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Andreev level spectrum and Josephson current in a superconducting ballistic point contact

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Using a transfer-matrix method, we study the Josephson current in a normal, clean, one-dimensional quantum channel in contact with two superconducting electrodes characterized by a finite value of the ratio between the gap Δ and the Fermi energy μ. Going beyond the "Andreev approximation," gaps open up in the quasiparticle level spectrum, the critical current is reduced, and the relationship between current and superconducting phase difference ϕ becomes more Josephson-like without any discontinuous jump at $\phi = \pi$. The mechanism behind these effects is the normal reflection at the interfaces due to a finite value of Δ/μ. The gap at $\phi = \pi$ goes to zero only at transmission resonances $k_F L = (2n+1)\pi/2$.

In recent years there has been a growing interest in superconducting-normal-superconducting (SNS) junctions from a mesoscopic point of view.[1–8] Concepts developed from investigations of electron transport in normal (nonsuperconducting) systems have been shown to have analogs in the superconducting case. One example is the discretization of the critical current in superconducting ballistic point contacts,[1] which is the analog of the quantization of conductance in normal ballistic point contacts. Another example is the critical current fluctuations in disordered SNS junctions,[2] which is the analog of universal sample-to-sample fluctuations in disordered metals. In this context one should also mention recent work on superconductor-semiconductor-superconductor (SSmS) junctions[7] and on the transient response of the Josephson current in an SNS junction to an applied electromagnetic field.[8]

Superconducting transport in SNS junctions is possible because of Andreev reflection,[9] which is a fundamental property of the superconducting state. SNS junctions could therefore be useful in studying the symmetry of the order parameter in cuprate superconductors.[10] There exist a few recent experiments on Y-Ba-Cu-O (YBCO) (Ref. 11) and Nb (Ref. 12) break junctions, where evidence for current transport through Andreev reflection is found.

In this paper we study Andreev levels and Josephson current in a one-dimensional *ballistic* (clean) SNS junction by solving the Bogoliubov–de Gennes (BdG) equation both analytically and numerically using a transfer-matrix technique. The new aspect is that we properly take into account the scattering associated with the localization of quasiparticles in the normal (N) region. The quasiparticles with energies in the superconducting gap Δ are decaying into the superconducting (S) region over a coherence length $\xi_0 \sim (1/k_F)\mu/\Delta$, where k_F is the Fermi wave vector and μ is the Fermi energy. In most investigations of electron transport in SNS junctions the "Andreev approximation" is used.[13] This means, crudely speaking, that only the wave functions, not the derivatives, are matched at the two SN interfaces. Using this approximation one leaves out effects of scattering and mismatch associated with a finite value of Δ/μ. In our formulation of the problem, the transfer matrix contains the matching of both wave functions and derivatives. In addition to numerical calculations, to explicitly show how Δ/μ corrections appear we also specialize to a highly idealized situation of a clean and short normal region and solve this problem analytically.

Our approach is equivalent to that of Ref. 7 for the SSmS junction in the sense that the BdG equation is solved without approximations. Technically, the difference is that we develop a transfer-matrix approach and present an analytic solution for the short junction. The difference in scope is that, in the present paper, we consider a ballistic junction and focus attention on wave vector mismatch and reflectivity that arise when going from the normal-normal-normal (NNN) to the superconducting-normal-superconducting (SNS) state. Related to the SNS junction is the problem of current transport in normal films sandwiched between superconducting sheaths. A detailed three-dimensional analysis beyond the Andreev aproximation of such a system was given in Ref. 14.

In a short SNS junction the main part of the current is carried by superconducting bound states associated with the normal region. The process that makes transport possible is *Andreev reflection*:[9,15] a right-going electron in the normal region, with energy in the superconducting gap, hits the SN interface and transforms into a left-going hole. The hole itself transforms into a right-going electron at the opposite interface, and so on. These processes will then constitute a state responsible for current transport in one direction, say positive direction. There is also a state responsible for current transport in the negative direction, namely, the time-reversed state made out of a left-going electron and a right-going hole. The essential point in the picture above is that the two states are degenerate (and uncoupled).

However, there is also a probability amplitude for the right-going electron being *normally reflected* into a left-going electron. The amplitude for normal reflection is of the order Δ/μ. This reflection couples the two states (different current directions). The effect of this coupling

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becomes noticeable when the the energy of the Andreev levels approaches zero, which happens for phase difference $\phi = \pm\pi$. We will show that this gives rise to energy gaps of magnitude Δ^2/μ in the Andreev level spectrum. As a result, the current-phase relationship will be continuous and the critical current diminished: even if the normal junction is ballistic (NNN), there is a residual, intrinsic reflectivity in the superconducting SNS case and one cannot reach the ballistic limit (except in special cases of transmission resonance). The behavior is therefore analogous, in principle, to the usual SNS case with finite reflectivity due to, e.g., barriers, different Fermi velocities or impurities.[1–8]

An equivalent view of the SNS junction is to consider both the positive and negative Andreev levels for a given phase difference (semiconductor picture). These levels become degenerate in a curve crossing at $\phi = \pi$. However, introduction of normal reflectivity induces level splitting, the energy-phase curves become noncrossing, and a gap appears at $\phi = \pi$.

The pair potential of the SNS-system, shown in Fig. 1, is assumed to take the form $\Delta_{1,3} = \Delta \exp(i\phi_{1,3})$ in the superconducting regions 1 and 3. In the normal region 2, $\Delta = 0$ by definition. This model of the pair potential does not take into account proximity effects occurring at the SN interfaces, but is often used in the literature.[13] The cross-section area of the junction is assumed to be less than the penetration depth, so that the vector potential can be neglected.[16] The wave functions are found from the time-independent Bogoliubov-de Gennes equation[17]

$$\begin{pmatrix} \mathcal{H}_0 & \Delta(x) \\ \Delta^*(x) & -\mathcal{H}_0 \end{pmatrix} \Psi = E\Psi, \tag{1}$$

where $\Psi(x)$ is a two-component wave function and $\mathcal{H}_0 = p_x^2/2m - \mu$ is the one-electron Hamiltonian. The positive eigenvalues E of Eq. (1), measured relative to the Fermi energy μ, determine the excitation spectrum of the system. In this paper we focus on the discrete part of the excitation spectrum, which means $E < \Delta$. The excitation spectrum E depends on the superconducting phase difference $\phi = \phi_3 - \phi_1$. From this dependence one calculates the current:[2]

$$I = -\frac{2e}{\hbar} \sum_n \tanh\left(E_n/2k_BT\right) \frac{dE_n}{d\phi}. \tag{2}$$

The sum in Eq. (2) is over the discrete energy levels determined from Eq. (1). In addition to Eq. (2) there is also a contribution to the current from the continuous part ($E > \Delta$) of the excitation spectrum, which we do not investigate here. The contribution from the continuous spectrum has been shown to be of minor importance in the case of short junctions.[1–3]

To produce wave functions Ψ of the whole SNS structure, we proceed in a way similar to Refs. 2, 3, 13, and 15. We differ by including the continuity condition for the derivative. Below we sketch the procedure.

In the normal region 2 the unnormalized eigenfunctions of Eq. (1) are

$$\begin{aligned} \Psi^{\pm}_{2,e} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(\pm i q_e x), \; q_e = k_F\sqrt{1 + E/\mu}, \\ \Psi^{\pm}_{2,h} &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(\pm i q_h x), \; q_h = k_F\sqrt{1 - E/\mu}, \end{aligned} \tag{3}$$

where $k_F = \sqrt{2m\mu}/\hbar$.

In the superconducting region 1 the unnormalized eigenfunctions of Eq. (1) are

$$\begin{aligned} \Psi^{+}_{1,e} &= \begin{pmatrix} u e^{i\phi_1} \\ v \end{pmatrix} \exp(ik_e x), \\ k_e &= k_F\left[1 + i\delta\sqrt{1 - (E/\Delta)^2}\right]^{1/2}, \\ \Psi^{-}_{1,h} &= \begin{pmatrix} v e^{i\phi_1} \\ u \end{pmatrix} \exp(-ik_h x), \; k_h = (k_e)^*, \end{aligned} \tag{4}$$

where

$$u^2 = \{1 + [1 - (\Delta/E)^2]^{1/2}\}/2,$$

$$v^2 = \{1 - [1 - (\Delta/E)^2]^{1/2}\}/2,$$

and $\delta = \Delta/\mu$. In the superconducting region 3 the eigenfunctions $\Psi^{-}_{1,e}$ and $\Psi^{+}_{3,h}$ are found by replacing 1 by 3 and x by $-x$ in Eq. (4). The form of the functions Ψ_1 and Ψ_3 guarantees a decaying behavior when $|x| \to \infty$. Now we make the ansatz

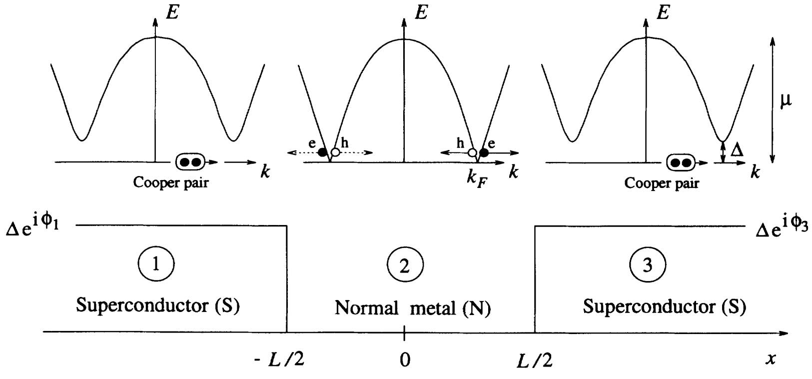

FIG. 1. Layout of the SNS system. A Cooper pair is incident from the left contact and transforms at the left SN interface to a right-going electron and a left-going hole (solid arrows). At the right interface the electron and the hole recombine to a Cooper pair. In addition, the electron/hole is normally reflected into an electron/hole (dashed arrows) at the interfaces, if Δ/μ is not low.

$$\Psi(|x| < L/2) = a_2\Psi^+_{2,e} + b_2\Psi^-_{2,h} + c_2\Psi^-_{2,e} + d_2\Psi^+_{2,h},$$
$$\Psi(x < -L/2) = c_1\Psi^+_{1,e} + d_1\Psi^-_{1,h}, \tag{5}$$
$$\Psi(x > L/2) = a_3\Psi^-_{3,e} + b_3\Psi^+_{3,h}.$$

The next step is to match both the ansatz and the derivative at the SN interfaces $|x| = L/2$. Eliminating a_2, b_2, c_2, and d_2, one ends up with the following equations for the coefficients c_1, d_1, a_3, and b_3:

$$\mathbf{M}\begin{pmatrix} 0 \\ 0 \\ c_1 \\ d_1 \end{pmatrix} = \begin{pmatrix} a_3 \\ b_3 \\ 0 \\ 0 \end{pmatrix}. \tag{6}$$

$\mathbf{M} \equiv \mathbf{M}(E)$ is the 4×4 transfer matrix associated with the clean SNS structure. Besides Eq. (6) there is also a normalization condition for the wave function, which in this work is of no interest. We do not need all the components M_{ij}, since the condition for having nonzero solutions c_1, d_1, a_3, and b_3 is

$$M_{33}M_{44} - M_{34}M_{43} = 0. \tag{7}$$

Equation (7) determines the excitation spectrum for the bound Andreev levels of the SNS structure. The matrix elements of Eq. (7) are given below:

$$\begin{aligned} M_{33} &= e^{ik_eL}(u^2 - v^2)^{-1}\left[u^2e^{-i\phi}A_{33} - v^2A^*_{44}\right], \\ M_{44} &= e^{-ik_hL}(u^2 - v^2)^{-1}\left[u^2A_{44} - v^2e^{-i\phi}A^*_{33}\right], \\ M_{34} &= e^{ik_eL}(u^2 - v^2)^{-1}uv\left[e^{-i\phi}A_{34} - A^*_{43}\right], \\ M_{43} &= e^{-ik_hL}(u^2 - v^2)^{-1}uv\left[A_{43} - e^{-i\phi}A^*_{34}\right], \end{aligned} \tag{8}$$

with

$$\begin{aligned} A_{33} &= \cos(q_eL) - i2^{-1}\left[\frac{q_e}{k_e} + \frac{k_e}{q_e}\right]\sin(q_eL), \\ A_{44} &= \cos(q_hL) + i2^{-1}\left[\frac{q_h}{k_h} + \frac{k_h}{q_h}\right]\sin(q_hL), \\ A_{34} &= 2^{-1}\left[1 - \frac{k_h}{k_e}\right]\cos(q_eL) \\ &\quad -i2^{-1}\left[\frac{q_e}{k_e} - \frac{k_h}{q_e}\right]\sin(q_eL), \\ A_{43} &= 2^{-1}\left[1 - \frac{k_e}{k_h}\right]\cos(q_hL) \\ &\quad +i2^{-1}\left[\frac{q_h}{k_h} - \frac{k_e}{q_h}\right]\sin(q_hL). \end{aligned} \tag{9}$$

In the following we study Eq. (7) together with Eq. (2).

First we note that the Andreev approximation means that k_e, k_h, q_e, and q_h in the prefactors multiplying the trigonometric functions in Eq. (9) are all put to k_F. We then get $A_{33} = \exp(-iq_eL)$, $A_{44} = \exp(iq_hL)$, and $A_{34} = A_{43} = 0$. We substitute these A coefficients into Eq. (8) and obtain the result of Kulik:[13]

$$2\arccos(E/\Delta) - (q_e - q_h)L \pm \phi = n2\pi, \tag{10}$$

where $n = 0, \pm1, \pm2, \ldots$. In the limit $L \to 0$ one gets the simple result $E = \Delta\cos(\phi/2)$.

How is Eq. (10) affected if the Andreev approximation is not used? To answer this question we first solve Eq. (7) for $L = 0$. In this case the solutions can be written down analytically, and we choose the solution that fulfills $0 \le E \le \Delta$:

$$E(\phi) = \frac{\Delta}{2}\left\{3 + \cos\phi + 2\delta^{-2}\left[1 - \sqrt{1 + \delta^2(1 - \cos\phi)}\right]\right\}^{1/2}. \tag{11}$$

The limit $L = 0$ is somewhat unphysical, since it means that L is smaller than the smallest length scale of the problem k_F^{-1}. However, we find Eq. (11) useful as a starting point for discussing the implications of a finite δ. Taking the limit $\delta \to 0$ of Eq. (11), $E = \Delta\cos(\phi/2)$ is reproduced. The dispersion relation of Eq. (11) exhibits a minigap feature at $\phi = \pi$, see Fig. 2. Evaluating Eq. (11) for small δ at $\phi = \pi$, we have directly for the minigap

$$E_g = E(\pi) = \frac{\Delta^2}{2\mu}. \tag{12}$$

This minigap is absent in treatments based on the Andreev approximation. It is interesting to compare with the bound quasiparticle states of vortices in type-II superconductivity.[18] Also in that case there is a minigap, which is of the same order of magnitude as in Eq. (12).

In the case of nonzero L, more Andreev levels are gradually trapped in the normal region with increasing L, see Fig. 2. The minigap oscillates on the scale of k_F^{-1},

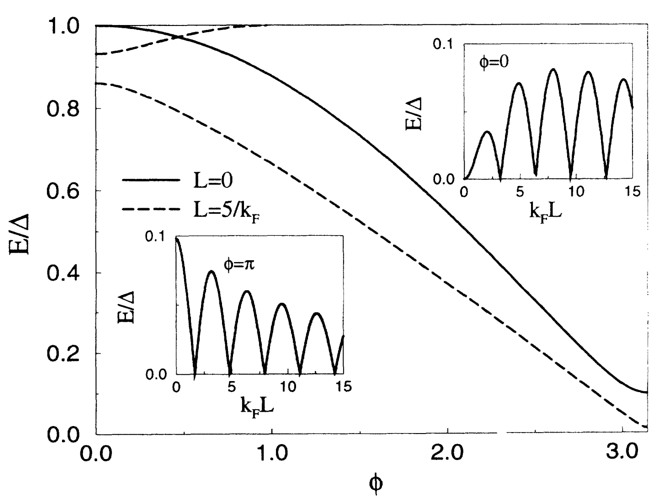

FIG. 2. The dispersion relation $E(\phi)$ is shown for $L = 0$ and $L = 5k_F^{-1}$. Also, $E(\phi = 0)$ and $E(\phi = \pi)$ as functions of L are shown as insets. For all curves we assume $\delta = 1/5$ and zero temperature.

$E_g \sim |\cos(k_F L)|$, as shown in the lower inset in Fig. 2. The amplitude of the oscillation decreases with length. However, considering the derivative of $E(\phi)$, the relation $dE/d\phi = 0$ at $\phi = \pi$ is always valid. Finally, at $\phi = 0$ there is a gap, which closes at $k_F L = n\pi$ (see the upper inset in Fig. 2).

Using Eq. (2) we calculate the current for zero temperature. The result is shown in Fig. 3. The dot-dashed curve gives the current-phase relationship for the case $\delta \to 0$, $I = (e\Delta/\hbar)\sin(\phi/2)$. In the case $\delta \neq 0$, three things can be pointed out for the current: first, the critical current I_c is reached for $\phi < \pi$; second, the value of the critical current is reduced from the value $e\Delta/\hbar$; third, the discontinuity at $\phi = \pi$ is absent.

It is possible to show how much I_c is reduced by expanding Eq. (11) in small δ and substitute into Eq. (2). The result of this algebra is

$$I_c = \frac{e\Delta}{\hbar}\left(1 - \frac{\Delta}{2\mu}\right), \tag{13}$$

correct to first order in δ.

Since $dE/d\phi = 0$ at $\phi = \pi$, the critical current is reached before $\phi = \pi$ for all L. The critical current calculated from the discrete energy spectrum decreases with length (see the inset in Fig. 3); however, at larger lengths, the continuous spectrum is known to contribute.[3] The $I_c(L)$ dependence tends to an oscillating behavior. These oscillations are not as large as in Ref. 19, where the authors studied a much stronger scattering mechanism due to large mismatch of the Fermi energies.

Our results, illustrated by Eqs. (12) and (13), are similar to the results of Bagwell in Ref. 3. In his treatment of an SNS system, Bagwell puts one impurity in the normal region. This impurity introduces normal reflection, which gives rise to energy gaps and suppression of the critical current. In our treatment these effects are present not because of impurity scattering but because of normal reflection at the SN interfaces due to a finite value of Δ/μ.

Finally, we speculate on a possible connection between our results and recent resistance measurements on short Al wires,[20] where the resistance was found to increase above the normal state value close to the critical temperature. Because of the appearance of superconducting fluctuations above the mean-field critical temperature, or inhomogeneities in the material, there will be a temperature region where superconducting regions coexist with normal regions. Then the scattering caused by the mismatch at the SN interfaces might explain why the resistance increases, *making it unnecessary to introduce a barrier at the SN interfaces* (as was done in Ref. 20). Moreover, since the scattering depends on Δ, the resistance

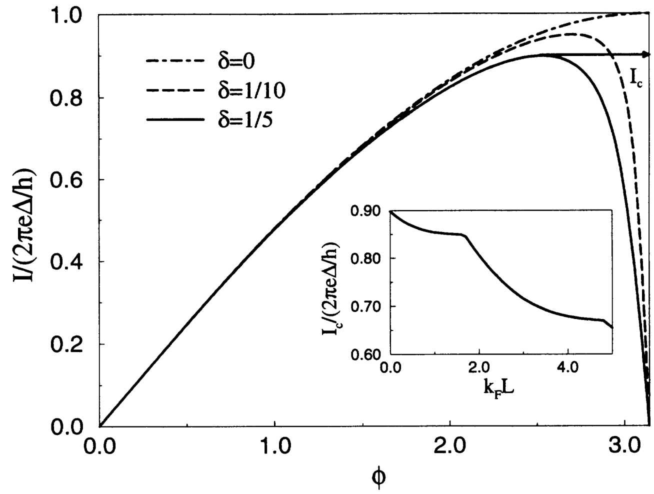

FIG. 3. The current-phase relationship is shown for different values of δ. As an inset, we show how the critical current I_c depends on L for $\delta = 1/5$. Zero temperature is assumed.

will depend both on temperature and magnetic field. In order not to spoil the effect by other stronger scattering mechanisms, low-resistance samples and contact arrangements should be used, in agreement with Ref. 20. The scattering from one single SN interface would certainly be small, especially close to T_c. However, in the case of many interfaces, the scattering is not necessarily small. During the preparation of this manuscript we became aware of very recent work along these lines,[21] where a mesoscopic system with many superconducting inclusions was studied.

In conclusion, we have studied the Josephson current in a clean (ballistic) SNS contact, where the superconducting electrodes are characterized by a finite value of Δ/μ. We have demonstrated the presence of energy gaps in the quasiparticle spectrum, the absence of kinks in the current-phase relationship and the reduction of the critical current. For high-T_c superconductors typically $\Delta/\mu \sim 0.1$ (see, e.g., Refs. 22 and 23) and this effect of short coherence length might be important. Even though s-wave BCS theory may not be directly applicable to the anisotropic high-T_c cuprate superconductors, accurate solutions of the BdG equation for large values of $\Delta/\mu \sim 0.1$–0.2 should be of considerable interest.

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