Wave instability in semiconductors without negative differential conductivity

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A traveling-wave instability at *finite critical wave number* is found in extrinsic semiconductors at the onset of impact ionization. A condition for this instability is a strong increase of the carrier-generation rate as a function of the electric field, e.g., due to a sufficiently low ionization energy of the impurities. The linear instability of the uniform steady state is discussed for a simple model, and traveling waves are shown to exist by solving numerically the basic partial differential equations.

A model for extrinsic semiconductors at low temperatures is presented, and an instability of the uniform state against perturbations of finite wavelengths and frequencies is found. This instability is not associated with negative differential conductivity (NDC), and it provides an alternative explanation of recently observed current instabilities at the onset of impact ionization at low temperatures.[1]

The complexity of charge transport in semiconductors driven far from equilibrium leads to a large number of possible mechanisms causing instabilities of the uniform stationary current-carrying state. Hence, current instabilities represent important examples of a great variety of typical nonlinear phenomena as self-sustained oscillations, chaotic dynamics, pattern formation, etc. Besides being technically useful, nonlinearities in semiconductors also provide simple experimental illustrations of several theoretical results in the field of nonlinear dynamics.[2–6]

A widespread classification of current instabilities is based on the shape of the current-field characteristic, $j(E)$, of the *uniform* state, j and E being the current-density and the electric field, respectively. This is because many of the known cases are associated with NDC,[7] where the characteristic has a part with a negative slope, $\sigma = dj(E)/dE < 0$. Usually, NDC is divided in the two cases of N- and S-shaped characteristics (NNDC and SNDC, respectively). NNDC arises from a zero of the differential conductivity σ at instability and corresponds, under current control, to a saddle-node bifurcation of the electric field E; NNDC can give rise to spatially uniform oscillations or to longitudinal space-time structures, e.g., electric high-field domains.[3] On the other hand, SNDC is characterized by a pole of the differential conductivity and corresponds, in the voltage-controlled case, to a saddle-node bifurcation of the current. At an SNDC instability the magnetic-flux diffusion constant $(\mu_0\sigma)^{-1}$, where μ_0 is the magnetic permeability, has a zero; this leads to the undamping of transverse fluctuations.[2,8] It is important to note that it is the kind of the connection to the external circuit which determines the actual behavior of the system. In current-controlled semiconductors exhibiting SNDC, for example, stable current filaments can exist;[2–4] in the voltage-controlled case, on the other hand, hysteresis of uniform states is expected.

Semiconductors dominated by impact ionization often show SNDC behavior caused by avalanche breakdown.[2,4] In contrast, in this paper an impact-ionization current-instability without NDC is investigated, where the uniform state turns out to be unstable against longitudinal waves with *finite* wave numbers. Other types of current instabilities without NDC can be found in, e.g., Refs. 5 and 10. The model is constructed in the framework of generation-recombination (g-r) kinetics of hot carriers; it is similar to some models in Refs. 2 and 9, where, however, traveling-wave instabilities at *finite* critical wave numbers in semiconductors with single-level impurities and without NDC have not been investigated.

Consider a bulk semiconductor doped with shallow identical single-level impurities (donors or acceptors) of density N. The sample is kept at low lattice temperatures in order that only carriers originating from impurities contribute to the conduction (extrinsic conduction). The description of current transport is based on Maxwell's equations for the electric field E and on a set of transport equations for appropriate transport variables. A hydrodynamic approach[3] leads to four transport equations for the carrier density n_0, the density of occupied impurities n_d, the mean drift velocity u and the mean carrier energy W, respectively. Assuming fast momentum and energy relaxation compared to carrier recombination, the carrier density n_0 and the density of occupied neutral impurities n_d become the only relevant dynamic transport variables, while u and W are eliminated adiabatically. At low lattice temperatures elastic impurity scattering is the relevant scattering process; hence, momentum relaxes faster than energy giving rise to hot carriers. In the specific model given below, impact ionization is assumed to increase as a function of the carrier energy being approximated by the relation $W = q\tau_w\mu E^2$,[11] where q, μ and τ_w are the charge, the mobility and the energy relaxation time of the carriers, respectively.

We prescribe a total dc current in x direction by $J_{\text{tot}} = \int dy\, dz\, j$, and restrict ourselves to longitudinal structures in this direction. Current control is achieved by connecting the sample in series to a large load resistor R and dc-bias voltage RJ_{tot}.[4]

The basic equations are[2]

$$\partial_t n_0 + q^{-1}\partial_x j = f(n_0, n_d, E) = -\partial_t n_d \ , \quad (1)$$

$$\epsilon\, \partial_t E = J - j \ , \quad (2)$$

$$\epsilon\, \partial_x E = q(n_0 + n_d - N) \ , \quad (3)$$

$$j = q\mu n_0 E - qD\partial_x n_0 \ , \quad (4)$$

where the total current per area J serves as the control parameter. For the sake of simplicity, the electric permeability ϵ, the mobility μ and the diffusion constant D are assumed to be constant. After the elimination of n_d with the help of Poisson's equation (3) and the transformation to dimensionless variables, one obtains

$$\partial_t n_0 = \partial_x^2 n_0 - \partial_x(n_0 E) + f(n_0, 1 - n_0 + \eta\partial_x E, E) \ , \tag{5}$$

$$\eta\partial_t E = J + \partial_x n_0 - n_0 E \ , \tag{6}$$

where the following rescaling is used: $t \to \tau_n t$, $x \to \sqrt{\tau_n D}x$, $n_{0,d} \to N n_{0,d}$, $f \to (N/\tau_n)f$, $E \to (\sqrt{D/\tau_n}/\mu)E$, and $J \to qN\sqrt{D/\tau_n}J$, with τ_n being the recombination time. Further, $\eta \equiv \tau_m/\tau_n$, where the dielectric relaxation time $\tau_m \equiv \epsilon/(q\mu N)$ characterizes the time scale for the decay of charge fluctuations. Usually, $\eta \ll 1$ holds.[2,9]

Let us now investigate the linear stability of a uniform solution of (5), (6). The addition of a small perturbation $\delta n_0, \delta E \propto \exp(\lambda t + ikx)$ to the stationary uniform state leads to a linear eigenvalue problem for the complex eigenvalues λ. The characteristic polynomial

$$n_0[\lambda - f' + v_0(E - ik)] + \eta\Big[\lambda(\lambda - f' + ikE + k^2) + \partial_{n_d} f(ikE + k^2)\Big] = 0 \tag{7}$$

must be solved for $\lambda(k)$, the real part of which is the growth rate of the small-amplitude wave with wave number k and a frequency corresponding to the imaginary part of $\lambda(k)$. In (7), we defined $f' \equiv \partial_{n_0} f - \partial_{n_d} f$ and $v_0 \equiv \partial_E f/n_0$. Assuming that $\partial_{n_d} f$, $-f'$, and v_0 are positive, one can show that in the limits $k \to 0$ and $k \to \infty$ the perturbation decays to zero. Hence, an instability can only occur at a finite wave number. In the following, the results are given for $\eta \ll 1$ instead of writing down the unwieldy exact solutions $\lambda_{1,2}(k)$ of (7) for finite k. Inspection of (7) suggests that the eigenvalues are of the form $\lambda_1 = g_1(z) + ikh_1(z)$ and $\lambda_2 = \eta^{-1}g_2(z) + ikh_2(z)$, where $g_{1,2}$ and $h_{1,2}$ are real functions of $z \equiv \eta k^2/n_0$, and can be expanded in power series with respect to η. The dependence on z reflects the well-known strong coupling of the carrier-density mode δn_0 to the dielectric relaxation mode δE on the Debye scale $\sqrt{\eta}k$, which is caused by the long-range Coulomb interaction between the carriers.[8]

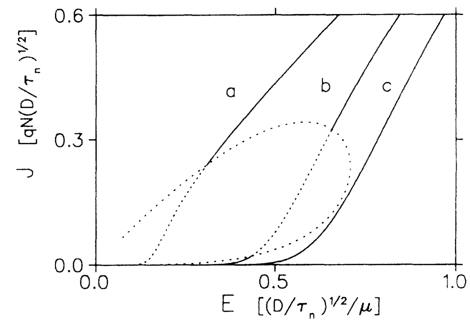

FIG. 2. J-E characteristics of uniform states at the onset of impact ionization [parameter values of Fig. 1; $\alpha = 1.7$ (c)]. The unstable parts and the stability boundary are indicated by dotted curves.

For λ_2, this ansatz yields $g_2(z) = -n_0(1+z)+O(\eta)$ and $h_2(z) = -[E+v_0/(1+z)]+O(\eta)$. Thus, λ_2 corresponds to a stable branch of the spectrum associated with dielectric relaxation at $k = 0$ and to carrier diffusion at $k \to \infty$. For λ_1 one finds

$$g_1(z) = \frac{f' - v_0 E + [h_1^2(z) + h_1(z)E - \partial_{n_d} f]z}{1+z} + O(\eta) \ , \tag{8}$$

$$h_1(z) = \frac{v_0}{1+z} + O(\eta) \ . \tag{9}$$

Marginal modes correspond to the positive solutions z of the equation $g_1(z) = 0$, which can be written as

$$\partial_{n_d} f z^3 + (2\partial_{n_d} f - f')z^2 + [\partial_{n_d} f - 2f' + v_0(E - v_0)]z + v_0 E - f' = 0 \ . \tag{10}$$

Plotting the solutions of this equation in the J-k plane, one obtains a curve (neutral curve) which represents at fixed control parameter J the boundary of the band of wave numbers associated with undamped waves. Because all the coefficients of the polynomial (10) in z are positive, except for large v_0, when the third one becomes negative, we conclude that an instability occurs at large values of $\partial_E f$.

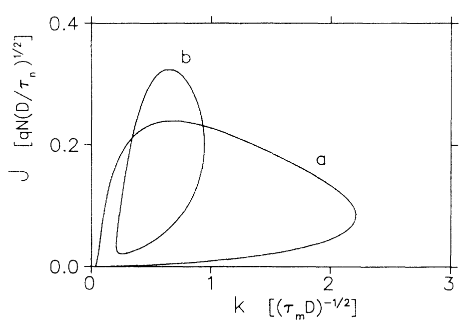

FIG. 1. Neutral curves for the parameter values $A_I^0 = 10$ and $\alpha = 0.1$ (a), 1.0 (b).

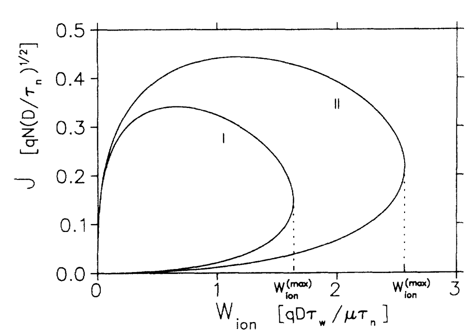

FIG. 3. Stability boundary in the J-$W_{\rm ion}$ plane; $A_I^0 = 10$ (I), $A_I^0 = 20$ (II).

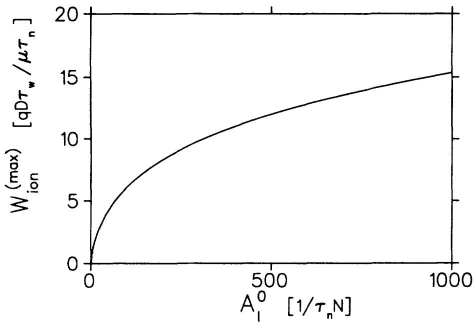

FIG. 4. Maximum values of $W_{\rm ion}$ as a function of A_I^0, where a traveling-wave instability can occur.

To give a specific example, we consider a typical semiconductor sample used in recent experiments concerning impact-ionization avalanche breakdown in In-doped p-Ge.[1] Typical values of the parameters are $N \approx 10^{14}$ cm^{-3}, $\tau_n \approx 10^{-8} - 10^{-7}$ s, $\tau_m \approx 10^{-11}$ s, $D \approx 10^3$ cm^2/s, and $\mu \approx 10^5$ cm^2/ V s.

The g-r function f is constructed in the usual way.[2] Generation of carriers consists of thermal (A_T) and impact (A_I) ionization processes at the impurities. Recombination of carriers corresponds to the inverse processes, which are single-carrier capture (B_T) and Auger recombination (B_I), respectively. With the approximation $A_T \approx 0$ (which is reasonable at low lattice temperatures) and neglecting Auger recombination,[2] the g-r function reads

$$f = A_I n_0 n_d - n_0(1 - n_d) \ , \tag{11}$$

where only the impact-ionization coefficient A_I is assumed to depend on E. This dependence is modeled by $A_I = A_I^0 \exp(-W_{\rm ion}/W)$ with a constant A_I^0. Typical ionization energies of shallow impurities are $W_{\rm ion} \approx 10$ meV.[1] The impact-ionization coefficient can be written in the form $A_I^0 \exp(-\alpha/E^2)$, where $\alpha \equiv W_{\rm ion}\tau_n\mu/(qD\tau_w)$. From the crude parameter values given above and $\tau_w \approx 10^{-9} - 10^{-8}$ s,[1,11] we conclude that α lies somewhere between 10^{-2} and 10^2.

The relevant uniform solution of $f = 0$ is $n_0 = A_I/(1+A_I)$, and it holds $\partial_{n_d} f = A_I > 0$, $f' = -A_I < 0$, and $v_0 = 2\alpha n_0/E^3 > 0$. This implies that the $J-E$ characteristic, $J = n_0 E$, is strictly monotonous, i.e., NDC does not occur.

The neutral curve is obtained from $z^3+3z^2+(3+c)z+1+b = 0$, where $b = 2\alpha n_d/E^2$ and $c = b(1 - bA_I/E^2)$; solutions are shown for two values of α in Fig. 1. Current-field characteristics at the onset of impact ionization are plotted in Fig. 2, where the unstable part is indicated by dotted curves. At low fields almost all carriers are bound to the impurities, representing the low-current state. At fields of the order $E \approx \sqrt{\alpha}$ impact ionization sets in, and the current increases in a nonlinear manner. In the unstable regime a band of finite wave numbers gives rise to the formation of traveling waves. Figure 3 shows the unstable regime in the J-$W_{\rm ion}$ plane. The instability vanishes for $W_{\rm ion} > W_{\rm ion}^{(\rm max)}$, when impact ionization is weak, e.g., in the case of deep impurity levels. This implies that a traveling-wave instability occurs in samples with impurities of rather low ionization energies. Figure 4 shows $W_{\rm ion}^{(\rm max)}$ as a function of A_I^0.

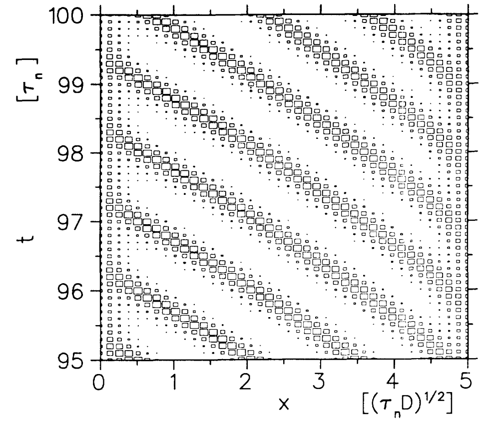

FIG. 5. Density plot of the electric field $E(x,t)$ obtained from a numerical solution with Neumann boundary conditions (and almost uniform initial conditions). The values of $E(x,t)$ approximately range between 0.7 (largest boxes) and 0.3 (zero boxes); $A_I^0 = 10$, $\alpha = 1$, $\eta = 5 \times 10^{-3}$, $L = 5$, and $J = 0.1$.

In order to confirm that a traveling wave forms in the linearly unstable regime, Eqs. (5) and (6) have been solved numerically[12] for Neumann boundary conditions ($\partial_x n_0 = \partial_x E = 0$ at $x = 0, L$). Choosing $\eta = 0.005$, $L = 5$, and parameter values in accordance with case b of Figs. 1 and 2, one expects from the neutral curve a traveling wave with a wavelength approximately between 0.5 and 1.8 in units of $\sqrt{\tau_n D}$. In Fig. 5, the space-time plot of the electric field shows a traveling wave with wavelength ≈ 1.5. Traveling waves of this kind should be observable in appropriate experiments.

In this paper, a model is investigated describing impact ionization in extrinsic semiconductors at low temperatures for the case of fast dielectric relaxation compared to carrier recombination. At the onset of impact ionization and at sufficiently low ionization energies, the uniform steady state turns out to be unstable against traveling waves with finite critical wave number. The neutral curve is derived and the instability is discussed in parameter space. Typical critical wavelengths are of the order of the Debye length, and typical frequencies are of the order $\sqrt{n_0/(\tau_n\tau_m)}/2\pi$. This simple model may also explain recent experimental results: assuming $n_0 \approx 10^{-8}$ at the onset of impact ionization leads to a frequency ≈ 10 kHz, in accordance with the values reported in Ref. 1.

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[1] U. Rau *et al.*, Phys. Rev. B **43**, 2255 (1991); U. Rau, Ph.D. thesis, Universität Tübingen, 1991.

[2] E. Schöll, in *Nonequilibrium Phase Transitions in Semiconductors*, edited by H. Haken, Springer Series in Synergetics Vol. 35 (Springer-Verlag, Berlin, Heidelberg, 1987).

[3] V.L. Bonch-Bruevich, I.P. Zvyagin, and A.G. Mironov, *Domain Electrical Instabilities in Semiconductors* (Consultants Bureau, New York, 1975).

[4] J. Peinke *et al.*, in *Spatio-Temporal Correlations in Semiconductors,* in Spontaneous Formation of Space-Time Structures and Criticality, Vol. C 349 of Nato Advanced Study Institute Series B: Physics, edited by T. Riste and D. Sherrington (Kluwer Academic Publishers, Dordrecht, 1991).

[5] J. Požela, *Plasma and Current Instabilities in Semiconductors* (Pergamon Press, Oxford, 1981).

[6] J. Peinke, J. Parisi, O.E. Rössler, and R. Stoop; *Encounter with Chaos* (Springer-Verlag, Berlin Heidelberg, 1992).

[7] B.K. Ridley, Proc. Phys. Soc. **82**, 954 (1963).

[8] M. Büttiker and H. Thomas, Z. Phys. B **33**, 275 (1979).

[9] L.L. Bonilla and S.W. Teitsworth, Physica D **50**, 545 (1991).

[10] S.W. Teitsworth, R.M. Westervelt, and E.E. Haller, Phys. Rev. Lett. **83**, 825 (1983); J. Peinke, J. Parisi, A. Mühlbach, and R.P. Huebener, Z. Naturforsch. A **42**, 441 (1987); J. Požela, A. Tamaševičius, and J. Ulbikas, Solid-State Electron. **31**, 805 (1988); J. Spinnewyn, H. Strauven, and O.B. Verbeke, Z. Phys. B **75**, 159 (1989); G. Hüpper, E. Schöll, and L. Reggiani, Solid-State Electron. **32**, 1787 (1989); E. Schöll, Phys. Scr. **T29**, 152 (1989).

[11] W. Clauss *et al.*, J. Appl. Phys. **67**, 2980 (1990).

[12] The program DMOLCH of the IMSL MATH/LIBRARY is used (method of lines with cubic Hermite polynomials; 40 mesh points in space).