Correlation function of finite two-dimensional superconductors

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The superconducting correlation function of finite two-dimensional systems is calculated within the framework of the Ginzburg-Landau theory in the Gaussian approximation. A crossover is found from zero-dimensional behavior at low temperatures to two-dimensional behavior at higher temperatures. The relation between this approach and the Kosterlitz-Thouless theory is briefly discussed.

I. INTRODUCTION

The discovery of the cuprate superconductors has renewed the interest in two-dimensional superconductivity since the highly anisotropic cuprates consist of weakly coupled superconducting planes. It was shown by Hohenberg[1] that true long range order of the superconducting order parameter does not exist in two dimensions. On the other hand, superconductivity was found experimentally in very thin films of both conventional and cuprate superconductors.

There are two major approaches to overcome this apparent paradox. The first one is based on the realization that superconductivity does not require long range order, but that quasi-long-range order suffices, characterized by a rational decay of the correlation function of the order parameter. Quasi-long-range order is brought about by vortex fluctuations, as described by the Kosterlitz-Thouless (KT) theory.[2,3]

The other approach is based on finite size effects. The basic idea is that in sufficiently small superconducting systems the order parameter varies only weakly over the whole system. The superconductor is then thought of as an effectively zero-dimensional system, which is exhibiting "long range order." Along this line of thought Bandte and Appel[4,5] investigate two superconducting planes with Josephson coupling, using the method of Hassing and Wilkins.[6] They consider fluctuations in the modulus of the order parameter up to fourth order in a self-consistent manner, taking also the Coulomb interaction, i.e., charging effect, into account. They ignore phase fluctuations in the planes. For increasing temperatures a crossover from zero-dimensional to two-dimensional behavior is found. It is shown that the results depend only weakly on Coulomb effects.

In this paper we discuss the size effects in the Gaussian approximation.

II. CALCULATION OF THE CORRELATION FUNCTION

We consider a two-dimensional square superconductor with linear dimensions L. The Ginzburg-Landau functional of the free energy is

$$\mathcal{F} = \int d^2r \left[\alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \gamma\left|\left(\nabla - \frac{2\pi i}{\phi_0}\mathbf{A}\right)\Psi\right|^2\right] + d\int d^2r\, \frac{B^2}{8\pi}, \tag{1}$$

where Ψ is the superconducting order parameter, α, β, and γ are the Ginzburg-Landau parameters, d is the film thickness, ϕ_0 is the flux quantum, $\mathbf{A}$ is the vector potential, and $\mathbf{B}$ is the magnetic field. The range of integration is over the square $-L/2 < r_1 < L/2$ and $-L/2 < r_2 < L/2$. We assume that there is no externally applied magnetic field. The fields $\mathbf{A}$ and $\mathbf{B}$ are thus entirely due to fluctuations. The typical length scale of spatial variations of these fields is the magnetic penetration depth λ, whereas the order parameter varies on the scale of the Ginzburg-Landau coherence length ξ. Since we are especially interested in thin films of cuprate superconductors, where $\lambda \gg \xi$, magnetic field fluctuations are ignored. Hence we replace the magnetic field by its average $\langle\mathbf{B}\rangle = 0$. We choose the gauge in such a way that $\langle\mathbf{A}\rangle$ also vanishes.

The free-energy functional then becomes

$$\mathcal{F} = \int d^2r \left(\alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \gamma|\nabla\Psi|^2\right). \tag{2}$$

The parameter α is negative below the Ginzburg-Landau critical temperature T_{c0} and becomes zero at T_{c0}. Below the critical temperature minimization of $\mathcal{F}$ with respect to Ψ yields the equilibrium value $\Psi_0 = -\alpha/\beta$. From hereon, we will only consider the case $T < T_{c0}$.

To investigate the fluctuations of the order parameter about Ψ_0 we use methods similar to those of Rice.[7] We define the fluctuations of the modulus $\psi(\mathbf{r})$ and of the phase $\varphi(\mathbf{r})$ by the equation

$$\Psi(\mathbf{r}) = [\Psi_0 + \psi(\mathbf{r})]e^{i\varphi(\mathbf{r})}. \tag{3}$$

Introducing this expression into Eq. (2) we obtain up to second order in the fluctuations

$$\begin{aligned}\mathcal{F} &= \int d^2r \left(\alpha\Psi_0^2 + \frac{\beta}{2}\Psi_0^4 - 2\alpha\psi^2 + \gamma(\nabla\psi)^2 + \gamma\Psi_0^2(\nabla\varphi)^2\right) \\ &= \text{const} + \int d^2r[-2\alpha\psi^2 + \gamma(\nabla\psi)^2 + \gamma\Psi_0^2(\nabla\varphi)^2] .\end{aligned} \tag{4}$$

We omit the third and fourth order terms, thereby restricting ourselves to Gaussian fluctuations.

We now proceed to calculate the correlation function

$$G(\mathbf{R}) = \langle \Psi^*(\mathbf{R})\Psi(0)\rangle, \tag{5}$$

where we omit all factors which are independent of $\mathbf{R}$, since one could easily normalize $G(\mathbf{R})$ afterwards by requiring $G(0) = 1$.

Keeping this in mind, we have

$$\begin{aligned} G(\mathbf{R}) &\propto \int D\Psi\; \Psi^*(\mathbf{R})\Psi(0) \exp\left(-\frac{F}{k_B T}\right) \\ &\propto \int D\psi\, D\varphi\, [\Psi_0 + \psi(\mathbf{R})][\Psi_0 + \psi(0)]\, e^{i[\varphi(0)-\varphi(\mathbf{R})]} \exp\left[-\frac{1}{k_B T}\int d^2r[-2\alpha\psi^2 + \gamma(\boldsymbol{\nabla}\psi)^2 + \gamma\Psi_0^2(\boldsymbol{\nabla}\varphi)^2]\right] , \\ &= G_\varphi G_\psi, \end{aligned} \tag{6}$$

where

$$G_\varphi(\mathbf{R}) = \int D\varphi\, e^{i(\varphi(0)-\varphi(\mathbf{R}))} \exp\left(-\frac{1}{k_B T}\int d^2r\, \gamma\Psi_0^2(\boldsymbol{\nabla}\varphi)^2\right) \tag{7}$$

and

$$G_\psi(\mathbf{R}) = \int D\psi\, [\Psi_0 + \psi(\mathbf{R})][\Psi_0 + \psi(0)] \exp\left[-\frac{1}{k_B T}\int d^2r[-2\alpha\psi^2 + \gamma(\boldsymbol{\nabla}\psi)^2]\right]. \tag{8}$$

Thus the correlation function factorizes into a phase part and a modulus part. These two factors are treated separately.

To derive the phase correlator G_φ, the phase is written in terms of its Fourier components $\varphi_{\mathbf{k}}$:

$$\varphi(\mathbf{r}) = \sum_{\mathbf{k}} \varphi_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{9}$$

where the sum is over all two-dimensional vectors $\mathbf{k}$ with $\mathbf{k} = (2\pi m/L, 2\pi n/L)$ (with integers m and n) and $|\mathbf{k}| \leq \pi/\xi_0$. The discreteness of $\mathbf{k}$ is due to the finite system size. The cutoff at large values of the wave vector is introduced, because fluctuations on length scales smaller than the coherence length at zero temperature, ξ_0, are not treated within the Ginzburg-Landau theory.

Since the phase $\varphi(\mathbf{r})$ is a real function, its Fourier components $\varphi_{\mathbf{k}}$ and $\varphi_{-\mathbf{k}}$ do not describe independent modes, but fulfill the equation $\varphi_{-\mathbf{k}} = \varphi^*_{\mathbf{k}}$. We may integrate over independent modes only and, therefore, the phase correlator is given by

$$\begin{aligned} G_\varphi \propto \int \left(\prod_{\mathbf{k}}{}' d^2\varphi_{\mathbf{k}}\right) \exp\Bigg[& i\sum_{\mathbf{k}} (\varphi_{\mathbf{k}} - \varphi_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{R}}) \\ & - \frac{\gamma\Psi_0^2}{k_B T}\int d^2r \sum_{\mathbf{k}} \varphi_{\mathbf{k}} \boldsymbol{\nabla} e^{i\mathbf{k}\cdot\mathbf{R}} \sum_{\mathbf{k}'} \varphi_{\mathbf{k}'} \boldsymbol{\nabla} e^{i\mathbf{k}'\cdot\mathbf{R}}\Bigg], \end{aligned} \tag{10}$$

where the primed product is only over the $\mathbf{k} = (k_1, k_2)$ with $k_1 > 0$ or $\mathbf{k} = (0, k_2)$ with $k_2 > 0$. The integration over the mode $\mathbf{k} = 0$ is neglected since it only contributes a constant factor. For every mode the integration is over the entire complex plane.

After carrying out the integration in the exponential, the off-diagonal terms in the double sum over $\mathbf{k}$ and $\mathbf{k}'$ are found to vanish and we obtain

$$\begin{aligned} G_\varphi \propto \int \left(\prod_{\mathbf{k}}{}' d^2\varphi_{\mathbf{k}}\right) \exp\Bigg[\sum_{\mathbf{k}}\Bigg(& i\left(1 - e^{i\mathbf{k}\cdot\mathbf{R}}\right)\varphi_{\mathbf{k}} \\ & - \frac{\gamma\Psi_0^2}{k_B T}|\varphi_{\mathbf{k}}|^2 k^2 L^2\Bigg)\Bigg]. \end{aligned} \tag{11}$$

This integral factorizes:

$$\begin{aligned} G_\varphi \propto \prod_{\mathbf{k}}{}' \int d^2\varphi_{\mathbf{k}} \exp\Bigg[& i\left(1 - e^{i\mathbf{k}\cdot\mathbf{R}}\right)\varphi_{\mathbf{k}} \\ & + i\left(1 - e^{-i\mathbf{k}\cdot\mathbf{R}}\right)\varphi^*_{\mathbf{k}} - \frac{2\gamma\Psi_0^2 L^2}{k_B T} k^2 |\varphi_{\mathbf{k}}|^2\Bigg]. \end{aligned} \tag{12}$$

Decomposition of the phase $\varphi_{\mathbf{k}}$ into its real and imaginary parts and subsequent integration yields

$$\begin{aligned} G_\varphi &\propto \prod_{\mathbf{k}}{}' \exp\left[-\frac{k_B T}{\gamma\Psi_0^2 L^2 k^2}\left(1 - \cos\mathbf{k}\cdot\mathbf{R}\right)\right] \\ &= \exp\left(-\frac{k_B T}{\gamma\Psi_0^2 L^2}\sum_{\mathbf{k}}{}' \frac{1 - \cos\mathbf{k}\cdot\mathbf{R}}{k^2}\right), \end{aligned} \tag{13}$$

where the primed sum is over the same $\mathbf{k}$ as in the product in Eq. (10).

We assume that the system is large compared to the coherence length at zero temperature, i.e., $L \gg \xi_0$. Therefore, the sum in Eq. (13) contains many terms. We first consider the case $R \gg \xi_0$. In this case we may approximate the sum by an integral over a semicircle,

$$\begin{aligned} \sum_{\mathbf{k}}{}' \frac{1 - \cos\mathbf{k}\cdot\mathbf{R}}{k^2} &\cong \int{}' \frac{d^2k}{\frac{4\pi^2}{L^2}}\, \frac{1 - \cos\mathbf{k}\cdot\mathbf{R}}{k^2} \\ &= \frac{L^2}{4\pi}\int_0^{\pi/\xi_0} dk\, \frac{1 - J_0(kR)}{k}. \end{aligned} \tag{14}$$

Utilizing the fact that $R \gg \xi_0$ we approximate this integral by

$$\frac{L^2}{4\pi}\int_0^{\pi/\xi_0} dk\, \frac{1 - J_0(kR)}{k} \cong \frac{L^2}{4\pi}\ln\frac{\pi R}{L} + \text{const}, \tag{15}$$

where $R = |\mathbf{R}|$. Substitution into Eq. (13) gives the phase correlator

$$G_\varphi(\mathbf{R}) \propto \left(\frac{\pi R}{L}\right)^{-k_B T/4\pi\gamma\Psi_0^2}. \tag{16}$$

This expression is in agreement with the result of Rice.[7]

We now consider the phase correlator at small values of R. For sufficiently small R the sum of Eq. (13) can be expanded and gives, to lowest order in R,

$$\sum_{\mathbf{k}}{}' \frac{1-\cos\mathbf{k}\cdot\mathbf{R}}{k^2} \cong \frac{1}{2}\sum_{\mathbf{k}}{}' \frac{(\mathbf{k}\cdot\mathbf{R})^2}{k^2}. \tag{17}$$

Since we are only interested in the dependence of G_φ on the modulus of $\mathbf{R}$, but not on the angle, we average over the angle to obtain

$$\overline{\sum_{\mathbf{k}}{}' \frac{1-\cos\mathbf{k}\cdot\mathbf{R}}{k^2}} \cong \frac{1}{2}\sum_{\mathbf{k}}{}' \frac{R^2(k_x^2+k_y^2)}{2k^2} = \frac{R^2}{4}\sum_{\mathbf{k}}{}' 1 \tag{18}$$

and

$$\overline{G_\varphi} \propto \exp\left(-\frac{k_BT}{\gamma\Psi_0^2L^2}\frac{R^2}{4}\sum_{\mathbf{k}}{}' 1\right) = \exp\left(-\frac{R^2}{R^{*2}}\right), \tag{19}$$

where the typical length scale is given by

$$R^{*2} = \frac{4\gamma\Psi_0^2L^2}{k_BT\sum_{\mathbf{k}}' 1}. \tag{20}$$

The phase correlator shows Gaussian behavior, Eq. (19), at small distances and a rational decay to zero, Eq. (16), for larger distances. Using straightforward algebra and utilizing the inequality $\xi_0 \ll L$, we obtain

$$\sum_{\mathbf{k}}{}' 1 \cong \frac{\pi L^2}{8\xi_0^2} \tag{21}$$

and, with Eq. (20),

$$R^* = 4\sqrt{\frac{2\gamma\Psi_0^2}{\pi k_BT}}\,\xi_0. \tag{22}$$

Before this result is discussed, we briefly turn to the modulus correlation function G_ψ, Eq. (8). Using similar techniques to those applied above, we obtain

$$G_\psi \propto \Psi_0^2 + \frac{k_BT}{L^2}\sum_{\mathbf{k}}{}' \frac{\cos\mathbf{k}\cdot\mathbf{R}}{\gamma k^2 - 2\alpha}. \tag{23}$$

We omit the derivation of Eq. (23), because the modulus part of the correlator does not fall to zero for large distances and thus does not destroy the long range order. The dimensional crossover discussed here is entirely due to the phase part, G_φ.

What is the physical meaning of the characteristic length R^*? If a given system is much smaller than R^*, the correlation function will vary only weakly over the whole system, and, therefore, the system is essentially zero dimensional. On the other hand, if the sample is much larger than R^*, the rational decay of the correlation function is evident from Eq. (16) and the system displays two-dimensional behavior. Hence we expect a dimensional crossover for linear dimensions L of the order of R^*.

The crossover length R^* depends on the temperature through the terms $\gamma\Psi_0^2$ and k_BT. Utilizing well-known equations from Ginzburg-Landau theory,[8] we may express R^* in terms of measurable quantities,

$$R^* = 2\sqrt{\frac{d}{\pi^2 k_BT}}\,H_c(T)\,\xi_0\xi(T), \tag{24}$$

where H_c is the (bulk) thermodynamic critical field, ξ is the temperature-dependent Ginzburg-Landau coherence length, and d is the thickness of the film. At temperatures not too far below T_{c0}, the temperature dependence of these quantities is given by[8]

$$H_c(t) \approx H_c(0)\,(1-t^2), \tag{25}$$

$$\xi(t) \approx \frac{\xi_0}{\sqrt{1-t}}, \tag{26}$$

where $t = T/T_{c0}$. Substitution into Eq. (24) yields

$$R^* \approx 2\xi_0^2 H_c(0)\sqrt{\frac{d}{\pi^2 k_BT_{c0}}}\,\frac{1-t^2}{\sqrt{(1-t)t}}. \tag{27}$$

In Fig. 1 a crossover diagram is given for parameters suitable for a one-unit-cell-thick $YBa_2Cu_3O_7$ film. The dashed line denotes the crossover region at $L \approx R^*$. This line does of cause not separate two thermodynamic phases. The very concept of phase transitions is meaningful only for infinite systems. Also, this line is based upon Ginzburg-Landau theory and, therefore, it is dubious at low temperatures. We would like to point out that Bandte[5] found a crossover in the same region of linear dimensions $L \sim 10^2$–10^3 Å.

Up to this point only Gaussian fluctuations were considered. It is well known that vortex fluctuations also play an important role in two-dimensional superconductors. This type of fluctuation is described by the KT theory.[2,3] Below the KT temperature T_{KT} all vortices and antivortices are bound in pairs, whereas above T_{KT} both

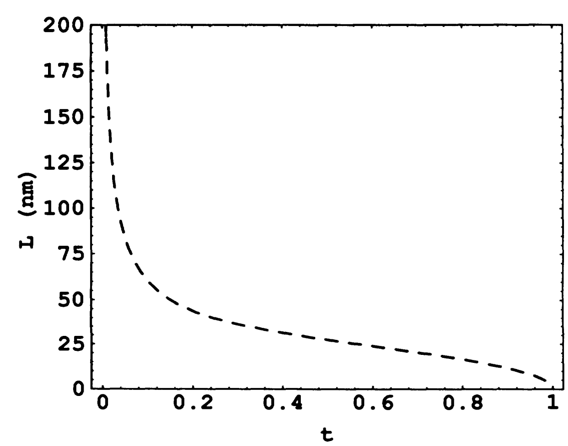

FIG. 1. "Phase diagram" for a finite, two-dimensional superconductor. The dashed line denotes the dimensional crossover region.

bound pairs and free (anti)vortices exist and the quasi-long-range order is destroyed. The bound pairs at lower temperatures lead to a renormalization of the thermodynamic properties, such as the exponent of the rational decay, Eq. (16). This renormalization is small, except very close to the phase transition. The KT temperature for most materials, probably including $YBa_2Cu_3O_7$ films, is lower than T_{c0} only by a few percent. Thus we do not expect the previous results to be changed qualitatively by fluctuating vortices, except for very small samples. For very small systems, however, the KT theory becomes increasingly inaccurate, because fewer and fewer vortices are present.

In summary, we calculate the superconducting correlation function of a finite, two-dimensional system. It is found to have a Gaussian form at small distances and to decay rationally at large distances. A crossover from effectively zero-dimensional behavior at low temperatures to two-dimensional behavior is obtained. At even higher temperatures, but below the bulk transition temperature, a smeared out Kosterlitz-Thouless transition destroys the rational decay of the correlation function and leads to short range order.

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