Model for fracture in fibrous materials

A. T. Bernardes
Departamento de Física, Instituto de Ciências Exatas e Biológicas, Universidade Federal de Ouro Preto, Campus do Morro do Cruzeiro, 35410-000 Ouro Preto, MG, Brazil

J. G. Moreira
Departamento de Física, Instituto de Ciências Exatas, Universidade Federal de Minas Gerais, Caixa Postal 702, 30161-970 Belo Horizonte, MG, Brazil

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A fiber-bundle model in (1+1) dimensions for the breaking of fibrous composite matrix is introduced. The model consists of N parallel fibers fixed in two plates. When one of the plates is pulled in the direction parallel to the fibers, these can be broken with a probability that depends on their elastic energy. The mechanism of rupture is simulated by the breaking of neighboring fibers that can generate random cracks spreading up through the system. Due to the simplicity of the model we have virtually no computational limitation. The model is sensitive to external conditions such as temperature and traction velocity. The energy versus temperature behavior, the diagrams of stress versus strain, and the histograms of the frequency versus the size of cracks are obtained.

I. INTRODUCTION

Fracture is an important problem in material sciences and engineering. The response of a solid under load depends on the features of the material, the external conditions (temperature, humidity, etc.), and how the load is applied (uniaxial, radial, shear, etc.). The main features of the fracture processes can be found in the classical Young's experiment. Let us consider a homogeneous bar of initial length L_0 and cross section S pulled by a uniaxial force F parallel to the length. In the diagram of $\sigma=F(t)/S(t)$ vs $\delta=\Delta L/L_0$ one can observe an elastic (linear and nonlinear) region and a plastic region. The elastic region occurs at the beginning of the traction, when the material returns to L_0 if the traction is stopped. On the other hand, the material acquires a permanent deformation when the force vanishes in the plastic region. If the material breaks in the elastic regime, the fracture is called brittle (like glass at room temperature). Otherwise, if the material breaks in the plastic region the fracture is called ductile (like an eraser).

The presence of disorder in the material is an important feature that determines the rupture processes.[1] These inhomogeneities strongly influence the mechanical behavior of the material and are responsible for the patterns obtained experimentally. In the last decade, some models taking into account the disorder were proposed to simulate the breaking processes of disordered media.[2] The material, in general, is represented by a network of structural units whose rate of rupture depends on the local conditions and inhomogeneities. These models, which were proposed to simulate the rupture of polymer fibers or thin films (models of lattice springs)[3–5] and to study the interface properties of breaking processes,[6,7] have been studied mostly by computational experiments. However, these models provide just a partial description of the problem. At most, only the fracture pattern and the stress vs strain diagram can be obtained. These models do not allow an analysis of the dependence of rupture features on traction velocity and temperature because they are sensitive to changes in only one of the external conditions.

In this paper, a fiber-bundle model to simulate the failure processes of fibrous material is introduced. Fracture of fiber-reinforced materials is an important field of investigation, because these materials have a higher Young's modulus and other different mechanical properties than unreinforced ones.[8,9] Fiber-bundle models were introduced to study the strength of material where fibers are held together by friction forces. They are also used to study the breaking of composite materials where the fibers of the material are joined together by other homogeneous material, such as a fiberglass-reinforced composite. When a fiber fails, the load that it carries is shared by intact fibers in the bundle. An important effect to study these models was carried out by calculation of the cumulative breaking probability of the chain of fiber bundles.[10–12]

Our model considers the amount of elastic energy in the material, the spread of a local crack, and the fusion of cracks, as the breaking mechanism. Some features already proposed in the literature are used in the definition of our model—the computation of the breaking probability from the elastic energy of a fiber[3–5] and a deformation limit for an isolated fiber like the threshold in the random-fuse-network model[6,7] In addition, we adopt the cascade of breaking fibers as the mechanism to form the cracks into the fiber bundle. This last characteristic is clearly inspired by self-organizing criticality.[13] Our attention is focused on computational simulation for the breaking of a fiber bundle when we have a uniaxial force

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(parallel to the fibers) in $(1+1)$ dimensions. The fracture processes are described by the energy of the rupture process vs temperature, the diagram of stress vs strain, and the size of the cracks that occur in the breaking. This paper is organized as follows: in Sec. II, the model is presented; the results of the computational experiments are shown and discussed in Sec. III; finally, the conclusions are given in the last section.

II. THE MODEL

Our model consists of N_0 parallel fibers, each of them with the same elastic constant k. These fibers are fixed in parallel as shown in Fig. 1. Note that the first and last fibers make contact with only one neighbor while the inner fibers have two neighbors. For convenience one plate is fixed and the other is pulled by a force F in the direction parallel to the fibers with constant velocity v. This means that at each time step τ the amount of deformation of the nonbroken fibers is equal to $(\Delta z = v \times \tau)$, where v is the velocity (in our units $\tau = 1$). When the deformation is z, the elastic energy for each fiber is given by

$$\epsilon = \tfrac{1}{2} k z^2 . \tag{2.1}$$

We define the critical elastic energy for each fiber as

$$\epsilon_c = \tfrac{1}{2} k z_c^2 , \tag{2.2}$$

where z_c is imposed as the maximum deformation supported by an individual fiber. We assume that an isolated fiber has a purely linear elastic behavior with a breaking probability which increases with the deformation z of the fiber, being equal to unity at $z = z_c$. The probability of rupture of the fiber i is

$$P_i(z) = \frac{1}{(n_i + 1)} \exp\left[\frac{1}{t} (\delta^2 - 1) \right] . \tag{2.3}$$

Here n_i is the number of nonbroken neighbor fibers of the fiber i (in this paper n_i could be 0, 1, or 2),

$$t = \frac{k_B T}{\epsilon_c} \tag{2.4}$$

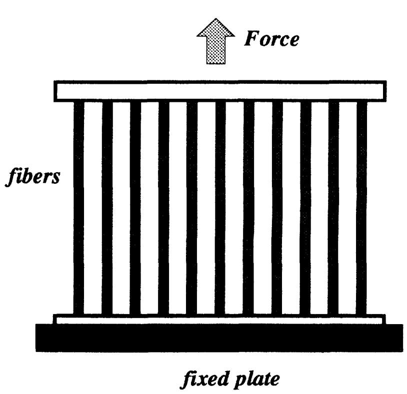

FIG. 1. Schematic representation of our model. We have shaded in deep gray the fibers on a plate at rest (lower plate), with the tops fixed on a moving plate, pulled with constant displacement.

is the normalized temperature, k_B is the Boltzmann constant (in our unity system it is equal to 1), and

$$\delta = \frac{z}{z_c} \tag{2.5}$$

is the strain of the material. The dependence on the nonbroken neighbors fibers simulates the existence of an interaction between the fibers. This dependence is responsible for the distribution of the load between neighboring fibers and allows a fiber having an elastic energy greater than ϵ_c. In this sense, one can observe fibers with $z > z_c$ if they have at least one nonbroken neighboring fiber.

Initially all the fibers have the same length and zero deformation. In each time step of the simulation the system is pulled by Δz, and we randomly choose $N_q = q \times N_0$ fibers that can be broken, where q is a positive number. This means that the probability of rupture for the material does not depend on the number of fibers in the fiber bundle. This assumption is in agreement with the observation that systems with different sizes must have the same rupture features for the same external conditions (temperature and traction velocity). Obviously the force and the energy needed to break the bundle must depend on the system size, but not the stress vs strain diagrams or the size of the cracks that arise in the breaking processes. This assumption also makes possible the appearance of cracks in different parts of the material for the same deformation. Let us consider a chosen fiber. The breaking probability is evaluated and compared with a random number in the interval [0,1). If the random number is less than the breaking probability, the fiber breaks. The load spreads to the neighbor fibers and their breaking probability increases because of the decrease of the parameters n_{i-1} and n_{i+1}. This procedure describes the propagation of the crack through the fiber bundle. Then, the same steps are performed for one of the neighboring fibers. Note that if it breaks, a cascade begins. It stops in a given fiber, when the test of the probability does not allow its rupture, or when a hole in the bundle is found (an old crack). The propagation of the crack is done in either "left" or "right" directions, perpendicular to the force applied on the system. When the cascade process stops, another fiber in the N_q set is chosen and all steps already described are repeated. After the N_q trials, we pull the system to a new displacement Δz and the breaking procedure begins again. The simulation continues until the rupture of the system, when no more entire fibers exist.

III. RESULTS AND DISCUSSION

At $t = 0$ it is easy to see that the model breaks at $\delta = 1.0$ with a maximum force $F = N_0 k z_c$. All the fibers break at the same time and we have just one crack spreading in the entire system (the limit of a brittle fracture). For finite temperatures different behaviors are observed when the traction velocity is varied. The number of fibers is chosen in such a way that it does not affect the propagation of the cracks. This means that a crack greater than or equal to the size of the system, for the values of t and v used in the simulation, occurs with a negligible probability. In order to investigate this picture, we have performed simulations in systems $N_0=10^3-10^6$. The probability is controlled by determining the distribution of cracks vs the sizes of the cracks arising in the process of fracture. We have used the following values for the parameters: $q=0.1$, $N_0=10^4$, $z_c=1$, and $k=1$.

As a preliminary, we have obtained the stress vs strain diagrams for different temperatures and traction velocities. When the deformation of the bundle is z and the number of nonbroken fibers is N, the stress σ is defined as

$$\sigma=\frac{Nkz}{N_0} . \tag{3.1}$$

The strain δ was defined in Eq. (2.5). We compare our results with the description obtained experimentally in order to classify the fracture as brittle or ductile.[14] Figure 2 shows the result of a computational simulation carried out in just one fiber bundle. In this case, averages are avoided. For $t=0.1$ one observes a brittle behavior, i.e.,

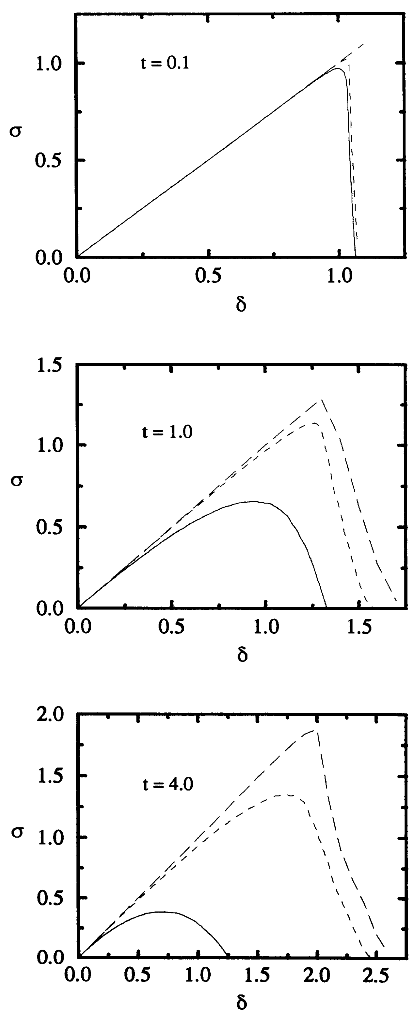

FIG. 2. Stress vs strain plots for different normalized temperatures (indicated in the diagrams) and velocities ($v=0.001$, full line; $v=0.01$, dashed line; $v=0.1$, long-dashed line). The plots were made with just one simulation with a sample of 10^4 fibers for each pair of parameters.

the fiber bundle breaks in the elastic region. Note that the σ vs δ plot is purely linear for the highest velocity ($v=0.1$). At $t=1.0$ and for high and intermediate velocities the fracture occurs in the brittle-ductile transition region. The rupture of the material is ductile for low velocities and it occurs in the plastic or deformation region. For high temperatures ($t=4.0$), the shape of the stress vs strain plot is typically ductile for intermediate and low velocities. For high velocities the fracture occurs in the brittle-ductile transition region.

Now let us discuss the behavior of the energy of rupture as a function of temperature. This energy is defined as the work done to break the material and it can be obtained from the stress vs strain diagrams. It is well known that the breaking of materials has a strong dependence on temperature. In general, some material undergo brittle fracture at low temperature and ductile fracture at high ones. This means that the energy of the fracture process has a small value in the brittle region and a greater value in the ductile region. The plots of the normalized averaged energy of the breaking process per fiber $\langle E_f \rangle$ vs the normalized temperature t are shown in Fig. 3. We have considered 10^3 samples with 10^4 fibers each, with velocities $v=0.001$, 0.002, and 0.005 in the simulations. At low temperatures the energy of the fracture becomes independent of the traction velocity. For velocity $v=0.005$ the energy increases with temperature. On the other hand, for slow traction ($v=0.001$) the energy increases up to a maximum (near $t\sim 0.5$) and for $t>0.5$ it decays smoothly. For an intermediate value of the velocity ($v=0.002$), the energy remains closely constant at high temperatures.

Figure 4 shows the frequency of the cracks H_c vs the size of the cracks S_c that arise in the breaking process. The frequency of the cracks is averaged over the samples (10^3 in this simulation). Two features can be observed in this figure. For low temperatures ($t=0.1$, typically brittle fracture), one observes cracks of very different sizes. For low velocities, one observes cracks with a maximum size $\sim 10^2$. For high velocities ($v\approx 0.1$), the size of the cracks tends to the entire system (10^4 fibers). This means

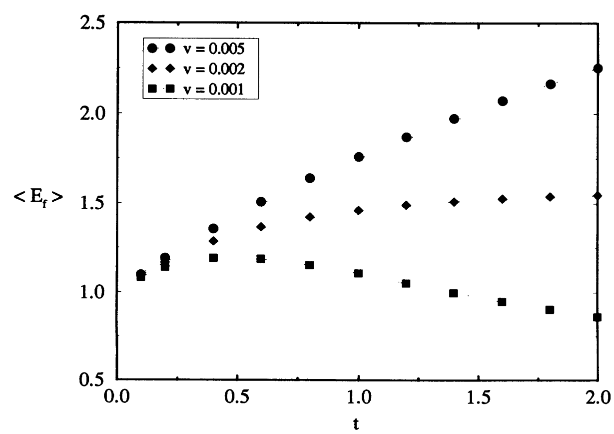

FIG. 3. Energy of fracture process fiber $\langle E_f \rangle$ vs normalized temperature t. The value of the velocities for each curve is shown in the inner box.

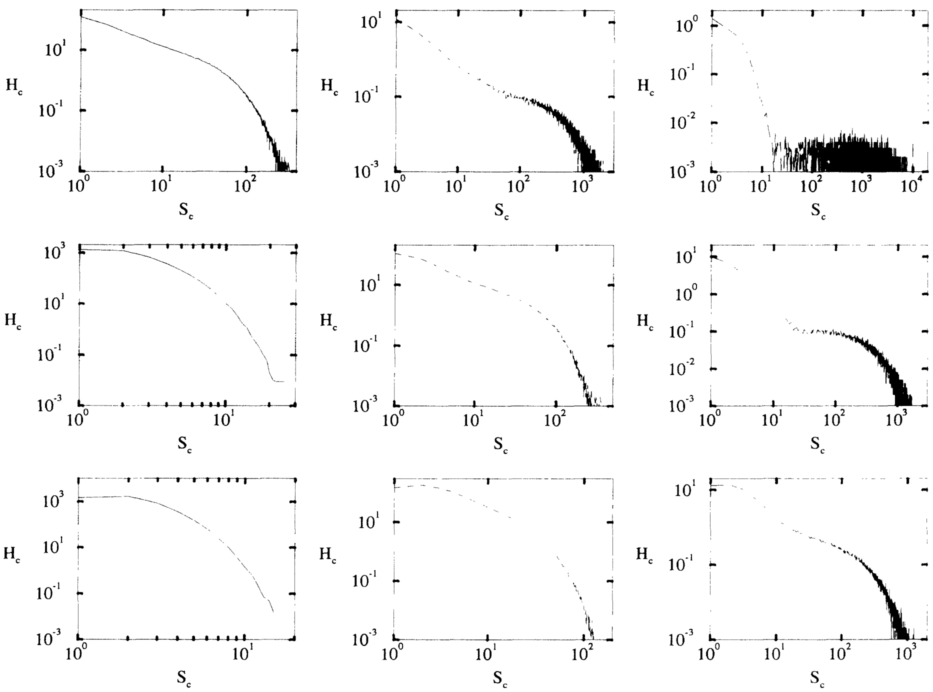

FIG. 4. Frequency of cracks H_c vs crack sizes S_c for different temperatures and velocities. The simulations were performed in 10^3 samples of 10^4 fibers each for each pair of parameters. Diagrams at the top have been calculated for $t=0.1$, in the middle for $t=1.0$, and at the bottom for $t=4.0$. The full line represents $v=0.001$, the dashed line $v=0.01$, and the long-dashed line $v=0.1$.

that the system is pulled essentially unbroken until a certain time when a large crack arises in the material. After this large crack, small ones are observed because of the rupture of the remaining fibers. The brittle process is characterized by the existence of cracks with different sizes, and a remarkable feature is the presence of cracks with sizes near to the system size. The curves that represent H_c vs S_c have a maximum at $S_c=1$. Note that at the beginning H_c decreases linearly. In order to verify this last feature, we adjust the data using

$$H_c \sim S_c^{\alpha} \; . \tag{3.2}$$

A good fit for this linear part is obtained with $\alpha \sim 1.02$ (see Fig. 5).

As long as the temperature is increased the size of the cracks becomes smaller. When one has many cracks of small sizes the fracture is clearly ductile. These cracks appear in different parts of the material and the shape of the curve of H_c vs S_c changes. The curve has a maximum at $S_c=2$, instead of $S_c=1$ as occurs for brittle fractures.

A quite different feature can be observed for slow traction ($v=0.001$) and high temperatures ($t>0.5$), when we have obtained a ductile fracture with an unusually low energy of rupture (see Fig. 3). Only cracks of small size are present in the system ($S_c<12$). Now the system fails when a force smaller than the one needed to break it at higher velocities is applied. This could indicate that the system is in a different state at high temperatures and that we have observed it in disaggregation.

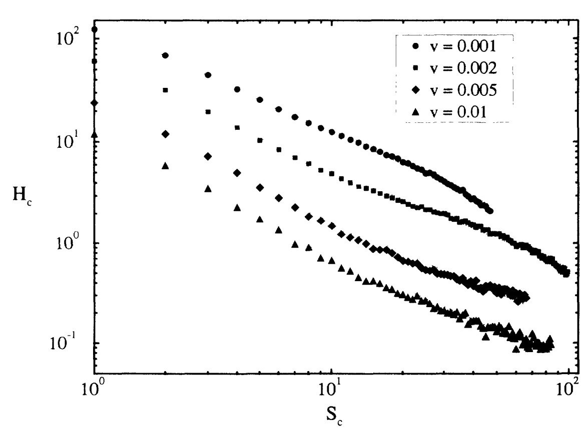

FIG. 5. Regression (dotted lines) for the diagrams of frequency of cracks H_c vs crack sizes S_c for $t=0.1$ for different velocities (indicated in the inner box).

IV. CONCLUSIONS

We have introduced a fiber-bundle model to simulate fractures in fibrous materials. The model is sensitive to external conditions which are present in some problems of material sciences: traction velocity and temperature. The simplicity of the model allows us to perform computations on very large systems. Because of this feature we can explore all diagrams of the failure processes.

We have obtained stress vs strain diagrams showing features of the two principal types of fractures: brittle and ductile. For low temperatures the system undergoes brittle fracture, independent of the traction velocity. When the temperature increases, the fracture is influenced by the traction velocity. We can observe a transition from the brittle to the ductile regime. The amount of energy needed to rupture the material is dependent on the traction velocity. For high velocities more energy is needed. This comes from the fact that the size of the cracks depends on the temperature. For high temperatures and low velocities we observe a curious behavior. In this case the energy is smaller than that needed to break the material in the brittle regime. This could indicate that we have a disaggregation process at this temperature, and the interaction between the fibers exerts a small influence in the rupture process. These results are independent of the number of fibers in the fiber bundle, because we have chosen values of the parameters v and t for which the maximum crack sizes obtained in our simulations are less than N_0.

Several questions remain open. The first one is the behavior of this model in $(2+1)$ dimensions for various lattice topologies. This could allow the comparison of our results with those observed in realistic systems. The behavior of the model at high temperatures and low velocities needs a more accurate investigation. It would be interesting to verify if those features remain in $(2+1)$ dimensions.

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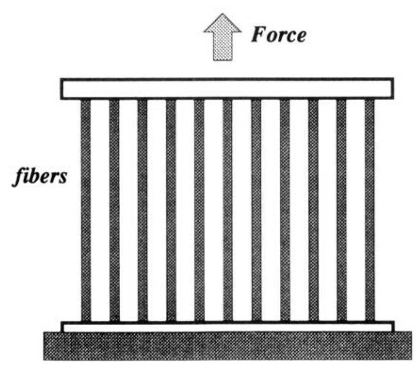

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