Quenching of resonant transmission through an oscillating quantum well

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The transmission probability of electrons traversing a quantum well subject to a harmonic driving force $V_1\cos(\omega t)$, matched between two static barriers, is shown to exhibit a rich spectrum of sidebands in addition to the common central resonance. The intensity of these bands is an oscillatory function of the driving amplitude V_1 and frequency ω, with a strong simultaneous reduction of the transmission probabilities of all bands at characteristic ratios $V_1/\hbar\omega$. It is shown that such a quenching is related to a vanishing amplitude of the central band, and analytical results, nonperturbative in V_1, are presented.

In 1982 Landauer and Büttiker[1] discussed electron transport through a harmonically oscillating barrier and showed that such an analysis can provide important insight into the mechanism of tunneling. The particular property they had in mind was the tunneling time, but since then a number of other effects related to oscillating potentials have been studied. All these effects rely on the fact that an oscillating potential can transfer an incoming electron of energy E—the so-called central band—with finite probability to sidebands at $E\pm n\hbar\omega$, where n is an integer and ω is the frequency of modulation. For instance, Jauho[2] considered a double-barrier resonant tunneling diode where either the barriers or the quantum well were subject to a periodic modulation of their potential, and by numerically solving the time-dependent Schrödinger equation he obtained information on the closely related process of phonon-assisted tunneling.[3] Also, he pointed out that oscillating quantum wells are much more effective than oscillating barriers in transferring electrons into the sidebands. Cai *et al.*[4] used a Green's function approach to numerically study photon-assisted resonant tunneling through a double-barrier structure for infrared-radiation detection. Very recently, Hu *et al.*[5] proposed a model of photon-assisted transport through quantum point contacts, the idea being that while electrons in the central band do not have an appreciable probability of tunneling through the point contact, electrons in higher sidebands do. For the oscillating-barrier turnstile device based on the Coloumb-blockade effect[6] it was recently shown that the ultimate accuracy achievable with these potential new current standards is limited by the formation of sidebands in the oscillating barriers.[7] Finally, a somewhat different approach based on Floquet states was chosen by Grossmann *et al.*[8] and by Holthaus and Hone[9] to study a harmonically driven double well and superlattice, respectively.

In this paper we present analytical results for the transmission probability of an electron traversing a double-barrier structure where the central quantum well is harmonically driven by an external force $V_1\cos(\omega t)$. Our results are perturbative in ω but nonperturbative in the modulation amplitude V_1, thus allowing us to access the regime $V_1 \gg \hbar\omega$. We start by solving the time-dependent Schrödinger equation for an isolated quantum well, and then proceed to apply the results obtained to a double-barrier resonant tunneling diode.

A time-dependent Hamiltonian of the form

$$H(x,t) = H_0(x) + V_1\cos(\omega t), \tag{1}$$

containing a position-independent oscillating potential, has already been studied some thirty years ago,[10] and the wave function solving the corresponding time-dependent Schrödinger equation was found to be

$$\begin{aligned}\psi(x,t) &= \psi_0(x)\exp(-\tfrac{iEt}{\hbar})\exp[-i\tfrac{V_1}{\hbar\omega}\sin(\omega t)]\\ &= \psi_0(x)\exp(-\tfrac{iEt}{\hbar})\sum_{n=-\infty}^{\infty} J_n(\tfrac{V_1}{\hbar\omega})\exp(-in\omega t),\end{aligned} \tag{2}$$

where J_n is the Bessel function of the first kind and $\psi_0(x)$ solves $H_0(x)\psi_0(x) = E\psi_0(x)$. At first, one might think that Eq. (2) means that due to the oscillating potential $V_1\cos(\omega t)$ electrons are being excited to the sideband n with probability $|J_n(\frac{V_1}{\hbar\omega})|^2$. This is, however, not true. As can be inferred from the first of these equations, an oscillating potential—uniform in space—simply gives rise to an overall phase factor to the wave function $\psi_0(x,t)$ of the unperturbed system, and hence can be eliminated by a gauge transformation. In order to see any effect of the oscillating potential, it is therefore vital to restrict the region of uniform modulation to an area smaller than the coherence length of the wave functions. We believe that this may be the reason for some experiments having failed so far in detecting photon-assisted tunneling.

We consider a quantum well of width d having walls of finite height V as depicted in Fig. 1, where only the quantum well is subject to a harmonic modulation of its potential, but not the adjoining regions. At the interfaces between the static regions I and III and the oscillating quantum well II at $x = 0$ and $x = d$, respectively, the wave function (2) and its flux have to be continuous. To satisfy these requirements, we need to superimpose many wave functions of the general form (2) inside the quantum well at energies $E + l\hbar\omega$,

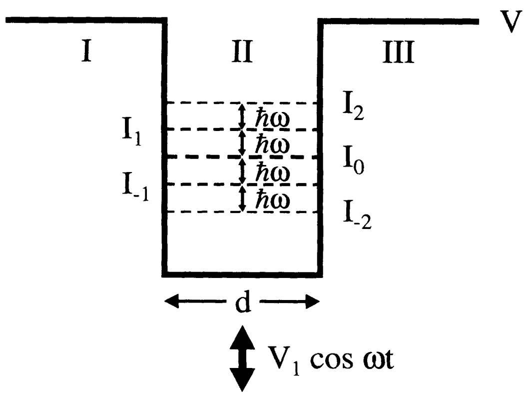

FIG. 1. A resonant level in a quantum well, oscillating as $V_1\cos(\omega t)$ and sandwiched between two static regions, splits up into a central band I_0 and sidebands I_n at $E + n\hbar\omega$.

$$\psi_{II}(x,t) = \sum_{l=-\infty}^{\infty} [A_{II}^l \exp(ik_{II}^l x) + B_{II}^l \exp(-ik_{II}^l x)] \times \sum_{n=-\infty}^{\infty} J_n(\tfrac{V_1}{\hbar\omega}) \exp\left(-\tfrac{i[E+(l+n)\hbar\omega]t}{\hbar}\right), \tag{3}$$

where the wave vector of each sideband l is defined as $\hbar k_{II}^l = \sqrt{2m_{II}(E + l\hbar\omega)}$. When matching this to a wave function in the right region,

$$\psi_{III}(x,t) = \sum_{l=-\infty}^{\infty} [A_{III}^l \exp(\kappa_{III}^l x) + B_{III}^l \exp(-\kappa_{III}^l x)] \times \exp\left(-\tfrac{i(E+l\hbar\omega)t}{\hbar}\right)$$

with $\hbar\kappa_{III}^l = \sqrt{2m_{III}(V_{III} - E - l\hbar\omega)}$, one finds that the expansion coefficients of the sideband amplitudes on both sides of the interface at $x = d$ have to satisfy the following set of equations,

$$A_{III}^n \exp(\kappa_{III}^n d) + B_{III}^n \exp(-\kappa_{III}^n d) = \sum_{l=-\infty}^{\infty} [A_{II}^l \exp(ik_{II}^l d) + B_{II}^l \exp(-ik_{II}^l d)]\, J_{n-l}(\tfrac{V_1}{\hbar\omega}),$$

$$\frac{\kappa_{III}^n}{m_{III}}[A_{III}^n \exp(\kappa_{III}^n d) - B_{III}^n \exp(-\kappa_{III}^n d)] = \sum_{l=-\infty}^{\infty} i\frac{k_{II}^l}{m_{II}} [A_{II}^l \exp(ik_{II}^l d) - B_{II}^l \exp(-ik_{II}^l d)] \times J_{n-l}(\tfrac{V_1}{\hbar\omega}). \tag{4}$$

A similar set of equations holds for the interface at $x = 0$. As usual, for a nondivergent, normalizable solution to exist, the coefficients B_I^n and A_{III}^n must all vanish. After some algebra to eliminate the A_{II}^l and B_{II}^l coefficients in the quantum well, the resulting system of equations for the remaining coefficients is given by

$$A_I^{n'} = \sum_{l,n=-\infty}^{\infty} \left[\cos(k_{II}^l d) + \frac{\kappa_{III}^n}{m_{III}}\frac{m_{II}}{k_{II}^l}\sin(k_{II}^l d)\right] \times J_{n-l}(\tfrac{V_1}{\hbar\omega}) J_{n'-l}(\tfrac{V_1}{\hbar\omega}) \exp(-\kappa_{III}^n d) B_{III}^n,$$

$$A_I^{n'} = \frac{m_I}{\kappa_I^{n'}} \sum_{l,n=-\infty}^{\infty} \left[\frac{k_{II}^l}{m_{II}}\sin(k_{II}^l d) - \frac{\kappa_{III}^n}{m_{III}}\cos(k_{II}^l d)\right] \times J_{n-l}(\tfrac{V_1}{\hbar\omega}) J_{n'-l}(\tfrac{V_1}{\hbar\omega}) \exp(-\kappa_{III}^n d) B_{III}^n. \tag{5}$$

This set of equations is still exact. As in the case of a static quantum well, it has nontrivial solutions only for some quantized values of the resonance energy $E = (\hbar k_{II}^0)^2/2m_{II}$. The essential physics can be captured by studying a symmetric quantum well where $V_I = V_{III}$, $m_I = m_{III}$, and $A_I^n = \exp(-\kappa_{III}^n d) B_{III}^n$ holds. Also, we require that $n\hbar\omega \ll E$ and $n\hbar\omega \ll V_I - E$ for all sufficiently populated sidebands, but no further assumptions are made about the modulation amplitude V_1. In this limit, the probabilities for the $n = \pm|n|$ sidebands will be the same, and guided by previously found numerical solutions of Eq. (5) we try an ansatz of the form $A_I^n = J_n(\gamma\frac{V_1}{\hbar\omega})$. In what follows, the parameter γ will be specified such that this ansatz is indeed a solution.

After expanding the wave vectors of the sidebands to lowest order in $\hbar\omega$ as $\kappa_I^n = \kappa_I + \kappa_I'\hbar\omega n$ and $k_{II}^l = k_{II} + k_{II}'\hbar\omega l$, and by employing the quantization conditions for the eigenenergy of the resonance,

$$1 = \cos(k_{II}d) + \frac{\kappa_I}{m_I}\frac{m_{II}}{k_{II}}\sin(k_{II}d) = \frac{m_I}{\kappa_I}\left[\frac{k_{II}}{m_{II}}\sin(k_{II}d) - \frac{\kappa_I}{m_I}\cos(k_{II}d)\right], \tag{6}$$

the first equation of (5) becomes

$$A_I^{n'} = \sum_{l,n=-\infty}^{\infty}\left[1 - \frac{\kappa_I}{m_I}\frac{m_{II}}{k_{II}}k_{II}'\hbar\omega l d + \left(\frac{\kappa_I'}{m_I}n - \frac{\kappa_I}{m_I}\frac{k_{II}'}{k_{II}}l\right)\frac{m_{II}}{k_{II}}\hbar\omega\sin(k_{II}d)\right] \times J_{n-l}(\tfrac{V_1}{\hbar\omega}) J_{n'-l}(\tfrac{V_1}{\hbar\omega}) J_n(\gamma\tfrac{V_1}{\hbar\omega}). \tag{7}$$

Now the summations over l and n can be performed exactly by utilizing von Neumann's addition theorems[11]

$$\sum_{l,n=-\infty}^{\infty} J_{n-l}(u)J_{n'-l}(u)J_n(\gamma u) = J_{n'}(\gamma u),$$

$$\sum_{l,n=-\infty}^{\infty} l J_{n-l}(u)J_{n'-l}(u)J_n(\gamma u) = n'\left(1 - \tfrac{1}{\gamma}\right) J_{n'}(\gamma u).$$

With the help of these theorems, Eq. (7) simplifies to $A_I^{n'} = [1 + n'\hbar\omega f(\gamma)]J_{n'}(\gamma\frac{V_1}{\hbar\omega})$, and in order for this to be consistent with our ansatz, we require that the coefficient of $n'\hbar\omega$, i.e., $f(\gamma)$, vanishes, leading to

$$\gamma = \frac{k_{II}d + \sin(k_{II}d)}{k_{II}d + 2\frac{k_{II}}{\kappa_I} + (1 - \frac{m_{II}}{m_I})\sin(k_{II}d)}. \tag{8}$$

It is easily checked that the second equation of (5) gives the same answer. Finally, the sideband amplitudes in the quantum well, $A^l_{\rm II}$ and $B^l_{\rm II}$, can be determined by inverting Eq. (4), and $\sum_n |J_n(u)|^2 = 1$ gives the normalization.

In Fig. 2 we have plotted a typical example for the probability $I_n = \int dx|\psi^n(x)|^2 = |J_n(\gamma\frac{V_1}{\hbar\omega})|^2$ of the electron being in a particular sideband n as a function of $V_1/\hbar\omega$. The parameters chosen are $m_{\rm I,II,III} = 0.067m_0$, $V_{\rm I,III} = 0.5$ eV, and $d = 5$ nm, yielding $\gamma = 0.93$. At zero modulation, only the central band is present, but with increasing modulation V_1 more and more sidebands become important, with the sideband probabilities starting to oscillate when V_1 becomes larger than a few $\hbar\omega$. The sideband probabilities vanish completely at zeros of $J_n(\gamma\frac{V_1}{\hbar\omega})$, which is a characteristic fingerprint of the wave function's phase coherence in time.

We can directly apply these results to the problem of calculating the probability of an electron traversing a double-barrier structure with a central oscillating quantum well, provided the energy of the incoming electron matches the energy $E + n_{\rm in}\hbar\omega$ of one of the bands of the quantum-well resonance. By picking up or losing photon quanta $\hbar\omega$ in the quantum well, the electron will emerge on the far side of the structure at some energy $E+n_{\rm out}\hbar\omega$, thus defining transmission channels $n_{\rm in} \to n_{\rm out}$. Before showing our numerical data based on an exact transfer-matrix solution, it is instructive to study a simple analytical model. We assume that the coupling strength to the quantum-well resonance for an incoming electron at energy $E + n_{\rm in}\hbar\omega$ is in first approximation proportional to the probability $I_{n_{\rm in}}$ of finding an electron in sideband $n_{\rm in}$ of the resonance. (This is exactly the quantity we have calculated above, see Fig. 2.) Similarly, for the outgoing channel $n_{\rm out}$, the coupling strength is proportional to $I_{n_{\rm out}}$. The total transmission probability from channel $n_{\rm in}$ to $n_{\rm out}$ is then given by $T_{n_{\rm in},n_{\rm out}} = T_0 I_{n_{\rm in}} I_{n_{\rm out}}$, where T_0 is the $n_{\rm in} = 0 \to n_{\rm out} = 0$ resonant transmission probability for a static quantum well, which for a symmetric structure equals unity. To illustrate this idea, we have studied a model double-barrier structure as depicted in Fig. 3 which has the same parameters as the quantum well considered above, except that now the barriers have a finite thickness of 5 nm each.

In Fig. 4(a) we have plotted the transmission probability $T_{n_{\rm in},n_{\rm out}} = I_{n_{\rm in}} I_{n_{\rm out}}$ for an incoming electron at the energy of the central resonance of the quantum well, i.e., for $n_{\rm in} = 0$. In contrast to the oscillating-barrier case we find that for an oscillating quantum well the sideband intensities do not monotonically increase with V_1 and decrease with $|n|$, but rather have a very rich $V_1/\hbar\omega$ and n dependence. In particular, a *simultaneous* quenching of the transmission probability in all bands is seen for some characteristic values of $V_1/\hbar\omega$. The origin

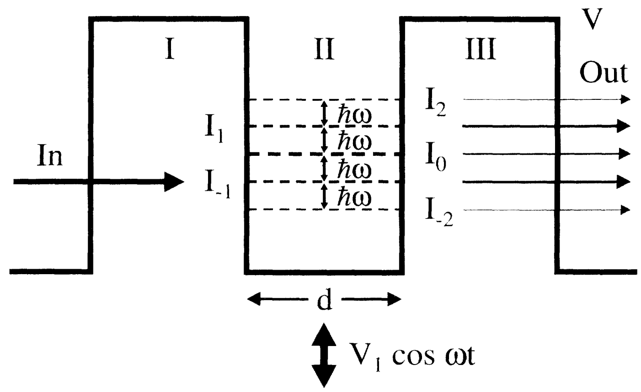

FIG. 3. Resonant transmission through a double-barrier structure consisting of an oscillating quantum well sandwiched between two static barriers. When the energy of the incoming electron matches one of the (side)bands, the outgoing wave will also be found at the energy of a (side)band, thus defining channels $n_{\rm in} \to n_{\rm out}$. Shown is $n_{\rm in} = -1$.

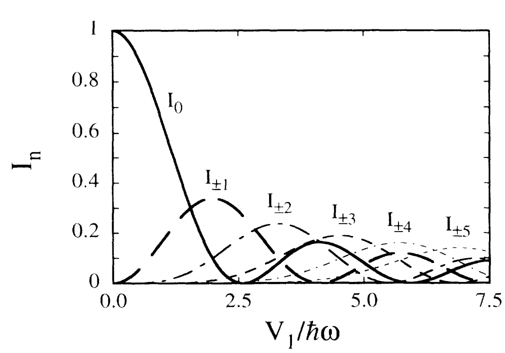

FIG. 2. Probability $I_n = |J_n(\gamma\frac{V_1}{\hbar\omega})|^2$ of the electron being in a particular sideband n of a quantum-well resonance as a function of $V_1/\hbar\omega$ (parameters chosen are $m = 0.067m_0$, $V = 0.5$ eV, and $d = 5$ nm, yielding $\gamma = 0.93$).

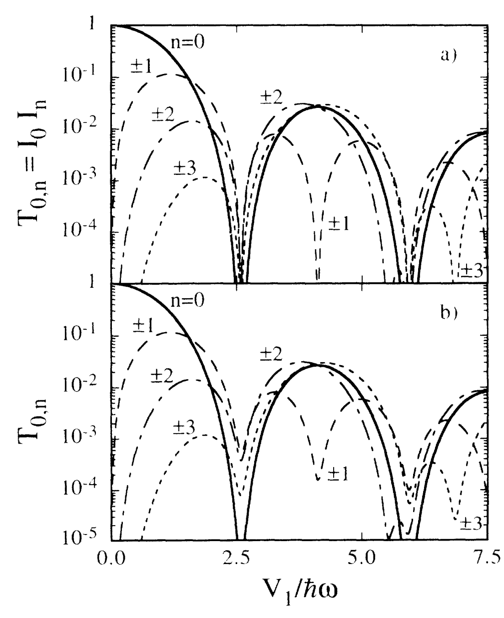

FIG. 4. Transmission probabilities in channels $n_{\rm in} \to n_{\rm out}$ for an electron impinging on the oscillating double-barrier structure of Fig. 3 at the energy of the central band ($n_{\rm in} = 0$) as a function of $V_1/\hbar\omega$. (Parameters as before.) Simultaneous quenching of all channels occurs when the amplitude of the central band vanishes at $V_1/\hbar\omega = 2.585, 5.936, \ldots$; (a) shows analytical results based on $I_n = |J_n(\gamma\frac{V_1}{\hbar\omega})|^2$ and Eq. (8), (b) numerical results based on a transfer-matrix calculation.

of this effect is the transmission probability being the combined probability of entering and leaving the quantum well. Hence, every time the amplitude I_0 coupling to the incoming electron vanishes, which is at zeros of $J_0(\gamma\frac{V_1}{\hbar\omega})$, a simultaneous quenching of all transmission channels occurs. In addition to this, a quenching of individual channels occurs when the amplitude of an outgoing channel vanishes (e.g., $I_{\pm 1}$ at $V_1/\hbar\omega = 4.12$). For comparison, we also show in Fig. 4(b) the results of a numerical transfer-matrix calculation for the same structure (details of which will be presented elsewhere). The agreement is extremely good, except that in the more precise numerical solution, maintaining higher orders of $\hbar\omega$, a small residual transmission probability remains at the points of quenching.[12]

And finally, Fig. 5(a) shows the transmission probability $T_{-1,n_{\rm out}}$ for an electron incident $\hbar\omega$ below the energy of the quantum-well resonance, which is the example commonly studied for photon-assisted tunneling. In this case, photons are necessary to achieve resonant tunneling, and in agreement with earlier work[4] we consequently observe a strong increase in the transmission probability of all channels as V_1 increases. However, for $V_1 > \hbar\omega$ we find that the transmission probabilities start oscillating, with a simultaneous quenching of all channels when the amplitude of the $n_{\rm in} = -1$ sideband vanishes at roots of $J_1(\gamma\frac{V_1}{\hbar\omega})$. Again, the agreement with the numerical solution of Fig. 5(b) is excellent except for the $T_{-1,-1}$ channel in the regime $V_1 < \hbar\omega$.

For all but the diagonal transmission channels we find the remarkable result that the reflection and transmission probabilites in each band are virtually identical for a symmetric double-barrier structure. This supports similar findings reported earlier by Jauho[2] for $V_1 \ll \hbar\omega$.

A similar quenching or "coherent destruction" of tunneling in driven double-well structures has recently been discussed by Grossmann *et al.*[8] in terms of level crossings of so-called Floquet states. There, as well as in similar work on driven superlattices,[9] it is essential to have at least two degenerate Floquet states, whereas in our case a single Floquet state—the driven quantum-well resonance—is sufficient for the quenching to occur. However, considering that in our double-barrier structure the quenching depends on the energy of the probing electron (i.e., on the input channel), we speculate that possibly the continuum of states provided by the open nature of our system acts as a second Floquet state. To clarify this, further work is needed.

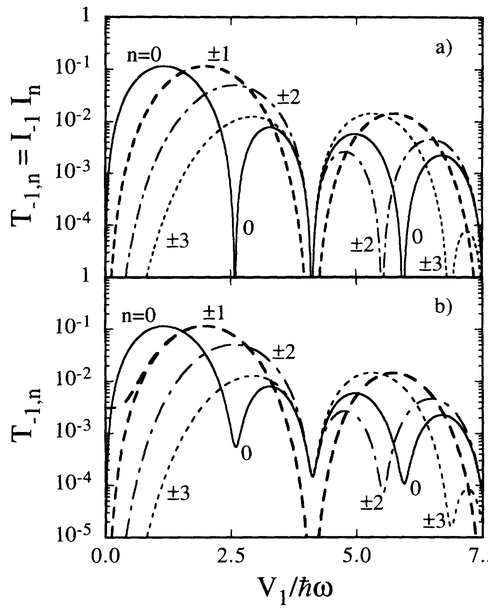

FIG. 5. Transmission probabilities in channels $n_{\rm in} \to n_{\rm out}$ for an incoming electron in the $n_{\rm in} = -1$ channel. Here the simultaneous quenching of all transmission channels happens when the amplitude of the $n = -1$ sideband vanishes at zeros of $J_1(\gamma\frac{V_1}{\hbar\omega})$; (a) analytical, (b) numerical results.

In conclusion, we have studied the transmission probability for an electron to resonantly traverse a double-barrier structure where the bottom of the quantum well is oscillating as $V_1 \cos(\omega t)$. The sidebands in the transmission probability, generated by such an oscillation, show a rich structure which can be understood by considering an isolated oscillating quantum well. At particular values of the ratio $V_1/\hbar\omega$ a simultaneous quenching of the resonant transmission probability in all sidebands is found. The analytical results presented are valid even in the regime $V_1 \gg \hbar\omega$ and show excellent agreement with numerical solutions. We believe that these results should be measurable with today's experimental technology and may lead to new detectors for infrared radiation. A central condition to be met in any such realization is an energetically sufficiently narrow stream of incoming electrons, as otherwise the effect will tend to be smeared out.

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[12] The same result is obtained in the present approach if higher orders of $\hbar\omega$ are retained when expanding Eq. (5).