

## Josephson coupling between a disk of one superconductor and a surrounding superconducting film of different symmetry

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A cylindrical Josephson junction with a spatially dependent Josephson coupling averaging to zero over the length of the junction is studied as a model for the physics of a disk of a  $d$ -wave superconductor embedded in a superconducting film of a different symmetry. It is found that the system always introduces Josephson vortices in order to gain energy at the junction. The critical current is calculated. It is argued that a recent experiment that was claimed to provide evidence for  $s$ -wave superconductivity in  $\text{YBa}_2\text{Cu}_3\text{O}_7$  may also be consistent with  $d$ -wave superconductivity.

In this paper I analyze a theoretical model of an experimental system recently proposed as a test for the presence of  $d$ -wave superconductivity in the cuprate high- $T_c$  superconductors.<sup>1</sup> The proposed experimental system is shown schematically in Fig. 1. (The inner region is drawn as a circle, the region used in Ref. 1 was a hexagon.) The inner region, labeled  $A$ , is a  $c$ -axis-oriented film of a high- $T_c$   $\text{CuO}_2$  superconductor with the  $\text{Cu-O}$  bonds running vertically and horizontally as shown. In the devices actually made<sup>1</sup> the outer region,  $B$ , is another  $c$ -axis film of a high- $T_c$  superconductor with the  $\text{Cu-O}$  bonds rotated by  $45^\circ$  with respect to region  $A$ , as shown. I will also consider the case in which region  $B$  is an  $s$ -wave superconductor. In either case the Josephson coupling alternates in sign as one proceeds around the perimeter of region  $A$ , and averages to zero. The experiment is to measure the critical current of the  $A$ - $B$  interface. A nonzero critical current was argued<sup>1</sup> to be evidence against  $d$ -wave pairing because if one chooses a constant phase  $\phi_A$  in region  $A$  and a constant phase  $\phi_B$  in region  $B$ , then the Josephson energy of the interface vanishes and so there can be no supercurrent across it. This argument is incorrect; the difficulty is the assumption of constant, spatially independent phases  $\phi_A$  and  $\phi_B$ . I shall show that for the two-dimensional geometry of interest, the system introduces Josephson vortices to gain energy at the boundary at the expense of introducing bulk su-

percurrents. The physics is similar to that of inductive effects in a superconducting quantum interference device (SQUID), where one balances the Josephson energy against the magnetic-field energy. The dimensionality is crucial to the argument; the results do not apply to the essentially one-dimensional "corner junction" SQUID experiments recently argued<sup>2</sup> to provide evidence of  $d$ -wave superconductivity in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ .

In the remainder of this paper I analyze the interplay between bulk currents and the Josephson energy more quantitatively, by minimizing the appropriate free energy. The free energy has three contributions: the Josephson energy, the supercurrent energy, and the energy in the magnetic field generated by the supercurrent flow. I first study the simplest case, that of a very thin film for which the penetration depth may be taken to be infinite and all magnetic effects are negligible. Then I consider the general case, and finally discuss the relation to experiments.

Assume first that the only contributions to the energy are the bulk supercurrents (i.e., the kinetic inductance) and the Josephson energy of the  $A$ - $B$  interface. The energy of a bulk supercurrent involves a superfluid stiffness  $\rho$  which I relate to a bulk critical current  $J_c$  by the argument that at the critical current the phase changes by an amount of order 1 in a distance of order the coherence length,  $\xi_0$ , i.e.,  $J_c \xi_0 = \rho$ . The energy  $E$  is then

$$E = \int_0^a r dr d\theta \frac{1}{2} J_c^A \xi_0^A (\nabla \phi^A)^2 + \int_a^\infty r dr d\theta \frac{1}{2} J_c^B \xi_0^B (\nabla \phi^B)^2 + I_J a \int_0^{2\pi} d\theta g_M(\theta) \cos[\phi^A(r=a, \theta) - \phi^B(r=a, \theta)]. \quad (1)$$

Here  $\phi^{A,B}$  are the superconducting phases in regions  $A$  and  $B$ , respectively,  $I_J$  is the maximum Josephson current of an infinitesimal region of the interface, and  $g_M(\theta)$  describes the angular dependence of the Josephson coupling. In the case where  $A$  is  $d$  wave and  $B$  is  $s$  wave, we must on general grounds have  $\int_0^{2\pi} d\theta g_M(\theta) = 0$  and  $g_M(\theta + \pi/2) = -g_M(\theta)$ . I call this case  $M = 2$ . In the case considered in Ref. 1 we must have  $\int_0^{2\pi} d\theta g_M(\theta) = 0$  and  $g_M(\theta + \pi/4) = -g_M(\theta)$ ; I call this  $M = 4$ . For explicit calculations I take the simplest forms consistent with these considerations,<sup>3,4</sup> namely  $g_2(\theta) = \sin(2\theta)$  and  $g_4(\theta) = \sin(4\theta)$ . I indicate the modifications arising for

the alternative case of a piecewise constant  $g_M(\theta)$ .

The theoretical problem is to minimize the energy in (1) subject to the constraints of local current conservation at each point of the  $A$ - $B$  interface and fixed total radial current. Local current conservation implies ( $\hat{n}$  is the outward normal to the  $A$ - $B$  interface at angle  $\theta$ )

$$J_c^A \xi_0^A \hat{n} \cdot \nabla \phi^A = J_c^B \xi_0^B \hat{n} \cdot \nabla \phi^B \quad (2a)$$

$$= I_J g(\theta) \sin[\phi^A(a, \theta) - \phi^B(a, \theta)]. \quad (2b)$$

In terms of the mean radial current density at the  $A$ - $B$  interface,  $I_0$ , the constraint on the total radial current is

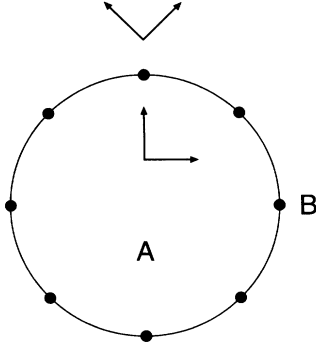


FIG. 1. System modeled in present paper shown for  $d$ -wave to  $d$ -wave case. Region A, inside the circle, is a film of a  $d_{x^2-y^2}$  superconductor with crystal axes running vertically, and horizontally as shown. Region B, outside the circle, is a film of a  $d_{x^2-y^2}$  superconductor with crystal axes rotated by  $45^\circ$ . The dots indicate places where the Josephson coupling across the circle vanishes by symmetry. In the  $d$ -s case the Josephson coupling would vanish only at  $0, \pi$ , and  $\pm\pi/2$ .

$$I_0 = \frac{I_J}{(2\pi)} \int_0^{2\pi} d\theta g(\theta) \sin[\phi^A(a, \theta) - \phi^B(a, \theta)]. \quad (3)$$

To perform the minimization, note that Eq. (1) implies that  $\phi^{A,B}$  satisfy  $\nabla^2\phi = 0$ . Assume that the current  $2\pi I_0 a$  is introduced symmetrically in the center of region A. The problem then has circular symmetry; the general solution to the Laplace equation may be written down, the solutions in regions A and B may be related using  $2a$ , and the result substituted into (1).

The result is

$$\frac{E}{\bar{J}_c \xi_0} = \frac{1}{2} \sum_n n(c_n^2 + d_n^2) - \bar{I} \int_0^{2\pi} \frac{d\theta}{2\pi} g_M(\theta) \cos[\phi(\theta)], \quad (4)$$

where

$$\begin{aligned} \phi(\theta) &= \phi^A(a, \theta) - \phi^B(a, \theta) \\ &= \phi^0 + \sum_n [c_n \sin n\theta + d_n \cos n\theta] \end{aligned} \quad (5)$$

and  $\bar{J} = J_c^A J_c^B / (J_c^A + J_c^B)$  and  $\bar{I} = \pi I_J a / \bar{J} \xi_0$ . Minimizing (5) with respect to the  $c_n$  and  $d_n$  at fixed  $\phi_0$  ensures that all of the  $n > 0$  Fourier components of the current conservation Eq. (2b) are satisfied.

From (4) it is clear that the important parameter is  $\bar{I}$ ; for  $\bar{I} \ll 1$  the Josephson energy is weak while for  $\bar{I} \gg 1$  it is strong. Further,  $\bar{I}$  increases linearly with the perimeter of the disk, because in two dimensions the energy of bulk superflow  $\int d^2r (\nabla\phi)^2$  is scale invariant. Consider first  $\bar{I} \ll 1$ . Clearly the free energy may be expanded in powers of the  $c_n$  and  $d_n$ . Expanding to leading nontrivial order and minimizing gives

$$I_0 = \frac{I_J \bar{I} \sin 2\phi_0}{2M} \quad (6a)$$

and

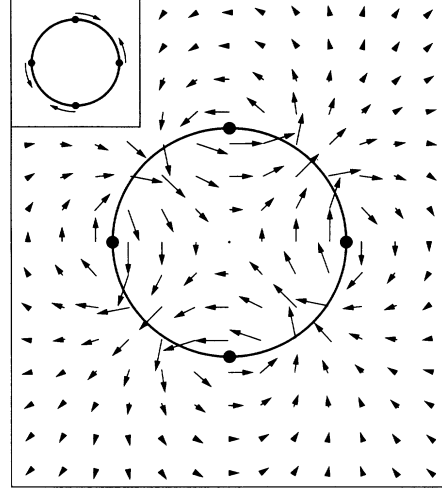


FIG. 2. Pattern of induced currents calculated at  $\phi_0 = \pi/2$  from Eq. (9) for weak Josephson coupling for the  $s$ -wave to  $d$ -wave case. Dots mark places where Josephson coupling vanishes. The inset shows motion of centers of vortices in the presence of imposed outward current.

$$E = -\frac{\bar{I}^2}{4M} \sin^2 \phi_0. \quad (6b)$$

Observe that  $E$  is minimized at a phase difference of  $\pi/2$ . The bulk supercurrents at  $\phi_0 = \pi/2$  are shown in Fig. 2, for  $M = 2$ . One sees that the solution corresponds to  $2M$  vortices, with alternating positive and negative circulation, centered at the points where  $g_M(\theta)$  vanishes. Because  $\bar{I} \ll 1$ , the “core” of the vortex is much larger than the vortex spacing  $a$ . If a mean radial current is imposed,  $\phi_0 \neq \pi/2$  and the centers of the vortices shift along the perimeter of the circle in such a way that the sum of the imposed radial current and the circulating current about each vortex vanishes at the points where  $g_M(\theta) = 0$ . The motion of the centers of the vortices is shown in the inset to Fig. 2. In the case of a piecewise constant  $g_M(\theta)$ , in which  $g_M(\theta) = 1$  ( $0 < \theta < \pi/M$ ),  $g_M(\theta) = -1$  ( $\pi/M < \theta < 2\pi/M$ ), etc., one finds that the energy is still proportional to  $\sin^2\phi_0$  but with a slightly different coefficient, and the qualitative structure of the vortices is not changed.

Note that the energy, Eq. 6(b), is periodic in  $\phi_0$  with period  $\pi$ , unlike a conventional junction which would have a  $2\pi$  periodicity. A  $\pi$  periodicity is unambiguous evidence of unconventional superconductivity and may in principle be measured in a SQUID experiment. Now the quantity  $\phi_0$  is the angular average of the phase drop across the A-B interface, but from the solution of the Laplace equation that led to Eq. (4) I also find that at a distance  $d \ll a$  from the center of the disk, the phase has a negligible angular dependence and is

$$\phi(d) = \phi_0 - (\bar{I}/\pi) \ln a/d. \quad (7)$$

Thus for  $\bar{I} \ll \pi/\ln(a/d) \sim 1$  the phase in the center of the disk is essentially  $\phi_0$  and so an experiment in which this phase is controlled would measure a  $\pi$  periodicity.

Now consider the opposite limit,  $\bar{I} \gg 1$ . Specialize first to the case  $\phi_0 = \pi/2$ ; then it follows from the min-

imization equations that all the  $d_n = 0$ . The Josephson term in Eq. (4) is then minimized by a piecewise constant  $\phi(\theta)$ , equal to  $\pm\pi/2$  according as  $g(\theta)$  is positive or negative. Now a piecewise constant  $\phi(\theta)$  implies  $c_n \sim 1/n$  and therefore a logarithmically divergent  $\sum_n n c_n^2$ . This divergence comes from the familiar  $1/r$  behavior of supercurrents outside the core of a vortex. To cut off the logarithm one must convert the steps in  $\phi(\theta)$  to smooth crossovers extending over a scale  $\epsilon$  which gives the core size of the vortex. For  $g_M(\theta) = \sin(M\theta)$  the cost in Josephson energy is  $\sim \bar{I}\epsilon^2$ , while the superfluid energy is  $\sim \ln 1/\epsilon$ . Optimizing gives  $\epsilon \sim \bar{I}^{-1/2}$  and  $E = -\bar{I} + O \ln \bar{I}$ . Thus at  $\phi_0 = \pi/2$  and  $\bar{I} \gg 1$  the system gains essentially the maximum Josephson energy and the vortices depicted in Fig. 2 are pinched at the  $A$ - $B$  interface into angular regions of size  $\bar{I}^{-1/2}$ . For a piecewise constant  $g$ , the core size would be  $\epsilon \sim \bar{I}^{-1}$  and the coefficient of the  $\ln$  would change by a factor of 2.

To treat general values of  $\phi_0$  at  $\bar{I} \gg 1$  one must write  $\phi(\theta)$  as a sum of two functions, one constant in the interval  $0 < \theta < \pi/M$  and the other piecewise constant and averaging to zero [these represent the sine and cosine Fourier components defined in Eq. (5)]. The details will be given elsewhere; the result is that up to terms of relative order  $[\ln(1/\bar{I})/\bar{I}]^{2/3}$  the energy is independent of the imposed phase  $\phi_0$ . This is related to the result of Eq. (7), namely that for  $\bar{I} \gg 1$ , the phase for any current of order  $I_J$  winds through many  $2\pi$  revolutions between the center of the disk and the perimeter, so the average phase difference  $\phi_0$  is not an easily controlled quantity. To determine the critical current one must subject the energy to a constraint on the total current (which is no longer fixed by  $\phi_0$ ); one finds that the critical current is  $2I_J/\pi$ .

Note that in the  $\bar{I} \gg 1$  case, unlike the  $\bar{I} \ll 1$  case, a solution with  $E < 0$  exists at  $\phi_0 = 0$ . Therefore, as  $\bar{I}$  is increased beyond some critical value,  $\bar{I}_c$ , presumably of order 1 the  $E < 0$  solution must appear. The transition region  $\bar{I} \sim 1$  requires numerical study, which has not been undertaken. However, by expanding Eq. (5) to quadratic order in the  $c$ 's and  $d$ 's I have determined that the solution  $c_n = d_n = 0$  at  $\phi_0 = 0$  becomes linearly unstable at  $\bar{I} = \bar{I}_c \cong 1.53$ . Note also that at large  $\bar{I}$  the  $\pi$  periodicity which still exists in principle is of no practical relevance because if one starts the system in the lowest-energy state at some initial phase  $\phi_0$  and then adiabatically increases the phase to  $\phi_0 + \pi$ , the system will end up in a nonoptimal state (higher than the optimum by a relative energy of order  $[\ln(\bar{I})/\bar{I}]^{2/3}$ ); to get to the optimal state would require overcoming an energy barrier of order  $I_J a$ , which will be much larger than  $k_B T$  for a large device.

I now extend the treatment to include the effects of a finite London penetration depth  $\lambda$  and of the energy stored in the magnetic field. One expects these effects to be important because currents in vortices decay exponentially on scales larger than the effective penetration depth  $\lambda_{\text{eff}}$  (which for a film of thickness  $d < \lambda$  may be greater than the microscopic penetration depth  $\lambda$ ), and because the magnetic field extends also in the third dimension. The most interesting case is  $\lambda_{\text{eff}} \ll a$ ; the currents in the

vortices will then be confined to the region close to the  $A$ - $B$  interface, so the precise geometry will not be important. It is convenient to study the linear interface shown in Fig. 3. Here regions  $A$  and  $B$  are sheets of thickness  $d$  lying in the  $x$ - $y$  plane and occupying the half planes  $x > 0$  and  $x < 0$ , respectively. I assume the Josephson coupling  $I_J(y)$  along the  $A$ - $B$  interface averages to zero, is periodic with period  $2a$ , and satisfies  $I_J(y+a) = -I_J(y)$ . To simplify the analysis I also assume that regions  $A$  and  $B$  have the same material parameters.

I proceed as before, by first determining the general form of the bulk currents and then forcing this form to be consistent with the Josephson relation across the interface. In the presence of a magnetic field  $\mathbf{h}$  one introduces a vector potential  $\mathbf{A}$  via  $\nabla \times \mathbf{A} = \mathbf{h}$  and uses the gauge invariant supercurrent  $\mathbf{J} = \nabla\phi - \frac{2\pi\mathbf{A}}{\Phi_0}$  instead of  $\nabla\phi$ . It is convenient to scale the field by the flux quantum  $\Phi_0 = hc/2e$  and work with  $\mathbf{H} = 2\pi\mathbf{h}/\Phi_0$ . In the volume occupied by the superconductor the equations governing the superflow and field are:  $\nabla \times \mathbf{J} = \mathbf{H}$  and  $\nabla \times \mathbf{H} = \mathbf{J}/\lambda^2$ . Outside the film,  $\mathbf{J} = \mathbf{0}$  and  $\nabla \times \mathbf{H} = \mathbf{0}$ . To simplify the analysis I assume that currents flow only in the  $x$ - $y$  plane. This is equivalent to assuming that  $d \gg \lambda$  or  $d \ll \lambda$ . I give the analysis for the  $d \gg \lambda$  case; the  $d \ll \lambda$  yields essentially identical results, but with  $\lambda$  replaced by  $\lambda_{\text{eff}} = \lambda^2/d$ .<sup>5</sup>

For  $d \gg \lambda$  the solution in the film is only weakly perturbed by the spreading of the field in the region above and below the film. I therefore eliminate  $\mathbf{H}$  from the in-film equations, solve for  $\mathbf{J}$ , compute  $\mathbf{H}$  in the plane of the film, and use this boundary condition for the equation for  $\mathbf{H}$  in the region outside the film. I find

$$J_x(x, y) = \frac{\pi}{2a} \sum_p \frac{b_p \exp\left[-\frac{|x|}{\lambda} \sqrt{1 + b_p^2}\right]}{\sqrt{1 + b_p^2}} \times \left[ c_p \sin \frac{p\pi y}{a} + d_p \cos \frac{p\pi y}{a} \right], \quad (8a)$$

$$J_y(x, y) = \frac{\pi}{2a} \text{sgn}(x) \sum_p \exp\left[-\frac{|x|}{\lambda} \sqrt{1 + b_p^2}\right] \times \left[ d_p \sin \frac{p\pi y}{a} - c_p \cos \frac{p\pi y}{a} \right], \quad (8b)$$

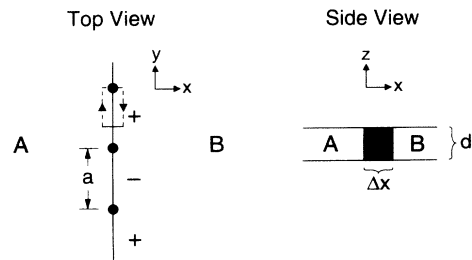


FIG. 3. Linear geometry used. Dots mark points at which a Josephson coupling changes sign. The + and - label the local of the Josephson coupling across interface. The dotted line is the contour used in the computation of  $\Delta\phi_{GI}$ .

where  $b_p = \pi p \lambda / a$ .

To implement the Josephson boundary condition one must compute the properly gauge-invariant phase drop across the interface  $\Delta\phi_{GI}$ . Now  $\Delta\phi_{GI}$  has a contribution  $A^x(\Delta x)$  from the change in the normal component of the vector potential across the interface. For a very thin junction,  $\Delta x \rightarrow 0$  and this contribution may be obtained from the condition that the flux enclosed within the contour shown in Fig. 3 is negligible. (The usual contour extending to  $|x| = \infty$  is not convenient because of the vortex currents.) I find  $\Delta\phi_{GI}(y_2) - \Delta\phi_{GI}(y_1) = \int_{y_1}^{y_2} dy [J^y(+\Delta x, y) - J^y(-\Delta x, y)]$ . The free energy per  $2a$  period, normalized to the film thickness and to  $J_c \xi_0$  is given by the sum of three contributions:  $F = f_{\text{current}} + f_{\text{field}} + f_{\text{Jos}}$  where  $f_{\text{current}} = \int dx dy [\mathbf{J}(x, y)]^2$ ,  $f_{\text{field}} = \frac{\lambda^2}{a} \int d^3r \mathbf{H}(\mathbf{r})^2$ , and  $f_{\text{Jos}} = \bar{I} \int_{-a}^a \frac{dy}{a} g(y) \cos[\Delta\phi_{GI}(\theta)]$ . I have found that  $f_{\text{field}} \leq f_{\text{current}}$ , so I neglect  $f_{\text{field}}$  henceforth. Using Eqs. (8) and the Josephson boundary condition one sees that the free energy has the same form as that previously considered in Eq. (5), except that the supercurrent term becomes

$$f_{\text{curr}} = \frac{\pi \lambda}{4a} \sum_p p^2 \frac{1 + 2b_p^2}{(1 + b_p^2)^{3/2}} [c_p^2 + d_p^2] \quad (9)$$

instead of  $\frac{1}{2} \sum_n n (c_n^2 + d_n^2)$ .

Note that for  $b_p^2 \gg 1$ , Eq. (9) reduces to the previous case, but for  $b_p^2 \ll 1$ , the supercurrent energy is much less than found previously. The previous analysis may now be applied with only minor modifications. The weak-coupling limit previously meant  $\bar{I} < 1$ ; now the condition is  $\bar{I} < \lambda/a$ , which is much more stringent in high- $T_c$  materials where  $\lambda \sim 1400 \text{ \AA}$  and it is difficult to imagine deliberately made devices with scales  $a < 1 \mu\text{m} \sim 10^4 \text{ \AA}$ , although as discussed below wandering of the  $A$ - $B$  interface may introduce a small length scale. In this weak-coupling limit the maximum current is  $\bar{I}^2(a/\lambda)$ , and the solution corresponds to vortices of extent  $\lambda$  in the  $x$  direction and  $a$  in the  $y$  direction. In the range  $\lambda/a < \bar{I} < (a/\lambda)$  a linearized equation cannot be used. Proceeding variationally one finds that modes with  $p < p_{\text{max}} = (\frac{a\bar{I}}{\lambda})^{1/2}$  have non-negligible amplitudes. (Note that  $b_{p_{\text{max}}} < 1$  if  $\bar{I} < a/\lambda$ .) This implies that the scale of a vortex in the transverse ( $y$ ) direction is  $\sim a/p_{\text{max}} \sim (\frac{\lambda a}{\bar{I}})^{1/2} \sim (\frac{\lambda \xi_0 J_c}{\bar{I}})^{1/2}$ , i.e., is the Josephson penetration depth of the junction. The critical current is of order  $I_J$ . Finally, for  $\bar{I} > a/\lambda$  the size of the vortex becomes less than the Josephson penetration depth and the problem becomes identical to the one previously considered; the solution corresponds to vortices of scale  $\lambda$  in the transverse direction, pinched where they cross the  $A$ - $B$  interface.

I now turn to the experiment reported in Ref. 1. For  $\text{YBa}_2\text{Cu}_3\text{O}_7$ ,  $\xi_0 \cong 10 \text{ \AA}$ .  $I_J$  was measured<sup>1</sup> to be about

$2 \times 10^3 \text{ A/cm}^2$ , and the commonly accepted value is  $J_c = 2 \times 10^7 \text{ A/cm}^2$ . Because the films were apparently several thousand angstroms thick, it seems reasonable to use the bulk value  $\lambda \sim 2 \times 10^{-5} \text{ cm}$  for the penetration depth. The circumference  $2\pi$  of the device used in Ref. 1, was  $0.3 \text{ cm}$ . Using these numbers I find  $\bar{I} \sim 10^2$  and  $a/\lambda \sim 2.5 \times 10^3$ . It thus seems that the Josephson energy is dominant, but that the vortex size along the interface is controlled by the Josephson penetration depth  $\lambda_J \sim 10^{-4} \text{ cm}$ . Now in Ref. 1 "region A" was a hexagon and the orientations were such that if the faces of the hexagon were flat on an atomic scale, then the Josephson coupling for two of the faces would vanish. The considerations presented here would then imply that it would carry no supercurrent; the other four would carry a large current. In addition, the vortices at the corners of the hexagons should produce a magnetic field which extends a distance  $\lambda$  away from the corner and  $\lambda_J$  along the edge, and which at the surface would be of order  $\Phi_0/4\lambda\lambda_J \sim 10 \text{ G}$ . In fact, the critical current through the individual faces was measured by destroying one face after another (via laser ablation) and monitoring the total critical current. The face-to-face variation of the critical current was found to be small, and for no face was the critical current found to be zero. This is not compatible with  $d$ -wave superconductivity if the interface is flat. However, if the interface wanders then the situation is different. In the rough interface case one might imagine that the Josephson coupling varies as one moves along the perimeter of the hexagon, with the rms value of the local coupling being similar for each face. If the length scale  $L$  over which the coupling varies is larger than the Josephson penetration depth then the  $\bar{I} \gg 1$  limit still applies with the scale  $a$  replaced by  $L$ , and the experimental results might be consistent with  $d$ -wave superconductivity.

Several extensions of the present work would be of use. A detailed examination of the hexagonal geometry used in Ref. 1 should be performed. Also, a numerical solution in the region  $\bar{I} \sim \bar{I}_c \cong 1.53$  would be of interest. Finally, the dynamics of the system might be worth examining. If the system is forced to have a current greater than the critical current, voltage drops must occur. Presumably the bulk superconducting regions will be equipotentials, so there will be an angle-independent voltage drop across the  $A$ - $B$  interface and the vortices shown in Fig. 2 might for small voltages precess about the circle.

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