Restoration of the continuous phase transition in the vortex state due to lattice translational symmetry: Large-N limit

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The critical behavior of type-II superconductors near the upper critical magnetic field $H_{c2}(T)$ is studied. The thermal fluctuations are shown to forbid a spontaneous breakdown of the U(1) symmetry in $d<4$ dimensions within the $1/N$ expansion, where N is the complex order parameter component number. The lattice effects such as Harper's broadening and splitting of Landau levels are found to restore the continuous phase transition in the conventional mixed state described by critical exponents which are identical to those of the Ginzburg-Landau model with $H=0$.

I. INTRODUCTION

The critical properties of type-II superconductors (SC) near the upper critical magnetic field $H_{c2}(T)$ have been extensively studied by various methods for many years.[1–9] All attempts to develop a consistent approach encountered serious difficulties. In contradiction to the zero-field case a dimensional reduction effect takes place in an external magnetic field. As a result, the order-parameter fluctuations have one-dimensional (1D) character and play an important role in a vicinity of the $H_{c2}(T)$ line. Apparently this means that a (continuous) phase transition cannot occur at all in such a system because the lower critical dimension equals four.[7,8] This in turn implies that theoretical approaches based on the mean-field approximation ignoring critical fluctuations are not reliable.

The renormalization-group analysis, having been carried out in Ref. 10, shows that the model under consideration is described by the nonrenormalizable complex scalar field theory Φ^4 with an infinite set of quartic coupling constants. The dimensional reduction and the nonrenormalizability of the effective Lagrangian were shown to result from the infinite degeneracy of Landau levels which is inherent to a charged particle in a homogeneous magnetic field.[10]

Another approach was suggested by Brezin, Fujita, and Hikami.[11] They calculated the specific heat in the vicinity of the $H_{c2}(T)$ line by summing of higher-order terms of the perturbation series with the Pade-Borel method. No specific-heat singularity was found.

In the recent paper of Moore[12] effects of phase fluctuations were examined in the flux-lattice state. It was shown that these effects destroy the off-diagonal long-range order (ODLRO) in $d<4$ dimensions. Thus, we face the serious controversy between the experimentally observed phase coherence in SC below T_c and theoretical results forbidding an existence of the mixed superconducting state in three dimensions.

A novel approach to the problem was developed by Tešanovic and Xing.[3,4] The essential points of this theory are as follows: (i) the dimensional reduction in a magnetic field is not complete due to the specific distribution of combinatoric coefficients of Feynman diagrams; (ii) nonperturbative effects were shown to be strongly relevant leading to the solid-liquid-like phase transition without a phase coherence.

In this paper we shall develop the $1/N$ expansion to show explicitly the absence of a continuous phase transition in the framework of the conventional Ginzburg-Landau (GL) theory with an N-component complex order parameter in an applied magnetic field B which is assumed to be spatially uniform. To resolve the abovementioned controversy we shall suggest one possible scenario of the phase transition restoration in the vortex lattice based on taking into account the lattice magnetic translational symmetry effects, in particular, Harper's broadening and splitting of Landau levels. In this approach we use the lattice version of the GL functional and other equivalent (at a critical domain) models.[13] We are forced to choose such a way because of the lack of a commonly accepted microscopic model for the high-T_c SC. The lattice model description seems also to be natural in view of the fact that the experimental coherence length values in high-T_c SC hardly exceed lattice spacing ones.[14]

At first glance, in a vicinity of a continuous phase transition point when the correlation length r_c diverges as T approaches T_c the system should "forget" the discrete nature of a lattice. That is why lattice terms in a Landau Hamiltonian are irrelevant near T_c. But if one studies a critical behavior of a SC in an applied magnetic field this is not the case. The physics of this phenomenon is rather transparent because lattice breaks the continuous translational magnetic symmetry reducing it to the infinite discrete one and suppresses the dimensional reduction effect.[13] From the renormalization-group point of view there is a strongly relevant operator constructed from lat-

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tice terms drastically changing the critical behavior of a SC in an external magnetic field. Moreover, these terms are known to lead to the appearance of Abrikosov's flux lattice below T_c being commensurate to the original lattice.[15] Thus, we may regard the above effects as one of the presumable mechanisms which is responsible for the existence of the ODLRO in the mixed state.

Therefore the essential difference in comparison with the Tešanovic's approach is the strong lattice pinning of the order parameter. The Tešanovic's theory is expected to be valid within the range of moderate magnetic-field values when the magnetic length is large enough in comparison with the lattice spacing. On the other hand our approach seems to be applicable in the limit of small magnetic length values comparable with the lattice spacing and the coherence length which is small in high-temperature superconductors. A detailed consideration of the crossover behavior between these two regimes is left for future studies.

This paper is organized as follows. In Sec. II the critical behavior of the conventional GL model is studied within the $1/N$ expansion. Section III is devoted to the treatment of the continuum limit and critical properties of lattice models of a SC. The main results are briefly summarized in Sec. IV.

II. THE $1/N$ EXPANSION FOR THE GINZBURG-LANDAU MODEL

We begin with the conventional fluctuation GL Hamiltonian

$$H=\int d^dx\left\{\frac{\hbar^2}{4m}\left|\left[i\partial_\mu+\frac{2\pi}{\Phi_0}A_\mu\right]\Phi_a\right|^2+\frac{1}{2}\kappa_0^2\Phi_a\Phi_a^*+\frac{u_0}{8}I_{abcd}\Phi_a\Phi_b\Phi_c^*\Phi_d^*\right\}, \tag{1}$$

where

$$\kappa_0^2-\kappa_{0c}^2\sim\tau=(T-T_c)/T_c\ . \tag{2}$$

Here $\Phi=\{\Phi_1,\ldots,\Phi_N\}$ is an N-component complex order parameter, u_0 may be considered as a positive constant near a transition temperature T_c without a magnetic field, A_μ is a vector potential; in the symmetric gauge we have

$$\mathbf{A}=\tfrac{1}{2}[\mathbf{B},\mathbf{r}]\ , \tag{3}$$

where $\mathbf{B}$ is taken to be along the z axis, $\phi_0=hc/2e$ is the magnetic flux quantum, $I_{abcd}=\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc}$; $a,b,c,d=1,\ldots,N$; the Einstein summation convention is used.

With the purpose to obtain an asymptotically exact solution of (1) in the large-N-limit let us consider the Dyson's equation for the two-point Green's function of the order parameter

$$G(\mathbf{r},\mathbf{r}')=G_0(\mathbf{r},\mathbf{r}')-\frac{1}{2}Nu_0\int d^dR\,G_0(\mathbf{r},\mathbf{R})G(\mathbf{R},\mathbf{R})G(\mathbf{R},\mathbf{r}')\ . \tag{4}$$

Since $G_0^{ab}(\mathbf{r},\mathbf{r}')=G_0(\mathbf{r},\mathbf{r}')\delta^{ab}$, flavor indices will be omitted throughout this paper. The bare gauge-noninvariant correlation function $G(\mathbf{r},\mathbf{r}')$ is the resolvent of the Schrödinger operator

$$\left[\frac{1}{2m}\left[-i\hbar\partial_\mu-\frac{e}{c}A_\mu\right]^2+\kappa_0^2\right]G_0(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}')\ . \tag{5}$$

An exact expression for $G_0(\mathbf{r},\mathbf{r}')$ can be obtained analytically in an arbitrary gauge

$$G_0(\mathbf{r},\mathbf{r}')=\exp\left\{-\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}'}dx_\mu A_\mu\right\}\int_0^\infty d\beta\, m\omega\left[4\pi\hbar\sinh\frac{\omega\beta}{2}\right]^{-1}\left[2\pi\hbar\frac{\beta}{m}\right]^{-1/2}\times\exp\left\{-\kappa_0^2\beta-\frac{m}{2\hbar\beta}(z-z')^2-\frac{m\omega}{4\hbar}\coth\frac{\omega\beta}{2}[(y-y')^2+(x-x')^2]\right\}. \tag{6}$$

Here $\omega=eB/mc$ is the Larmor frequency, z and z' stand for coordinates in $(d-2)$ longitudinal directions. From Eq. (6) it follows that $G_0(\mathbf{r},\mathbf{r})$ and, hence, $G(\mathbf{r},\mathbf{r})$ being gauge-invariant quantities do not depend on $\mathbf{r}$. The integral in the exponent is taken along the straight line connecting points $\mathbf{r}$ and $\mathbf{r}'$.

This important conclusion also results from the local gauge invariance of the theory (1) which implies that under the translation $r\Longrightarrow r+a$, the normal Green's function transforms as

$$G(\mathbf{r}+\mathbf{a},\mathbf{r}'+\mathbf{a})=\exp\left\{\frac{ie}{2\hbar c}[\mathbf{B},\mathbf{a}](\mathbf{r}-\mathbf{r}')\right\}G(\mathbf{r},\mathbf{r}')\ . \tag{7}$$

Therefore it is clear that $G(\mathbf{r},\mathbf{r})$ is a constant.

Thus, we may regard the Dyson's equation as a linear integral one. However, the solution of this equation cannot be directly obtained by means of the Fourier transformation because both Green's functions in Eq. (4) do not depend on differences $x-x'$ and $y-y'$.

One easily gets the solution of Eq. (4) by doing the Fourier transformation with respect to longitudinal coordinates and using the lowest Landau level approximation for the $x-y$ part of the Green's function. One believes that near T_c only this level gives the dominant contribution to $G(\mathbf{r},\mathbf{r})$. Using some algebra, we arrive at the expression

$$G(x,x',y,y';k)=C\exp\left\{-\frac{ie}{\hbar c}\int_{\mathbf{r}}^{\mathbf{r}'}dx_\mu A_\mu-\frac{m\omega}{4\hbar}[(y-y')^2+(x-x')^2]\right\}\frac{1}{k^2+M^2}\ , \tag{8}$$

where C is an inessential constant factor. Having been equipped with this result we may immediately turn Eq. (4) into the equation for a "physical" mass M (an inverse correlation length $r_c^{-1}=M$), familiar from the phase transition theory[16]

$$M^2=\kappa_0^2+\frac{1}{2}Nu_0\int\frac{d_k^{d-2}}{(2\pi)^J}\frac{1}{k^2+M^2}\ . \tag{9}$$

We see that an external magnetic field inevitably leads to the dimensional reduction which forbids a spontaneous breakdown of the continuous $U(1)$ symmetry in $d<4$ dimensions. This assertion can be shown to be valid in the framework of $1/N$ expansion. Equation (9) is in good agreement with Moore's results[6] and the conjecture given by Ruggeri and Thouless[7] that the physical properties of (1) for $d=3$, $N=2$ are identical to those of the 1D GL model with $B=0$.

III. THE CRITICAL BEHAVIOR OF LATTICE MODELS

Now we turn to study how lattice terms in the GL action affect a critical behavior of a SC. There are two completely equivalent ways to incorporate lattice translational symmetry effects into a theory. The most straightforward approach is to treat the lattice Hamiltonian of the uniformly frustrated $O(2N)$-symmetric nonlinear σ model from the very beginning in the spirit of the paper of Choi and Doniach[15]

$$H=-\sum_{\langle i,j\rangle}J_{ij}n_i^a n_j^{a*}\exp\left[i\frac{2\pi}{\Phi_0}\int_i^j dx_\mu A_\mu\right] \tag{10}$$

with n_i^a being an N-component complex unit vector, $n_i^a n_i^{a*}=1$, J_{ij} equals J for nearest neighbors and zero otherwise; the gauge-invariant sum around a plaquette $\sum A_{ij}=2\pi f$, where A_{ij} denotes the internal in the exponent Eq. (10); $f=p/q$ is the frustration; p and q are relatively prime integers.

The Hubbard-Stratonovich transformation was shown to provide a systematic prescription for obtaining an infinite set of effective GL Hamiltonians being the continuum limits of Eq. (10) with different internal symmetries corresponding to different rational values of f.[15]

Within the other approach being somewhat artificial but leading to the identical expressions for the effective GL action one deals with the GL free energy density which is different in quadratic terms from that given by Eq. (1),[13]

$$H=\int d^dx\left\{\frac{1}{2}\left|\varepsilon\left[-i\hbar\partial_\mu-\frac{e}{c}A_\mu\right]\Phi_a\right|^2+\frac{1}{2}\kappa_0^2\Phi_a\Phi_a^*+\frac{u_0}{8}I_{abcd}\Phi_a\Phi_b\Phi_c^*\Phi_d^*\right\}\ , \tag{11}$$

where $\varepsilon(k)$ is the band spectrum; without lattice effects this function turns into the standard expression $\varepsilon(k)=\hbar^2k^2/2m$,[17]

$$B=\phi_0\frac{a_z}{v}f\ . \tag{12}$$

Here v is the unit-cell volume and a_z is the lattice spacing in the z direction.

Provided N tends to infinity, $G(\mathbf{r},\mathbf{r}')$ satisfies Eq. (4) where $G_0(\mathbf{r},\mathbf{r}')$ is the solution of the Harper's equation[18]

$$\left[\varepsilon\left[-i\hbar\partial_\mu-\frac{e}{c}A_\mu\right]+\kappa_0^2\right]G_0(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}')\ . \tag{13}$$

It is helpful to consider the associated eigenvalue problem

$$\left[\varepsilon\left[-i\hbar\partial_\mu-\frac{e}{c}A_\mu\right]+\kappa_0^2\right]\Psi_{nk\alpha}(x)=E_{nk}\Psi_{nk\alpha}(x)\ , \tag{14}$$

where k is a quasimomentum defined in a magnetic Brillouin zone, n is a magnetic band number, and $\alpha=1,\ldots,s$ where $s=q(q/2)$ if q is an odd (even) integer, respectively. Although the explicit expressions of Harper's eigenfunctions and eigenvalues are not known, its general properties are well understood.

The energy spectrum is known to be degenerated with regard to the quantum number α labeling eigenfunctions $\Psi_{nk\alpha}(x)$ which form the basis of the s-dimensional irreducible projective representation of the magnetic translational group.

Near the band's bottom the spectrum E_{nk} can be written in the form

$$E_{nk}=\frac{\hbar^2}{2m_1}(k_x^2+k_y^2)+\frac{\hbar^2}{2m_2}k_z^2+\kappa_0^2 \tag{15}$$

with $m_{1,2}$ being effective masses in the x-y plane and along the z direction, respectively. Substitution of the resolvent spectral expansion in basis functions (14)

$$G_0(\mathbf{r},\mathbf{r}')=\sum_{n,k,\alpha}E_{nk}^{(0)-1}\Psi_{nk\alpha}(\mathbf{r})\Psi_{nk\alpha}^*(\mathbf{r}')\ , \tag{16}$$

$$G(\mathbf{r},\mathbf{r}')=\sum_{n,k,\alpha}E_{nk}^{-1}\Psi(\mathbf{r})\Psi^*(\mathbf{r}') \tag{17}$$

into Eq. (4) yields

$$E_{nk}^{-1}=E_{nk}^{(0)-1}+\tfrac{1}{2}u_0Ns\sum_{n,k}E_{nk}^{-1}\ . \tag{18}$$

To obtain Eq. (18) we have used the relation

$$G(0)=s\sum_{n,k}E_{nk}^{-1}\ . \tag{19}$$

The summation over index α gives rise to the factor s in Eqs. (18) and (19). Combining Eqs. (15) and (18), one obtains the familiar equation for the correlation length coinciding with Eq. (9) apart from the spatial dimension value. Thus, due to the disappearance of the dimensional reduction the continuous phase transition occurs in the 3D model (11) described by universal critical exponents which are identical to that of the spherical model and do not depend on f.

Inserting the order-parameter expansion in eigenfunctions (14)

$$\Phi_a(x)=\sum_{n,k,\alpha} u_\alpha^a(k;n)\Psi_{nk\alpha}(x) \qquad (20)$$

into Eq. (11) and using the effective-mass approximation (15) one rewrites the GL functional in terms of the new sN-component complex order parameter $u_\alpha^a(k;n)$:[13]

$$H=\frac{1}{2}\sum_{k,\alpha,a} E_{nk}|u_\alpha^a(k)|^2 + \frac{u_0}{8}\sum_{\substack{\{k_i\}\\ a,b,c,d\\ \alpha,\beta,\mu,\nu}} I_{abcd}\, g_{\alpha\beta\mu\nu}(k_1,k_2,k_3,k_4) \times u_\alpha^a(k_1)u_\beta^b(k_2)u_\mu^{c*}(k_3)u_\nu^{d*}(k_4) . \qquad (21)$$

In Eq. (21) contributions of higher Landau modes $n=1,2,\ldots$ have been neglected; $\alpha,\beta,\mu,\nu=1,\ldots,s$; coefficients g are defined by the relation

$$g_{\alpha\beta\mu\nu}(k_1,k_2,k_3,k_4) = \int d^3x\,\Psi_{k\alpha}^*(x)\Psi_{k\beta}^*(x)\Psi_{k\mu}(x)\Psi_{k\nu}(x) . \qquad (22)$$

In accordance with the conventional field-theoretical approach to phase transitions and critical phenomena we may ignore the $\{k_i\}$ dependence of $g_{\alpha\beta\mu\nu}(k_1,k_2,k_3,k_4)$ near T_c with the exception of δ-function factor

$$g_{\alpha\beta\mu\nu}(k_1,k_2,k_3,k_4)=g_{\alpha\beta\mu\nu}\delta(k_1+k_2-k_3-k_4) . \qquad (23)$$

Using Eqs. (15), (21), and (23), we are directly led to the effective local GL Hamiltonian[7,9]

$$H=\int d^dx\left\{\frac{1}{2}|\partial_\mu u_\alpha^a|^2+\frac{1}{2}\kappa_0^2|u_\alpha^a|^2 + \frac{u_0}{8}g_{\alpha\beta\mu\nu}I_{abcd}u_\alpha^a u_\beta^b u_\mu^{c*}u_\nu^{d*}\right\} \qquad (24)$$

where

$$u_\alpha^a(x)=\sum_k u_{k\alpha}^a\exp(ikx) . \qquad (25)$$

The Hamiltonian (24) is known to describe phase transitions in anisotropic SC with d-wave pairing of charge carriers [which is supposed to occur in heavy-fermion SC (Ref. 13)] for $N=2,3$ as well as the superconductor-insulator phase transitions at $T=0$ in Josephson-junction arrays in an external magnetic field for $N=1$.[19,20]

The critical behavior of the renormalizable multicomponent scalar field theory Φ (Ref. 4) can be studied by virtue of the well-known standard approach. The renormalization-group analysis of (24) cannot be done in the general case due to the following reasons: (i) since $g_{\alpha\beta\mu\nu}$ is a highly discontinuous function of the frustration f, it's explicit expression has been found only in simple particular cases;[15] (ii) at the present time the reliable methods of the calculation of renormalization-group functions in the 3D space through the perturbation theory combined with the Pade-Borel resummation technique of asymptotic series are well developed only for models with a few independent quartic coupling constants.[21] The detailed treatment having been done in Ref. 13 shows that the critical behavior of (25) is governed by the superfluid (Bose) fixed point[22] for $q=1,2,3,4$ and is the same as for the usual SC with $H=0$.

It is tempting to assume that the critical behavior of (24) is universal and can be actually described by the Bose critical exponents. Although we are not able to give rigorous proof of this point , we present some arguments in favor of this plausible possibility: (i) the previous analysis in the $1/N$ expansion and results obtained in Refs. 12 and 13 strongly confirm this hypothesis; (ii) since the numerical value of the specific-heat critical exponent α is negative,[23] the Harris criterion implies that the Bose fixed point is stable with respect to turning on a small perturbation produced by local operators which are given by $|u_\alpha|^2|u_\beta|^2(N=2,\alpha\neq\beta=1,\ldots,s)$. In particular, quenched nonmagnetic impurities of a random-temperature type do not affect critical properties of a SC.

IV. CONCLUDING REMARKS

Let us consider in brief the situation below T_c. In this case thermodynamic averages of order-parameter components $\langle u_\alpha^a(x)\rangle$ are expected to take constant values. According to Eqs. (20) and (25) the spatially uniform ordering in the effective theory (24) corresponds to the inhomogeneous periodical ordering in the original model reflecting the formation of the regular flux superlattice which is commensurate with the original lattice for arbitrary rational values of the frustration f.[15] So, we expect that the modified GL theory (11) gives the consistent description and truly captures the essentials of the phase transition in the conventional mixed state.

We have shown above that thermal fluctuations do indeed destroy the ODLRO and the lower critical dimension of the conventional GL theory in an external magnetic field equals four in the $1/N$ expansion in accordance with Ref. 12. The lattice effects were shown to restore the ODLRO, at least, in the large-N limit. The continuous phase transition occurs on the $H_{c2}(T)$ line provided an applied magnetic field takes values given by Eq. (12). The critical behavior of our model is predicted to be universal and identical to that of a superconductor with $H=0$ irrespective of the value of f.

As for intrinsic magnetic-field fluctuations,[24–25] one can easily check that they cannot suppress the dimensional reduction effect in the conventional GL theory, at least, in the lowest order in the $1/N$ expansion and do not change the nature of a phase transition and the critical behavior of lattice models. This conclusion is consistent with the Monte Carlo simulation results.[25] A detailed investigation of the theory taking into account fluctuations of A_μ and lattice effects is left to future studies. Notice that while different versions of continuum theory predict destruction of a continuous phase transition by fluctuations (it is true within the $1/N$ approximation, for instance), pinning of fluctuations by a lattice restores the second-order phase transition in the limit of extremely strong magnetic fields. To investigate a crossover between these two regimes and, in particular, to estimate a critical value of the field is the extremely intriguing problem.

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