure has been reported recently.[7] Third, the problem is thought to be relevant for the quantum Hall effect in the limit of the half-filled Landau level.[9,10]

A number of numerical investigations have been performed, with conflicting results. In Ref. 9 it was argued on the basis of results of numerical diagonalization on square lattices of up to 10^4 sites for zero-average random flux per plaquette and, in addition, site-diagonal disorder and a uniform magnetic field that localization can be suppressed by the random flux. In Ref. 5 the conductance of a square lattice of quantum wires subject to a random magnetic flux per plaquette, distributed uniformly between $-\phi_0/2$ and $\phi_0/2$ was calculated numerically (ϕ_0 is the flux quantum). Although no definite conclusion could be drawn, the results were found to be consistent with the existence of extended states and a mobility edge. In contrast, the results of applying the finite size scaling method of MacKinnon and Kramer to the random magnetic field problem reported in Ref. 4 suggested that all states are localized by a random magnetic field. Since the localization length for a two-dimensional disordered system may be very large, it is obvious that numerical studies of systems of finite size are of limited value in deciding the principal question whether there exist extended states in these systems.

In this paper we show that the problem of charged particles in a static random magnetic field can be mapped onto a nonlinear σ model of unitary matrices. The latter model has been proposed for disordered systems containing random spin scattering centers as well as models featuring random phase fluctuations of the hopping matrix elements of a tight-binding Hamiltonian.[12,13] Perturbation theory for this model yields a divergent quantum correction to the conductance in two-loop order.[14] As a consequence the scaling function in two dimensions remains negative, leading to the result that all states are localized for these models.

We study the transport properties of a charged spinless quantum particle (mass m, charge e) in two dimensions in a static random magnetic field $\vec{H}(\vec{r}) = \vec{\nabla} \times \vec{A}(\vec{r})$ normal to the plane, as defined by the Hamiltonian

$$H = \frac{p^2}{2m} - \frac{e}{mc}\vec{p}\cdot\vec{A} + \frac{e^2}{2mc^2}\vec{A}^{\,2}, \quad (1)$$

where $\vec{p} = -i\vec{\nabla}$ is the momentum operator and the Coulomb gauge ($\vec{\nabla}\cdot\vec{A} = 0$) has been used ($\hbar = 1$). The magnetic field is assumed to be Gaussian distributed and δ-correlated, with vanishing mean and variance in Fourier space

$$\langle H(\vec{q})H(-\vec{q})\rangle = \langle h^2\rangle. \quad (2)$$

Accordingly, the variance of the vector potential is $\langle A_\alpha(\vec{q})A_\beta(-\vec{q})\rangle = (1/q^2)\langle h^2\rangle\ \delta^T_{\alpha\beta}(\hat{q})$ where $\delta^T_{\alpha\beta}(\hat{q}) = \delta_{\alpha\beta} - \hat{q}_\alpha\hat{q}_\beta$ accounts for the transverse character of the field ($\hat{q} = \vec{q}/\mid\vec{q}\mid$). By contrast, an independent random distribution of phases of the hopping matrix elements corresponds to a δ-correlated distribution of vector potentials given by $\langle A_\alpha(\vec{q})A_\beta(-\vec{q})\rangle = \langle a^2\rangle\delta^T_{\alpha\beta}(\hat{q})$. We will comment on this case later.

Assuming the fluctuation strength of the magnetic field, $\langle h^2\rangle$, to be weak, we first consider perturbation theory. The standard Feynman diagram language for impurity scattering may be employed, with the impurity line describing scattering of a particle from momentum state $\mid\vec{p}+\vec{q}/2\rangle$ into state $\mid\vec{p}\,'+\vec{q}/2\rangle$ and a hole from $\mid\vec{p}-\vec{q}/2\rangle$ into $\mid\vec{p}\,'-\vec{q}/2\rangle$ being given by

$$w_{\vec{p}\vec{p}\,'}(q) = v_0^2k^{-2}\{(\vec{p}+\vec{p}\,')^2 - q^2 - [(\vec{p}+\vec{p}\,')\cdot\hat{k}]^2 + (\vec{q}\cdot\hat{k})^2\}\ , \quad (3)$$

where $\vec{k} = \vec{p} - \vec{p}\,'$ is the transferred momentum and $v_0^2 = e^2\langle h^2\rangle/4m^2c^2$ is a velocity squared characteristic of the strength of disorder. Note that $w_{\vec{p}\vec{p}\,'}$ is strongly singular in the forward direction ($\vec{p} = \vec{p}\,'$), due to the long range of the vector potential fluctuations, even though the magnetic field fluctuations are assumed to be short ranged. In lowest order the imaginary part of the (advanced) single-particle self-energy $\Sigma^A(\vec{p}, E)$ on the energy shell ($E = p^2/2m$) is given by

$$\mathrm{Im}\Sigma^A \equiv \frac{1}{2\tau} = \pi \int (dp\,')w_{pp\,'}(0)\delta\left(\frac{p^2}{2m} - \frac{p\,'^2}{2m}\right) = \pi N_0 v_0^2 \int_0^{2\pi} \frac{d\phi}{2\pi} \cot^2\frac{\phi}{2} \; . \quad (4)$$

Here, $(dp) = d^2p/(2\pi)^2$, $\hat{p}\cdot\hat{p}\,' = \cos\phi$, and $N_0 = \frac{m}{2\pi}$ is the density of states. The ϕ integral in (4) is strongly divergent at $\phi = 0$, which may be traced to the contribution of vector potential fluctuations in the limit $q \to 0$. A self-consistent treatment of the divergence leads to a much weaker dependence as discussed below after (23). Nonetheless, the single-particle relaxation rate $1/2\tau$ can be expected to diverge, which might lead to the generation of a branch cut in the single-particle Green's function, as argued in Refs. 1 and 2 (see, however, Ref. 18). We will show below that the transport relaxation rate and the diffusion constant are nonsingular, in agreement with results of a naive perturbation theory.[2] This may be done by regularizing the infrared divergence in (4) in a convenient way. We will use a soft cutoff, replacing $\cot^2\frac{\phi}{2}$ in (4) by $w(\phi) = \cos^2\frac{\phi}{2}/(\sin^2\frac{\phi}{2} + \gamma^2)$, although the precise form of the cutoff is not important. At the end of the calculation we take the limit $\gamma \to 0$.

Next, let us consider the diffusion propagator Γ ("diffuson"), obtained by summing the particle-hole ladder diagrams

$$\Gamma_{\vec{p}\vec{p}\,'}(\vec{q},\omega) = w_{\vec{p}\vec{p}\,'}(\vec{q}) + \int (dp'')w_{\vec{p}\vec{p}\,''}(\vec{q})G^R_{\vec{p}\,''_+}(E_+) \times G^A_{\vec{p}\,''_-}(E_-)\Gamma_{\vec{p}\,''\vec{p}\,'}(\vec{q},\omega) \; , \quad (5)$$

where

$$G^{R,A}_{\vec{p}}(E) = \left[E - \frac{p^2}{2m} \pm \frac{i}{2\tau}\right]^{-1} \quad (6)$$

and $\vec{p}\,''_\pm = \vec{p}\,'' \pm \vec{q}/2$, $E_\pm = E \pm \omega/2$. The solution of (5) may be easily obtained in terms of the eigenfunction of the operator $w_{\vec{p}\vec{p}\,'}(q = 0)$ (see Ref. 15). In two dimensions and for $|\vec{p}| = |\vec{p}\,'|$ one has

$$w_{\vec{p}\vec{p}\,'}(\vec{q} = 0) = \sum_n w_n \chi_n^*(\hat{p})\chi_n(\hat{p}\,') \quad (7)$$

with

$$w_n = v_0^2 \int_0^{2\pi} \frac{d\phi}{2\pi} w(\phi) e^{-in\phi} \quad (8)$$

and

$$\chi_n(\hat{p}) = e^{in\phi} \; . \quad (9)$$

Here, ϕ is the polar angle of $\hat{p}$. The leading contribution to $\Gamma_{\vec{p}\vec{p}\,'}$ is obtained as

$$\Gamma_{\vec{p}\vec{p}\,'}(q,\omega) = \frac{1}{-i\omega + Dq^2}\frac{1}{2\pi N_0 \tau^2} \; , \quad (10)$$

where the diffusion coefficient D is given by

$$D = \frac{1}{2}v^2\tau\left(1 + \frac{w_1}{w_0 - w_1}\right) = \frac{1}{2}v^2\tau_{\mathrm{tr}} \quad (11)$$

and $v = p/m$. Here we have defined the transport relaxation time τ_{tr} as

$$\frac{1}{\tau_{\mathrm{tr}}} = mv_0^2 \int \frac{d\phi}{2\pi}\cot^2\frac{\phi}{2}(1 - \cos\phi) = mv_0^2 \; . \quad (12)$$

In contrast to τ, the transport time τ_{tr} is finite in the limit $\gamma \to 0$.

In a time reversal invariant system, the coherent backscattering described by a diffusion pole in the particle-particle ladder diagrams, the so-called Cooperon, plays a dominant role. The Cooperon $C_{\vec{p}\vec{p}\,'}(\vec{q},\omega)$ obeys the integral equation (5), with $w_{\vec{p}\vec{p}\,'}(\vec{q})$ replaced by $w_{\vec{k}\vec{k}\,'}(\vec{Q})$, where $\vec{k} = \frac{1}{2}(\vec{p} - \vec{p}\,' + \vec{q})$, $\vec{k}\,' = \frac{1}{2}(\vec{p}\,' - \vec{p} + \vec{q})$, and $\vec{Q} = \vec{p} + \vec{p}\,'$. The fact that the vector potential $\vec{A}$ couples to the particle momentum $\vec{p}$ [see (1)], which changes the sign under time reversal, leads to the relation $w_{\vec{k}\vec{k}\,'}(\vec{Q}) = -w_{\vec{p}\vec{p}\,'}(\vec{q})$. Correspondingly, the Cooperon is finite in the limit $\vec{q},\omega \to 0$ and cannot play any role in bringing about localization in the present case.

In the following, we map the problem onto a nonlinear σ model of unitary symmetry. The generating functional for two-particle Green's function of the retarded-advanced type may be represented in terms of a functional integral over a supersymmetric field $\psi = (\varphi_1, \chi_1, \varphi_2, \chi_2)$, where $\varphi_{1,2}$ are bosonic and $\chi_{1,2}$ are fermionic components as[11]

$$Z = \int D[\psi]\exp -(S_0 + S_1), \quad (13)$$

where

$$S_0 = -i\int d^2r\left\{\bar{\psi}\Lambda\left(E + \frac{1}{2m}\nabla^2\right)\psi - i\frac{\omega}{2}(\bar{\psi}\psi)\right\} \quad (14)$$

and

$$S_1 = -i\frac{e}{mc}\int d^2r\{\bar{\psi}\Lambda(-i\vec{A}\cdot\vec{\nabla})\psi\}. \quad (15)$$

The 4×4 matrix Λ is diagonal, $\Lambda = \mathrm{diag}(1,1,-1,-1)$.

Averaging over the vector potential one finds that S_1 in (15) has to be replaced by

$$S_1^{\mathrm{eff}} = -i\sum_{\substack{\vec{k},\vec{k}\,' \\ q<q_0}} \Lambda_\alpha \bar{\psi}^\alpha_{\vec{k}+\vec{q}}\psi^\beta_{\vec{k}} w_{\vec{k}\vec{k}\,'}\bar{\psi}^\beta_{\vec{k}\,'-\vec{q}}\psi^\alpha_{\vec{k}\,'}\Lambda_\beta \; , \quad (16)$$

where $w_{\vec{k},\vec{k}\,'}$ is defined in (3), and only long-wavelength fluctuations are considered ($q < q_0$). It is useful to introduce the representation of $w_{\vec{k},\vec{k}\,'}$ in terms of eigenfunctions (7), and to define "density" fields[15]

$$\rho^{\alpha\beta}_{n,\vec{q}} = \sum_{\vec{k}} \Lambda_\alpha \bar{\psi}^\alpha_{\vec{k}+\vec{q}} \psi^\beta_{\vec{k}} \chi_n(\hat{k}) \ , \quad (17)$$

in terms of which $S_1^{\rm eff}$ can be written as

$$S_1^{\rm eff} = -i \int d^2r \sum_n w_n \bar{\rho}^{\alpha\beta}_n(\vec{r}) \rho^{\beta\alpha}_n(\vec{r}). \quad (18)$$

As usual, the interaction term may be decoupled with the aid of Hubbard-Stratonovich fields $Q_n(r)$, which are (4×4) supermatrices of unitary symmetry. The functional integration over the primary fields may be performed, yielding

$$Z = \int D[Q] \exp -\tilde{S}\{Q\} \ , \quad (19)$$

where the effective action of the Q fields is at first given by

$$\tilde{S} = \int d^2r \Big\{ {\rm Str} \ \ln \ G^{-1} - \frac{1}{2} \sum_n w_n \ {\rm Str} \ Q_n^2 \Big\}, \quad (20)$$

where Str denotes the supertrace and

$$G_p^{-1} = \hat{\epsilon} - \frac{p^2}{2m} + i \sum_n w_n \chi_n(\hat{p}) Q_n \Lambda \quad (21)$$

and $\hat{\epsilon} = {\rm diag} \ (E + \frac{\omega}{2}, E + \frac{\omega}{2}, E - \frac{\omega}{2}, E - \frac{\omega}{2})$. The saddle point of $\exp(-\tilde{S})$ is at $Q_n = Q_n^{(0)}$, where $Q_n^{(0)}$ is

$$\begin{aligned} Q_n^{(0)} &= i \int (dp) \chi_n(\hat{p}) G(p) \Lambda \\ &= i\delta_{n0} \int (dp) \Big[\hat{\epsilon} - \frac{p^2}{2m} + i w_0 Q_0^{(0)} \Lambda\Big]^{-1} \Lambda \ , \end{aligned} \quad (22)$$

so that the Green's function at the saddle point is given by

$$G(p) = \Big[\hat{\epsilon} - \frac{p^2}{2m} + \frac{i}{2\tau}\Lambda\Big]^{-1} \ . \quad (23)$$

Equations (22) and (23) are statements of the self-consistent Born approximation for the single-particle relaxation rate $\frac{1}{\tau}$. In contrast to the lowest order expression (4) for $\frac{1}{\tau}$, which scales with the cutoff γ as $\tau^{-1} \propto \gamma^{-1}$, the self-consistent value is given by $\tau^{-1} = [\frac{4}{\pi} p^2 v_0^2 \ \ln \ (1/E\tau\gamma)]^{1/2}$ and, hence, shows a weaker divergence as $\gamma \to 0$.[18]

We now expand the action around the saddle point:

$$\begin{aligned} \tilde{S} = \tilde{S}_0 + \frac{1}{2} \int (dq) \Big\{ &\int (dp) \sum_{n,m} w_n \chi_n(\hat{p}) w_m \chi_m(\vec{p} + \vec{q}) \\ &\times {\rm Str}[G(\vec{p}) \Lambda \delta Q_n(\vec{q}) G(\vec{p} + \vec{q}) \Lambda \delta Q_m(-\vec{q})] \\ &- \sum_n w_n \ {\rm Str}[\delta Q_n(q) \delta Q_n(-q)\Big\}. \end{aligned} \quad (24)$$

The p integral is only finite for the products $G^R G^A$, which are generated by the off-diagonal components of δQ, denoted $\delta\tilde{Q}$. In the limit of small $\vec{q}, \omega$, using

$$\begin{aligned} &\int (dp) \chi_n(\hat{p}) \chi_m(\hat{p}) G^A\Big(E - \frac{\omega}{2}, \vec{p}\Big) G^R\Big(E + \frac{\omega}{2}, \vec{p} + \vec{q}\Big) \\ &= 2\pi N_0 \tau \Big[\delta_{nm} + i\tau\Big(\omega \delta_{nm} - \frac{pq}{m} b^{(1)}_{nm}\Big) - \Big(\frac{pq\tau}{m}\Big)^2 b^{(2)}_{nm}\Big], \end{aligned} \quad (25)$$

where $b^{(\ell)}_{nm} = \langle \chi_n \chi_m \cos^\ell \phi \rangle$, $\ell = 1, 2$, we find

$$\begin{aligned} \tilde{S} = \tilde{S}_0 - \frac{1}{2} \int (dq) \Big\{ &\sum_n w_n \Big(1 - \frac{w_n}{w_0}\Big) \\ &\times {\rm Str} \ [\delta\tilde{Q}_n(q) \delta\tilde{Q}_n(-q)] \\ &+ w_0 \tau (-i\omega + D_0 q^2) \ {\rm Str} \ [\delta\tilde{Q}_0(q) \delta\tilde{Q}_0(-q)] \\ &+ \frac{i}{2} w_1 \tau \Big(\frac{pq}{m}\Big) \sum_{n=\pm 1} {\rm Str} \ [\delta\tilde{Q}_0(q) \delta\tilde{Q}_n(-q)] \Big\} \ . \end{aligned} \quad (26)$$

As expected, the $n = 0$ mode is massless and describes interacting diffusons. The coefficient of the q^2 term is the bare diffusion constant, $D_0 = \frac{1}{2} v^2 \tau$, which tends to zero if the infrared cutoff is taken to zero. However, the bare diffusion constant gets dressed by the coupling to the massive $n = \pm 1$ modes. Indeed, integrating out δQ_1 and δQ_{-1} produces a renormalization term

$$\Delta\tilde{S} = -\frac{1}{2} \int (dq) D_0 \tau \frac{w_0 w_1}{w_0 - w_1} q^2 \ {\rm Str} \ [\delta\tilde{Q}_0(q) \delta\tilde{Q}_0(-q)] \ , \quad (27)$$

which combined with the bare diffusion term has the effect of changing the bare diffusion constant D_0 into the renormalized D as defined in (12). The final result for the effective action is

$$\tilde{S} = \tilde{S}_0 - \pi N_0 \int (dq) \{(Dq^2 - i\omega) \ {\rm Str} \ [\delta\tilde{Q}_0(q) \delta\tilde{Q}_0(-q)]\} \ . \quad (28)$$

We note in passing that the coefficients of terms with higher spatial derivatives of Q will be renormalized in a similar way and may be expected to be finite as well.

The expansion (28) of the action in terms of $\delta\tilde{Q}_0$ serves to determine the coefficients of the two terms in the nonlinear σ model obtained from (20) by keeping only the integration over the saddle point manifold:

$$S_\sigma = \frac{\pi N_0}{4} \int d^2r \{-D \ {\rm Str} \ (\vec{\nabla} Q \cdot \vec{\nabla} Q) - 2i\omega \ {\rm Str} \ (\Lambda Q)\} \ , \quad (29)$$

where the rescaled field $Q(\vec{r})$ is constrained by $Q^2(\vec{r}) = 1$.

We have, thus, shown that the problem of a charged quantum particle in a static random magnetic field is equivalent to a nonlinear σ model of interacting Q matrices with unitary symmetry. This model has been studied extensively.[12–14,16] It is known that the Gell-Mann-Low β function describing the scaling behavior of the dimensionless conductance g (in units of e^2/h) with the length of the sample is given in leading order for large g by

$$\frac{d\ln g}{d\ln L} = \beta(g) = -\frac{1}{2\pi^2 g^2} + O\left(\frac{1}{g^4}\right) . \tag{30}$$

It follows then that all states are localized, and that the localization length in the weak disorder regime is given by

$$\xi = \xi_0 \exp(\pi^2 g_0^2) , \tag{31}$$

where $g_0 = mv^2\tau_{tr}/2 = (v^2/4v_0^2)$ is the Drude conductance, and $\xi_0 \simeq v\tau_{tr}$. For the case of a δ-correlated distribution of vector potentials the single-particle relaxation rate $\frac{1}{2\tau}$ does not show an infrared divergence, and the nonlinear σ model may be derived in the usual way. The more complete derivation given here leads to a renormalization of the diffusion constant as $D = 2D_0 = (4\pi N_0 e^2\langle a^2\rangle/c^2)^{-1}$, where $\langle a^2\rangle$ was defined after (2).

Our results are in good agreement with the available numerical data.[4,5] The authors of these papers studied the lattice version of the problem with the maximum possible disorder corresponding to $v_0 \sim v$. Accordingly, the typical values of the "bare" conductance g_0 are of order of unity. However, when approaching the center of the band (i.e., when g_0 increases), the localization length was found to grow exponentially,[4] in agreement with (31). The finite size scaling analysis[5] yields for the localization length Λ of a quasi-one-dimensional strip of width M:

$$\Lambda(M) = M f(\xi/M) . \tag{32}$$

Calculating the scaling function $f(x)$ by using (31) and comparing with the known result for the localization length of a quasi-one-dimensional system,[16] we obtain:[17]

$$f(x) \simeq \begin{cases} \frac{2}{\pi}\sqrt{\ln\ x}, & x \gg 1 \\ x, & x \ll 1. \end{cases} \tag{33}$$

This agrees well with the asymptotic behavior of $f(x)$ as obtained by numerical means in Ref. 4 for both $x \gg 1$ and $x \ll 1$ (see Fig. 1). Scaling behavior of the conductance $g(L/\xi)$ obtained in Ref. 5 (see Fig. 4 of Ref. 5) is also compatible with the scaling law $g(L) \sim (1/\pi)\sqrt{\ln(\xi/L)}$ ($\xi \gg L$), which follows from Eqs.(30) and (31).

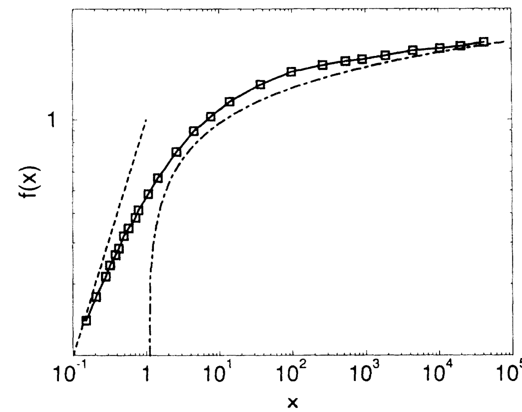

FIG. 1. The scaling function $f(x)$, Eq. (32), as obtained by numerical study of the problem in Ref. 4. The dashed and dot-dashed lines represent the asymptotical behavior for $x \ll 1$ and $x \gg 1$, respectively, given by Eq. (33).

To summarize, we have shown here that the nonlinear σ model description of disordered systems with broken time reversal invariance holds true even in the case of long-ranged fluctuations of the vector potential when the single-particle properties are dominated by infrared divergencies. This is true provided the magnetic field fluctuations are short ranged. In the opposite case of long-ranged magnetic field fluctuations it is conceivable that even the transport relaxation rate diverges, signaling a different physical regime. One may speculate that then the topological excitations governing the behavior in the quantum Hall effect, where the average magnetic field is finite and large, will play a role. However, for short-ranged magnetic field fluctuations the topological term is absent. Finally, we emphasize that our analysis is restricted to quenched random magnetic fields. To what extent this model applies to dynamical gauge field models remains to be seen.

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[17] We used the definition of the localization length adopted in Ref. 4, which differs by a factor 2 from the one used in Ref. 16.
[18] The question of the physically meaningful definition of the single-particle relaxation time will be discussed elsewhere [E.L. Altshuler, A.G. Aronov, A.D. Mirlin, and P. Wölfle, (unpublished)].