Tunneling in quantum-wire superlattices with random layer thicknesses

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The electron tunneling of a $GaAs/Ga_{1-x}Al_xAs$ superlattice with randomly distributed layer thicknesses is studied within the framework of the effective-mass approximation in the Wannier representation. The transfer-matrix method and Landauer formula are used to calculate the electron transmission coefficient and tunneling conductance, respectively. Analytical and numerical calculations are performed for the tunneling conductance with different values of potential barrier (different x for $Ga_{1-x}Al_xAs$) and layer thicknesses. The present calculations show that the peak positions of the tunneling conductance are shifted to lower energy as x decreases. The number of peaks can be increased by increasing the layer thicknesses. Therefore, optical and electronic semiconductor devices may be artificially prepared by the proper choice of these parameters for quantum-wire superlattices with random layer thicknesses.

Experimental advances in submicrometer physics naturally lead to an increasing interest in their physical properties, especially those related to transport phenomena, because they allow the fabrication of nearly ideal quantum-wire superlattices.[1] These structures are important for device application, as the one-dimensional (1D) carrier confinement reduces scattering and results in higher mobility.[2,3] In particular, the propagation of electrons along quantum wires of various geometries has been considered extensively.[4–6] Ulloa, Castano, and Kirczenow[7] and Wu *et al.*[8] have considered a linear array of mesoscopic potential wells separated by square potential barriers. Propagation through such an array reveals an interesting structure of tunneling plateaus, which could form the base for a quite different type of transistor action. Experiments with such geometries have also been undertaken.[9]

At the same time, transmission through the 1D quasiperiodic system has also attracted some attention.[10,11] For instance, Avishai and Berend[10,11] have discussed scattering from an infinite system of δ-function potentials located on the quasiperiodic numbers. Singh, Tao, and Tong[12] have studied the electron tunneling in quasiperiodic superlattice systems, such as Fibonacci and Thue-Morse quantum-wire superlattices, and compared their tunneling results of quasiperiodic system with results of periodic system. On the other hand, superlattices with distributed randomly layer thicknesses have been fabricated, and the degree of randomness in the material can be artificially controlled.[13] Experimental measurements on the photoabsorption and photoluminescence of these materials have revealed the optical properties to be quite different from those of bulk alloys as well as ordinary superlattices. In our previous paper,[14] we have studied the superlattices with randomly distributed layer thicknesses. In dealing with the breaking of the periodic symmetry in the growth direction, a model based on the effective-mass approximation was expressed by the Wannier-Bloch mixing representation with a Winnier tight-binding form in the growth direction and a Bloch form in the in-plane directions. Some results are in qualitative agreement with experiments.[13]

However, there is not much work on the study of electron and hole tunneling in semiconductor quantum-wire superlattices with randomly distributed layer thicknesses. In present paper, we study the electron tunneling in the semiconductor quantum-wire superlattices with random layer thicknesses. We consider a quantum-wire superlattice fabricated by means of alternative deposition of two compounds layers GaAs and $Ga_{1-x}Al_xAs$. For simplicity, we assume an infinitely confining potential in the y direction. A finite potential $V(x)$ should be presented in the x direction to allow interwire coupling. If the quantum wire has complete periodic symmetry, the layers are arranged as

$$\underset{L_A}{\mathrm{GaAs}}\ \ \underset{L_B}{\mathrm{Ga_{1-x}Al_xAs}}\ \ \underset{L_A}{\mathrm{GaAs}}\ \ \underset{L_B}{\mathrm{Ga_{1-x}Al_xAs}}\ \ \underset{L_A}{\mathrm{GaAs}}\ \ \underset{L_B}{\mathrm{Ga_{1-x}Al_xAs}}\ldots,$$

where $L_A(L_B)$ is the thickness of layer $GaAs(Ga_{1-x}Al_xAs)$, so the system has a period of L_A+L_B in the growth direction (x direction). If the randomness in the layer thicknesses is introduced, the thickness of a special layer becomes a random variable. Thus, the distribution of the thicknesses can be expressed by the following stochastic functions:

$$P(L_A)=\sum_{i=1}^{N_A}\rho_{Ai}\delta(L_A-l_{Ai})\ ,$$
$$P(L_B)=\sum_{i=1}^{N_B}\rho_{Bi}\delta(L_B-l_{Bi})\ , \tag{1}$$

with

$$\delta(l)=\begin{cases}1, & l=0\\ 0, & l\neq 0,\end{cases}$$

where l_{Ai} and l_{Bi} $(i=1,2,\ldots,N_{A(B)})$ are the possible values of the thickness L_A and L_B, respectively, $\rho_{Ai(Bi)}$ is the probability for finding the thickness of a special layer of compound GaAs($Ga_{1-x}Al_xAs$) equal to $l_{Ai(Bi)}$. A special thickness L_A or L_B is generated from a random-number generator according to this distribution. The whole quantum wire is formed by alternating piling of layers GaAs and $Ga_{1-x}Al_xAs$. Thus, the degree of the randomness is controlled by the values ρ_{Ai} and ρ_{Bi}.

Since the wire has a uniform width and the potential $V(x)$ depends only on the longitudinal coordinate x, the envelope of the wave function in one-band effective-mass approximation can be expressed as[14]

$$\psi(x,y,z)=Ae^{ikz}\sin\left[\frac{p\pi y}{b}\right]\phi(x)\ , \tag{2}$$

where A is a normalization constant, p is an integer, and b is the wire width. The function $\phi(x)$ is the envelope function along the direction of growth and is the solution of the effective mass Schrödinger equation. We neglect spatial variations of the electronic effective mass and assume a constant mass in the x direction. Owing to the randomness in the thickness, these connection conditions at the interfaces cannot be expressed in the same periodic form as that used in the study of the periodic quantum-wire superlattices. This creates an infinite number of connection conditions, which should be individually taken into account in solving the envelope functions. To deal with this difficulty, we introduce a tight-binding form by replacing the continuous media in the effective-mass equation with a 1D lattice in the x direction, with site spacing d.[14] Then, one can write the effective-mass Schrödinger equation as a transfer matrix form

$$\Phi(l+1)=\widehat{M(l)}\Phi(l)\ , \tag{3}$$

where $\Phi(l)$ is a column vector

$$\begin{bmatrix}\phi(l)\\ \phi(l-1)\end{bmatrix}$$

and $\widehat{M(l)}$ is a transfer matrix

$$\begin{bmatrix}\dfrac{E-E(l)}{t} & -1\\ 1 & 0\end{bmatrix},$$

with

$$E(l)=\frac{\hbar^2}{2m^*}\left[k^2+\left[\frac{p\pi}{b}\right]^2+\frac{2}{d^2}\right]+V(l)$$

$$\text{and } t=\frac{\hbar^2}{2m^*d^2}\ .$$

The relation which connects both ends of the random layer thickness quantum wire is

$$\Phi(N+1)=\sum_{i=1}^{N}\widehat{M(i)}\Phi(1)=P_N\Phi(1)\ . \tag{4}$$

We consider that these quantum-wire superlattices containing $N+1$ layers are sandwiched between GaAs. If a particle is injected into the system from left with unit incident amplitude, then we have

$$\phi(l)=\begin{cases}e^{ikl}+r_Ne^{-ikl}, & l\le 0\ ,\\ t_Ne^{ikl}, & l\ge N+1\ .\end{cases} \tag{5}$$

According to Landauer formula, the conductance of periodic and aperiodic system is related simply to the electron-transmission coefficient t_N through these systems[15]

$$g(E)=\left[\frac{2e^2}{\hbar}\right]t_N\ . \tag{6}$$

For simplicity, we give the parameters in Eq. (1) $N_A=1$ and $N_B=n$, so L_A is a constant as taken to be m and L_B is considered to be l_{Bi} $(i=1,2,\ldots,n)$ randomly.

For the above-mentioned lattice, the product of matrices in Eq. (4) becomes

$$\ldots\hat{M}_B^{L_{Bi}}\hat{M}_A^{m}\hat{M}_B^{L_{Bi+1}}\hat{M}_A^{m}\ldots\ . \tag{7}$$

From the theory of matrices, the mth power of the 2×2 unimodular matrix $\hat{M}_A$ can be written as[16]

$$\hat{M}_A^m=u_{m-1}(x)\hat{M}_A-u_{m-2}(x)\hat{I}\quad\text{for } m\ge 2\ , \tag{8}$$

where $x=\frac{1}{2}\mathrm{Tr}(\hat{M}_A)$, $u_m(x)$ is the mth Chebyshev polynomial of the second kind. If $|x|\le 1$,

$$u_{m-1}(x)=\frac{\sin(m\cos^{-1}(x))}{\sin(\cos^{-1}(x)}\ . \tag{9}$$

If $x_l=\cos(l\pi/m)$, $(l=1,2,\ldots,m-1)$, we have $u_{m-1}(x_l)=0$ and $u_{m-2}(x_l)=(-1)^{l+1}$. The existence condition of the existence of the electronic-tunneling peaks and their exact location in the energy band can be written as

$$E_l=E(k)+2t\cos\left[\frac{l\pi}{m}\right]\ ,\quad (k=1,2,\ldots,m-1) \tag{10}$$

From Eq. (10), for energies E_l, the matrix string of the 1D quantum wire is only composed of matrices $\hat{M}_B$ and $(-1)^l\hat{I}$. Since matrix $(-1)^l\hat{I}$ has no influence on the amplitude of wave function, this matrix string is equivalent to the periodic one, which only consists of matrix $\hat{M}_B$. For this periodic system, the exact electronic tunneling positions exist in the energy range $|E_l-E_B|\le 2t$. Therefore, if E_l is located within the energy range, the Furstenberg's theorem[17] implies that the electron at E_l can tunnel through the quantum-wire superlattice.

In our calculation, we let $k=0$ and $p=1$ in Eq. (3), while th parameters of GaAs and $Ga_{1-x}Al_xAs$ are chosen to be the same as those used in Ref. 18,

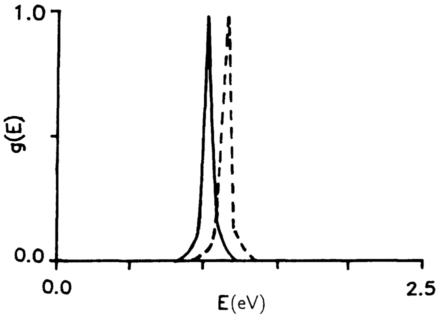

FIG. 1. Tunneling conductance $g(E)$ in unit $2e^2/\hbar$ vs electron energy with $N = 2500$, $m = 2$, $k = 0$, and $p = 1$. Solid line, $x = 0.5$; dashed line, $x = 1.0$.

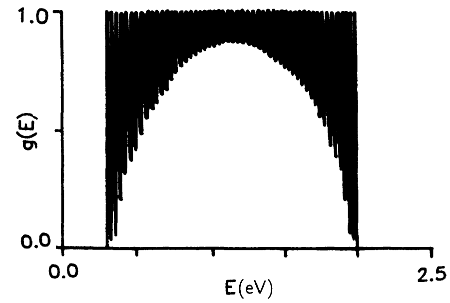

FIG. 3. Tunneling conductance $g(E)$ in unit $2e^2/\hbar$ vs electron energy in the periodic quantum-wire superlattice with $N = 110$, $m = 1$, $k = 0$, and $p = 1$.

$$E_{\rm GaAs} = 1.43 ev,$$

$$E_{{\rm Ga}_{1-x}{\rm Al}_x{\rm As}} = E_{\rm GaAs} + (1.155x + 0.37x^2) ev ,$$

$$m_e = 0.067 .$$

The electronic-subband offset of the superlattice potential is taken to be 60% of the difference in the band gap between bulk GaAs and $Ga_{1-x}Al_xAs$.[19]

From Eq. (10), we can analytically give the electronic tunneling energy positions and the number of the tunneling peaks. By modulating the ratio between species Ga and Al in the compound $Ga_{1-x}Al_xAs$ (i.e., x takes different values), the tunneling energy positions may be shifted. According to the transfer-matrix relation, the transmission possibility is calculated for the samples with different x values. Figure 1 shows the tunneling conductance of the quantum wire with $x = 1$ and that with $x = 0.5$, where the solid line and dashed line represent the tunneling peak with $x = 0.5$ and $m = 2$, and $x = 1.0$ and $m = 2$, respectively. From Fig. 1, it can be seen that tunneling peak appears in different location and the location of tunneling peak can be shifted to higher energy side as x increases. On the other hand, from Eq. (10), we find $m - 1$ tunneling energy positions if m is larger than 2. By taking different m values, we obtain the devices with a different number of tunneling peaks with $m = 2$ and $m = 3$, shown in Fig. 2, where only one tunneling position with solid line and two tunneling positions with dashed line appear. Therefore, by varying the thickness of the layers with $N_A = 1$, one can obtain the different number of tunneling conductance peaks for these quantum wire superlattices. In order to compare the tunneling conductance of random layer thickness quantum-wire superlattice with that of periodic quantum-wire superlattice, we have taken $N = 110$ and $m = 1$ to calculate the tunneling conductance with a series of periodic structures, which is shown in Fig. 3. From Fig. 3, one can see that the tunneling peaks appear in a range of energy by considering only $E < (\hbar^2/2m^*)[k^2 + (p\pi/b)^2 + 2/d^2] - V_B$. When N increases, the tunneling peaks become denser so that they become the continuous plateaus. The tunneling conductance with random layer thickness quantum-wire superlattice is obviously different with that of periodic quantum wire. The very narrow tunneling peaks with random layer thickness quantum-wire superlattice appear in the tunneling plateaus for periodic quantum wire and the tunneling energy positions can be shifted by modulating the ratio between species Ga and Al in the compound $Ga_{1-x}Al_xAs$. Moreover, by choosing certain values of m, one can obtain the different number of tunneling conductance peaks.

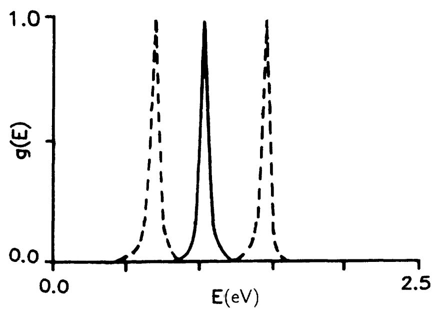

FIG. 2. Tunneling conductance $g(E)$ in unit $2e^2/\hbar$ vs electron energy with $N = 2500$, $x = 0.5$, $k = 0$, and $p = 1$. Solid line, $m = 2$; dashed line, $m = 3$.

In summary, we have investigated the transmission probability and tunneling conductance of semiconductor quantum-wire superlattice with special constructed randomness in the layer thicknesses. On the basis of an effective-mass model, the Hamiltonian has been expressed in the representation with a tight-binding form in the growth direction. Thus, the model is reduced to 1D lattice form. The transfer-matrix method and Landauer formula are used to calculate the electron-transmission coefficient and tunneling conductance, respectively. Both analytical and numerical calculations are performed for the tunneling conductance for different values of potential barrier (different x for $Ga_{1-x}Al_xAs$) and layer thicknesses. The present calculations demonstrate that the peak positions of tunneling conductance are shifted by varying x value. The number of peaks increases as the thickness of the layer with $N_A = 1$ increases. Therefore, it is possible that some new semiconductor optical and electronic devices can be artificially fabricated by the proper choice of those parameters for random layer thickness quantum wire superlattices. These calculations can be extended to hole tunneling in these systems.

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