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Intrinsic resistance fluctuations in mesoscopic superconducting wires

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The puzzling appearance of a large resistance peak in the superconducting state near T_c of mesoscopic superconducting wires is analyzed. It is shown that this resistance anomaly can be explained in terms of thermal fluctuations producing phase slips of the superconducting order parameter in quasi-one-dimensional wires. Good quantitative agreement with the experimental observations is obtained within the framework of the modified Langer-Ambegaokar and McCumber-Halperin models.

A typical resistive transition into the superconducting state, $R(T)$, is usually described as a slow but *monotonic* decrease of the resistance due to fluctuations of the superconducting order parameter ψ above T_c, followed by a sharp drop at the transition temperature $T = T_c$. Recent experiments on *mesoscopic* superconducting samples revealed, however, a surprising $R(T)$ behavior characterized by the presence of an anomalous resistance peak in the vicinity of T_c.[1,2] The measured resistance typically becomes 10 to 20% higher than the normal-state resistance value R_n which is extracted from the plateau in the $R(T)$ curve above T_c. In an attempt to understand the origin of this phenomenon, Santhanam *et al.*,[1] as well as Vloeberghs *et al.*,[2] suggested several physical mechanisms based on the ideas discussed in Refs. 3–5 which, however, failed to *explain quantitatively* the experimental observations.

In this paper we show that the anomalous resistance peak in mesoscopic superconducting samples, with a width w much smaller than the temperature-dependent coherence length $\xi(T)$ and the charge imbalance relaxation length $\lambda_Q^*(T)$,[6] is related to intrinsic resistive fluctuations which can be analyzed within the framework of the Langer-Ambegaokar[7] (LA) and McCumber-Halperin[8] (MH) models. These models have to be modified to take into account the confinement of the superconducting current by the extremely narrow lines forming a mesoscopic sample.

Typical experimental zero-field $R(T)$ curves, clearly showing a resistance peak above T_c, are plotted for different transport currents I in Fig. 1. These data were obtained on a 1×1 μm^2 square mesoscopic Al loop having a thickness $t = 25$ nm and a width $w = 0.15$ μm (see Fig. 1 inset). The details of the sample preparation and characterization have been described in detail elsewhere.[2,9] Different samples all have a sheet resistance ranging between 1.5 and 2.0 $\Omega/\Box$ at 4.2 K, indicating a well-defined metallic character. These values are nearly 4 orders of magnitude smaller than the typical sheet resistance $R_\Box \simeq h/4e^2 \simeq 6.45$ k$\Omega/\Box$ for which electron localization phenomena become important.[10] Therefore, the existence of the anomalous $R(T)$ peak above T_c (Fig. 1) cannot be related to a resistance increase due to a pronounced granularity or disorder-induced effects.[11,12] The value of the mean free path $l \simeq 15$ nm is derived from the residual resistance ($R_{300\ K}/R_{4.2\ K} \simeq 2.1$). The latter value for l is much smaller than the coherence length $\xi_0 \simeq 1.6$ μm for pure bulk Al, indicating that the superconducting properties of the Al structures have to be treated within the "dirty limit" regime with $\xi(0) = \xi(T \to 0) = 0.85(\xi_0 l)^{1/2} \simeq 130$ nm. This value for $\xi(0)$ is in good agreement with the value $\xi(0) \simeq 120$ nm derived from the change of T_c of the Al thin films in an applied magnetic field H.

The most important experimental features of the anomalous $R(T)$ peak, shown in Fig. 1, can be summarized as follows:

(i) A marked resistance increase $[(\Delta R/R) \sim 10^{-1}]$ is observed in samples where the temperature-dependent coherence length $\xi(T)$ is of the order of or larger than the overall size of the sample and where the width w is much smaller than the loop size $L \simeq 1$ μm$< \xi(T)$, i.e., in quasi-one-dimensional mesoscopic superconducting microcircuits (see also Refs. 1 and 2).

(ii) The anomalous $R(T)$ peak is suppressed by increasing the transport current I through the loop as illustrated in Fig. 1. For higher currents the effect is smeared

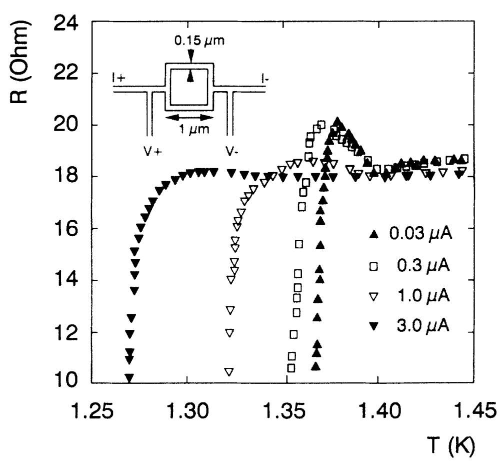

FIG. 1. Temperature dependences of the resistance for a mesoscopic Al loop, measured for different transport currents in zero magnetic field. The inset shows a scheme of the square loop with a side of 1 μm and a width of 0.15 μm. The measured Al loop has a thickness $t = 25$ nm.

out and the superconducting transition is shifted towards lower temperatures.

(iii) The amplitude ΔR of the $R(T)$ peak becomes smaller in larger samples,[1,13] i.e., the ΔR value correlates with the inverse volume of the sample. The $R(T)$ peak is clearly related to the mesoscopic nature of the samples, although a weak increase of $R(T)$, related to some kind of semiconducting or granular behavior, has been observed in larger superconducting samples.[12,14]

It is well known that in quasi-one-dimensional superconducting wires the product of the phase gradient $d\varphi/dx$ with the square of the modulus of the order parameter $|\psi|^2$ [$\psi = |\psi|\, exp(i\varphi)$] is a constant proportional to I:[15]

$$\frac{d\varphi}{dx}|\psi(x)|^2 = \text{const} \propto I. \tag{1}$$

A phase slip, corresponding to a substantial increase of $d\varphi/dx$ in Eq. (1), should therefore be accompanied by a proper reduction of $|\psi(x)|^2$, inducing an intrinsic resistance in the superconducting state.[6,16]

In what follows we will present a model which is based on the fact that the resistance of one-dimensional (1D) wires in the superconducting state [at $T < T_{c0}$, where T_{c0} is the mean-field Ginzburg-Landau (GL) transition temperature] is mainly determined by the rate of phase slip events.[6,8,17,18]

The average voltage V_s, arising from the phase slip events, is determined by the number N of these events in the sample [$N(T) \simeq L/\xi(T)$], a characteristic time $\tau(T)$,[7,8] an overlap factor $(\Delta F_0/k_BT)^{1/2}$, the Boltzmann factor $\exp[-\Delta F(T)/k_BT]$, and the factor $\sinh(I_s\phi_0/2k_BT)$ derived from the difference ΔF in the energy barrier for $+2\pi$ and -2π phase jumps. For intermediate transport currents I_s in the superconducting state

$$I_1 \equiv k_BT/\phi_0 < I_s < I_c, \tag{2}$$

the voltage V_s is determined by the following expression:[7,8,18]

$$V_s = 2\phi_0 \frac{N(T)}{\tau(T)} \left(\frac{\Delta F_0}{k_BT}\right)^{1/2} \left(1 - \frac{2I_s}{3I_c}\right)^{15/4} \times \exp\left[-\frac{\Delta F(T)}{k_BT}\right] \sinh\left(\frac{\phi_0 I_s}{2k_BT}\right). \tag{3}$$

Here $\phi_0 = h/2e$ is the superconducting flux quantum, ΔF_0 is the current independent difference in free energy between the normal and superconducting states, and I_c is the mean-field critical current. The characteristic value for I_1 is $I_1[A] \simeq 0.7 \times 10^{-8} T[K]$, i.e., for $T = 1$ K, $I_1 \sim 0.01$ μA. The free energy difference ΔF_0 in the volume $V = A\xi(T)$ (with A denoting the sample cross section) is given by

$$\Delta F_0 = \tfrac{8}{3}\sqrt{2}[A\xi(T)H_c^2(T)/8\pi]. \tag{4}$$

The critical current I_c is related to ΔF_0 by

$$I_c = (\tfrac{2}{3})^{1/2}\pi\Delta F_0/\phi_0. \tag{5}$$

The current-dependent energy difference $\Delta F(T)$ can be calculated from the time-dependent GL equation:[7,8]

$$\Delta F(T) = \Delta F_0 + I_s^2\phi_0^2/3\pi^2\Delta F_0. \tag{6}$$

The total resistance of the macroscopic superconducting system in the vicinity of T_c, where Cooper pairs coexist with normal-state quasiparticles, may be calculated by assuming that the supercurrent I_s and the normal current I_n are running in parallel: $R^{-1} = R_s^{-1} + R_n^{-1}$.[7,8] This procedure is used by Newbower, Beasley, and Tinkham[18] to describe the $R(T)$ curve for resistance values close to R_n. A nonmonotonic $R(T)$ dependence could be simulated, though it was not possible, in principle, to obtain a total resistance R higher than R_n for the currents I_s and I_n running in parallel. In the calculations below we shall consider such a temperature interval close to T_c, where the Andreev reflection is negligible.

Before applying the LA and MH models[7,8] to the mesoscopic superconducting wires, with the size smaller than both the coherence length $\xi(T)$ and the charge imbalance relaxation rate $\lambda_Q^*(T)$, the following important modifications have to be made:

(i) In wires, where the width is very small, $w \ll \xi$, the currents I_n and I_s cannot run in parallel for any fixed time t_0. Indeed, the coexistence of the parallel normal and superconducting currents leads to a tremendous increase in energy due to the variation of the modulus of the order parameter $|\psi|$ on a very short length scale $w \ll \xi$. The required coexistence of superconducting pairs and normal-state quasiparticles expected in the vicinity of T_c, may be achieved by the proper time averaging of the two possible states, superconducting and normal. Basically, we are assuming that there are two ways to provide the coexistence of normal and superconducting carriers close to T_c: (a) via their static mixture, in case of bulk samples or (b) via their dynamic mixture, in case of mesoscopic samples, when the coexistence appears as a result of the time averaging. This is one of the important assumptions used in our calculations which produce a total resistance R higher than R_n. Therefore we assume that in quasi-1D superconducting wires the instantaneous currents $I_n(t_0)$ and $I_s(t_0)$ can only flow *in series* and the Kirchoff law is still valid [$I_s(t_0) = I_n(t_0) = I$ and $R = R_n + R_s$ with $R_s = V_s/I_s$ and V_s given by Eq. (3)].

(ii) The number of phase slip events $N(T) \simeq L/\xi(T)$ for the mesoscopic regime is a constant $N(T) \simeq 1$. Under this condition, the characteristic time τ [Eq. (3)] has the meaning of the switching time between the superconducting (τ_s) and normal (τ_n) states, like switching in a quantum two-level system. What we essentially use here, is, in fact, the similarity between a mesoscopic superconducting sample with the size $L < \xi(T)$ and the behavior of a quantum system with well-defined energy levels. In the latter, the coexistence of the two allowed levels implies a proper time dependence of their occupation numbers. In a similar way, the coexistence of the supercurrent I_s with the normal-state quasiparticle current I_n may be treated as a time-dependent switching between the two states ("two levels"). This results in a total resistance

$$R = \frac{R_n(\tau_n/\tau_s) + R_s}{(\tau_n/\tau_s) + 1}, \tag{7}$$

where τ_s is the characteristic time in the time-dependent GL theory given by

$$\tau_s = \tau_{s0}\left(\frac{T_c}{T_c - T}\right) \tag{8}$$

with

$$\tau_{s0} \simeq (\pi\hbar)/(8k_BT_c) \tag{9}$$

and

$$\tau_n = \gamma\tau_s \exp\{-\Delta F(T)/k_BT\}, \tag{10}$$

where $\gamma \simeq \tau_n/\tau_{s0}$.

(iii) In mesoscopic superconducting samples where not only the sample width is small ($w \ll \xi$), but also the overall sample dimensions ($L \leq \xi$), the product $A\xi(T)$ in Eq. (4) should be substituted by the sample volume $V = AL$. In this case

$$\Delta F_0(\text{meso}) \simeq \frac{\sqrt{2}}{3\pi} V H_c^2(0)(1 - T/T_c)^2\delta. \tag{11}$$

Here δ is the normalized actual volume of a phase slip center. For bulk samples the exponential dependence in Eq. (3) is sharper since the free energy difference $\Delta F_0(\text{bulk}) \propto (1 - T/T_c)^{3/2}$.

The calculation of the $R(T)$ values using Eq. (7) has been carried out with two adjustable parameters: γ [Eq. (10)] and δ [Eq. (11)]. The parameter δ corresponds to the ratio of the actual volume of the phase slip center to the total volume of the sample. Since in superconducting networks the order parameter at nodes is larger than in the individual connecting stripes,[3] it is reasonable to expect that phase slip centers will be formed in the stripes between the voltage probe and the loop itself. It implies that we should introduce the parameter $\delta < 1$ in Eq. (11). For the geometry of the sample used in our experiment, we find that $\delta_{\text{calc}} \simeq 0.0604$. As shown in Fig. 2, excellent agreement between the experimental data and the calculated $R(T)$ is obtained over the whole transition region.

To check the quality of the fit, we used the parameters $\gamma = 6500$ and $\delta = 0.0565$ (to be compared with $\delta_{\text{calc}} \simeq 0.0604$) obtained from fitting the experimental data for the measuring current $I = 0.03$ μA [Fig. 2(a)] and calculated the $R(T)$ curve for a different current $I = 0.1$ μA [Fig. 2(b)]. Again, we have obtained quite a good correspondence between the model calculation and the experimental data covering the whole range $0 < R < R_n$. Due to the limitations, imposed on the validity of Eq. (3),[7,8] we cannot use our model to fit the experimental $R(T)$ curves measured at higher current values ($I > 0.3$ μA).

The second crucial test to check the validity of our model is the reproduction of the qualitative changes of the differential resistance dV/dI measured by Santhanam *et al.*[1] The calculated current dependence of dV/dI, at different temperatures and using the same parameter $\delta = 0.0565$, is shown in Fig. 3. The calcu-

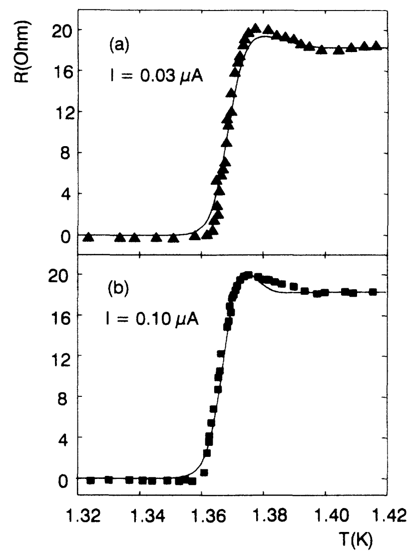

FIG. 2. (a) Temperature dependence of the 1×1 μm^2 loop resistance measured for $I = 0.03$ μA. The solid line shows the fit of the resistance R with the parameters $\gamma = 6500$ and $\delta = 0.0565$. (b) Temperature dependence of the resistance for the Al loop, measured for $I = 0.10$ μA. The solid line shows the calculated curve with the same parameters as used for (a), for the different current $I = 0.10$ μA.

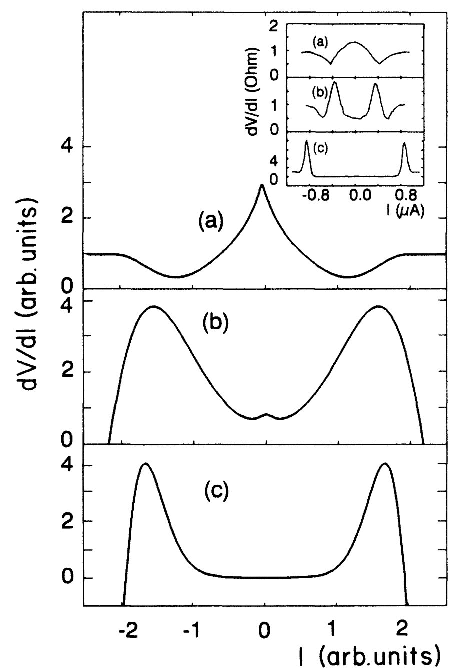

FIG. 3. Qualitative calculation of the current dependence of the differential resistance dV/dI, at fixed temperatures (a) above the anomalous $R(T)$ peak, (b) at the peak, and (c) below the peak. Note that due to the limitations for the current [Eq. (2)], the calculations cannot be expanded to higher currents. The inset shows the dV/dI curves measured by Santhanam *et al.* (Ref. 1) at the same fixed temperatures.

lated curves clearly reproduce all anomalous features, observed by Santhanam *et al.*,[1] including the suppression of the central dV/dI peak [Fig. 3(a)] at lower temperatures [Figs. 3(b) and 3(c)].

In conclusion, we have demonstrated that in mesoscopic superconducting wires the anomalous $R(T)$ peak near the Ginzburg-Landau temperature can be related to the appearance of intrinsic phase slips, below the critical regime. In mesoscopic quasi-1D wires and due to the confinement of the current path, this dominant dissipation mechanism may cause the appearance of resistance values exceeding those of the normal state. In macroscopic wires the temperature range ΔT, where the $R(T)$ peak may be observed, is extremely narrow $[\Delta T \sim (10^{-10}\text{–}10^{-12})T_c]$. In mesoscopic wires ΔT substantially increases and it becomes possible to observe experimentally the puzzling $R(T)$ peak just near T_c.

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[1] P. Santhanam, C. C. Chi, S. J. Wind, M. J. Brady, and J. J. Bucchignano, Phys. Rev. Lett. **66**, 2254 (1991).

[2] H. Vloeberghs, V. V. Moshchalkov, C. Van Haesendonck, R. Jonckheere, and Y. Bruynseraede, Phys. Rev. Lett. **69**, 1268 (1992).

[3] H. J. Fink and V. Grünfeld, Phys. Rev. B **31**, 600 (1985).

[4] A. M. Kadin, W. J. Skocpol, and M. Tinkham, J. Low Temp. Phys. **33**, 481 (1978).

[5] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).

[6] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988).

[7] J. S. Langer and V. Ambegaokar, Phys. Rev. B **164**, 498 (1967).

[8] D. E. McCumber and B. I. Halperin, Phys. Rev. B **1**, 1054 (1970).

[9] A. H. Verbruggen, H. Vloeberghs, P. A. M. Holweg, C. Van Haesendonck, S. Radelaar, and Y. Bruynseraede, Phys. Rev. B **45**, 8799 (1992).

[10] H. M. Jaeger, D. B. Haviland, and A. M. Goldman, Physica B **152**, 218 (1988).

[11] H. Ebisawa, H. Fukuyama, and S. Maekawa, J. Phys. Soc. Jpn. **54**, 2257 (1985).

[12] A. Nordström and Ö. Rapp, Phys. Rev. B **45**, 12 577 (1992).

[13] S. Rubin, T. Schimpfke, B. Weitzel, C. Vossloh, and H. Micklitz, Ann. Phys. (Leipzig) **1**, 492 (1992).

[14] W. E. Masker, S. Marčelja, and R. D. Parks, Phys. Rev. **188**, 745 (1969).

[15] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).

[16] B. I. Ivlev and N. B. Kopnin, Adv. Phys. **33**, 47 (1984).

[17] W. A. Little, Phys. Rev. **156**, 396 (1967).

[18] R. S. Newbower, M. R. Beasley, and M. Tinkham, Phys. Rev. B **5**, 864 (1972).