Calculation of quantum-limit cyclotron-resonance linewidths in Ge and Si by the isolation-projection technique

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On the basis of the line-shape theory introduced by the isolation-projection technique, we calculate the cyclotron-resonance half-linewidths for intravalley-deformation-potential scattering in the quantum limit. For comparison of the theoretical values with the experimental data, the effective deformation-potential constant (E_1) is chosen as the fitting parameter, as in the works of Bagguley, Flaxen, and Stradling [Phys. Lett. **1**, 111 (1962)], Stradling and Zhukov [Proc. Phys. Soc. **87**, 263 (1966)], Ito, Kawamura, and Fukai [Phys. Lett. **13**, 26 (1964)], and Murase, Enjouji, and Otsuka [J. Phys. Soc. Jpn. **29**, 1248 (1970)]. In the best fitting, we obtain $E_1=12.7$ eV for Ge and $E_1=7.90$ eV for Si, which are similar to the values obtained by the other authors. In the quantum limit, the half-linewidths in Ge and Si are in good agreement with the experimental data of Kobori, Ohyama, and Otsuka [J. Phys. Soc. Jpn. **59**, 2141 (1990)], except for the extremely low-temperature region.

I. INTRODUCTION

The study of cyclotron resonance is well known to be a powerful tool for investigating band structures of solids and scattering mechanisms of electronic carriers. Cyclotron-resonance linewidth (CRLW) may give direct information about the transport behavior of the materials. Recently many theoretical studies have been reported on cyclotron-resonance line shapes for systems of electrons in interaction with phonon and impurity backgrounds.[1–21] To the knowledge of the present authors, however, agreement of these theories with the available experiments[22–25] is limited.

It is well known that acoustic deformation potential scattering is considered dominant in comparison with other scatterings below room temperature for pure Ge, and below 100 K for pure Si. Meyer[1] indicated that at low temperatures, the relaxation time for acoustic deformation potential scattering could not be explained in terms of the classical relaxation time, and obtained the temperature dependence of the relaxation time using Fermi's golden rule and the elastic scattering approximation. Suzuki and co-workers[6(b),13] obtained the energy-dependent relaxation rate approximately in low- and high-temperature regions, and indicated that in the low-temperature region electron-phonon inelastic processes play an important role. Kobori, Ohyama, and Otsuka[25(c)] obtained theoretical results for the temperature and magnetic-field dependence of the half-CRLW for various materials, and compared them with their experimental data. In these formalisms, the absorption power $P(\omega)$ for the incident electromagnetic wave of frequency ω is given by

$$P(\omega)=\int_{-\infty}^{\infty} dk_z\, A(\omega,k_z)\frac{\Gamma(\omega,k_z)}{(\omega-\omega_c)^2+[\Gamma(\omega,k_z)]^2}\ , \quad (1.1)$$

where ω_c is the cyclotron frequency, k_z is the z component of electron wave vector, and $A(\omega,k_z)$ and $\Gamma(\omega,k_z)$ are proper functions of ω and k_z. Note that $\Gamma(\omega,k_z)$ is the energy-dependent relaxation rate and not the linewidth. The linewidth γ can be obtained if $P(\omega)$ can be plotted. Since the shape is not symmetric with respect to ω in general, we suggest obtaining γ as follows. First, we define the left halfwidth γ_L and the right halfwidth γ_R as

$$P(\omega_{\max}-\gamma_L)=P(\omega_{\max}+\gamma_R)=(\tfrac{1}{2})P(\omega_{\max})\ , \quad (1.2)$$

where $\omega_{\max}$ is the frequency at the maximum absorption. Then we may have

$$\gamma=\gamma_L+\gamma_R \quad (1.3)$$

as our linewidth. However, in the works of Suzuki and co-workers and Kobori, Ohyama, and Otsuka, $1/\gamma$ is calculated approximately by the Boltzmann average of $1/\Gamma(\omega,k_z)$ at $\omega=\omega_c$. This way of calculation is considered to be the second-best method that can be utilized when no other methods are available.

In a previous paper[14(d)] we introduced a theory of cyclotron-resonance line shape for electron-phonon systems on a quantum-statistical basis, which is valid at the resonance peak. The line-shape function, however, was given in the general form of the electron-phonon-interaction matrix element. Thus, for further calculation, specification of the interaction is required. In this paper, we will calculate the line-shape function further by assuming that the interaction is isotropic. The purpose of this paper is to complete the theory in such a way that the line-shape formula is written in a tangible form, and to obtain theoretical CRLW's through the above-mentioned plotting method. In Sec. II, the line-shape formula reported above will be expressed in terms of the so-called K matrices, which can be calculated easily if the

scattering mechanism is specified. In Sec. III, the energy-dependent relaxation rate for intravalley acoustic-phonon scatterings will be derived. In Sec. IV, CRLW's for pure Ge and Si will be calculated in the quantum limit and compared with the experimental data of Kobori, Ohyama, and Otsuka.[25(c)] Finally Sec. V is devoted to summary and discussions.

II. LINE-SHAPE FUNCTION

For the Faraday configuration with a static magnetic field **B** along the z direction, the absorption power density for the electromagnetic radiation of angular frequency ω, which is circularly polarized in the xy plane, is given by[14(d)]

$$P(\omega)=\frac{E_0^2}{2\hbar\omega}\sum_{\alpha}\mathrm{Re}\left[\frac{(f_\alpha-f_{\alpha+1})|j_\alpha^+|^2}{i(\omega-\omega_c)+\Gamma_\alpha(\omega)}\right] \tag{2.1}$$

in the parabolic band approximation, where E_0 is the amplitude of the electric field, Re means "the real part of," and $\Gamma_\alpha(\omega)$ is the line-shape function which is given, in the weak scattering approximation, by

$$\begin{aligned}i\hbar\Gamma_\alpha(\omega)=&\sum_q\sum_{\mu\neq\alpha+1}C_{\alpha+1,\mu}^\dagger(C_{\mu,\alpha+1}-C_{\mu-1,\alpha}j_{\mu-1}^+/j_\alpha^+)\left[\frac{1+N_{-q}-f_\mu}{\hbar\bar\omega-\mathcal{E}_{\mu,\alpha}-\hbar\omega_{-q}}+\frac{N_q+f_\mu}{\hbar\bar\omega-\mathcal{E}_{\mu,\alpha}+\hbar\omega_q}\right]\\&+\sum_q\sum_{\mu\neq\alpha}C_{\mu,\alpha}^\dagger(C_{\alpha,\mu}-C_{\alpha+1,\mu+1}j_\mu^+/j_\alpha^+)\left[\frac{1+N_q-f_\mu}{\hbar\bar\omega-\mathcal{E}_{\alpha+1,\mu}+\hbar\omega_q}+\frac{N_{-q}+f_\mu}{\hbar\bar\omega-\mathcal{E}_{\alpha+1,\mu}-\hbar\omega_{-q}}\right],\end{aligned} \tag{2.2}$$

where the line shift due to the self-energy [Eq. (3.28) in Ref. 14(d)] and the extra term in the line-shape function [the last term of Eq. (3.29) in Ref. 14(d)] have been neglected because their contribution is very small in the weak scattering case. Note that $N_{-q}=N_q$ and $\omega_{-q}=\omega_q$, since the phonon energy depends only on the magnitude of the momentum ($|\mathbf{q}|$). In Eq. (2.2), however, ω_{-q} and N_{-q} are left as they are, without being changed into ω_q and N_q, respectively, for the reason that the present forms are more convenient for further calculation, as will be seen soon. Here $N_{\pm q}=[\exp(\beta\hbar\omega_{\pm q})-1]^{-1}$ is the Bose-Einstein distribution function for phonons with energy $\hbar\omega_{\pm q}$, where $\beta=(k_BT)^{-1}$ for the temperature T, $\bar\omega\equiv\omega-i\eta(\eta\to0^+$ in the final stage), and $\pm q\equiv(s,\pm\mathbf{q})$, s being the polarization index and $\mathbf{q}$ being the wave vector. $f_\alpha\equiv f_{N_\alpha,k_{z\alpha}}=\{\exp[\beta(\mathcal{E}_\alpha+\mathcal{E}_C-\mathcal{E}_F)]+1\}^{-1}$ is the Fermi-Dirac distribution function for electrons with energy $\mathcal{E}_\alpha\equiv(N_\alpha+\frac{1}{2})\hbar\omega_c+\hbar^2k_{z\alpha}^2/2m$ in the state $|\alpha\rangle\equiv|N_\alpha,\mathbf{k}_\alpha\rangle$ where N_α is the Landau index, $\mathbf{k}_\alpha\equiv(k_{y\alpha},k_{z\alpha})$ the electron wave vector, $\omega_c\equiv eB/m$ the cyclotron frequency, $\mathcal{E}_c$ the minimum energy of the conduction band, and $\mathcal{E}_F$ the Fermi energy. $|\alpha+1\rangle\equiv|N_\alpha+1,\mathbf{k}_\alpha\rangle$, $\mathcal{E}_{\alpha,\mu}\equiv\mathcal{E}_\alpha-\mathcal{E}_\mu$, $X_\alpha\equiv\langle\alpha+1|X|\alpha\rangle$, and $j^+\equiv j_x+ij_y$ for one-electron current operator **j**. $C_{\alpha,\mu}\equiv V_q\langle\alpha|\exp(i\mathbf{q}\cdot\mathbf{r})|\mu\rangle$ is the electron-phonon-interaction matrix element, where V_q is the coupling factor which depends on the mode of the phonons and **r** is the electron position vector. $\mu\neq\alpha+1$ in the first summation in Eq. (2.2) means that the terms $(N_\mu,\mathbf{k}_\mu)=(N_\alpha+1,\mathbf{k}_\alpha)$ are excluded, and $\mu\neq\alpha$ in the second summation in Eq. (2.2) has the same meaning. So far we have introduced the line-shape function for electron-phonon systems obtained by the present authors. We will now change these expressions into more convenient form.

Using the explicit form of the electron state function $|\alpha\rangle$ given by Eq.(2.6) in Ref. 14(d), we can calculate the electron-phonon-interaction matrix elements $C_{\alpha,\mu}$ in Eq. (2.2) in the isotropic interaction approximation. Then, after some manipulation, we have[14(a)]

$$\begin{aligned}i\hbar\Gamma_\alpha(\omega)=&\sum_q\sum_{\mu\neq\alpha+1}|V_q|^2K(N_\alpha,N_\mu;t)\left[\frac{1+N_q-f_\mu}{\hbar\bar\omega-\mathcal{E}_{\mu,\alpha}-\hbar\omega_q}\delta_{\mathbf{k}_\mu,\mathbf{k}_\alpha-\mathbf{q}_{yz}}+\frac{N_q+f_\mu}{\hbar\bar\omega-\mathcal{E}_{\mu,\alpha}+\hbar\omega_q}\delta_{\mathbf{k}_\mu,\mathbf{k}_\alpha+\mathbf{q}_{yz}}\right]\\&+\sum_q\sum_{\mu\neq\alpha}|V_q|^2K(N_\alpha,N_\mu;t)\left[\frac{1+N_q-f_\mu}{\hbar\bar\omega-\mathcal{E}_{\alpha+1,\mu}+\hbar\omega_q}\delta_{\mathbf{k}_\mu,\mathbf{k}_\alpha-\mathbf{q}_{yz}}+\frac{N_q+f_\mu}{\hbar\bar\omega-\mathcal{E}_{\alpha+1,\mu}-\hbar\omega_q}\delta_{\mathbf{k}_\mu,\mathbf{k}_\alpha+\mathbf{q}_{yz}}\right]\end{aligned} \tag{2.3}$$

where $\delta_{\mathbf{k}_\mu,\mathbf{k}_\alpha\pm q_{yz}}\equiv\delta_{k_{y\mu},k_{y\alpha}\pm q_y}\delta_{k_{z\mu},k_{z\alpha}\pm q_z}$, and the K matrices are defined as

$$K(N,N',t)=\begin{cases}-\dfrac{N!}{N'!}t^{N'-N}\exp(-t)L_N^{(N'-N)}(t)L_{N+1}^{(N'-N-1)}(t) & (N<N')\\ \dfrac{N'!}{(N+1)!}t^{N-N'+1}\exp(-t)L_{N'}^{(N-N')}(t)L_{N'}^{(N-N'+1)}(t) & (N\geq N')\,.\end{cases} \tag{2.4}$$

Here $t\equiv\hbar q_\perp^2/2m\omega_c$, $q_\perp\equiv(q_x^2+q_y^2)^{1/2}$, and $L_n^{(m)}(t)$ is the associated Laguerre polynominal. In Eq. (2.3), the Kronecker's δ's and the energy denominators yield the momentum and energy conservation in the transitions, respectively.

So far we have derived the cyclotron-resonance line-shape function for the isotropic electron-phonon interactions. We have seen that the function can be expressed in terms of the K matrices as in some other theories.[4,6,13,14(a),14(c)]

III. RELAXATION RATE FOR ACOUSTIC-PHONON INTERACTION

In the quantum limit ($\hbar\omega_c \gg k_B T$), we assume that conduction electrons are mostly populated in the lowest Landau level and are then transferred to a higher-energy state by absorbing or emitting an acoustic phonon. Therefore we shall consider transitions between the nearest-neighboring Landau levels. In the following notations, we shall abbreviate $k_{z\alpha}$ and N_μ to k_z and N, respectively, for brevity. When η is taken to be zero, the absorption power in Eq. (2.1) is given by

$$P(\omega) \propto \mathrm{Re}[\sigma_{+-}(\omega)] = \frac{e^2}{\pi^2\hbar}\left[\frac{\omega_c^2}{\omega}\right]\int_{-\infty}^{\infty} dk_z \frac{(f_{0,k_z}-f_{1,k_z})\gamma(\omega,\omega_c;k_z)}{(\omega-\omega_c)^2+\gamma(\omega,\omega_c;k_z)^2} \ . \tag{3.1}$$

Here the energy-dependent relaxation rate $\gamma(\omega,\omega_c;k_z)$ is the real part of Eq. (2.3), or

$$\gamma(\omega,\omega_c;k_z)=\gamma_{ab}^{(+)}+\gamma_{ab}^{(-)}+\gamma_{em}^{(+)}+\gamma_{em}^{(-)} \ , \tag{3.2}$$

where

$$\gamma_{ab}^{(\pm)}=(1/2\pi\hbar)\sum_{N=0,1}\int_0^\infty dq_\perp \int_{-\infty}^{\infty} dq_z q_\perp |V_q|^2 K(0,N;t)\left[\begin{bmatrix}0\\1\end{bmatrix}+N_q \pm f_{N,k_z\pm q_z}\right]\delta(\hbar\omega-\mathcal{E}_{N,k_z\pm q_z}+\mathcal{E}_{0,k_z}\pm\hbar\omega_q) \tag{3.3}$$

and

$$\gamma_{em}^{(\pm)}=(1/2\pi\hbar)\sum_{N=0,1}\int_0^\infty dq_\perp \int_{-\infty}^{\infty} dq_z q_\perp |V_q|^2 K(0,N;t)\left[\begin{bmatrix}0\\1\end{bmatrix}+N_q \pm f_{N,k_z\pm q_z}\right]\delta(\hbar\omega-\mathcal{E}_{1,k_z}\mathcal{E}_{N,k_z\pm q_z}\mp\hbar\omega_q) \ . \tag{3.4}$$

Considering the energy conservation in the δ functions and the electron and phonon distribution functions $\gamma_{ab}(\gamma_{em})$ is due to the absorption (emission) of photons, and the $+$ $(-)$ sign of the superscript represents the absorption (emission) of phonons.

In the resonance absorption, the electron absorbing an incident photon makes a transition to an excited state, and later the excited electron emits a photon as it decays to a state of lower energy. The emitted photon is uncorrelated with the incident photon. In solids, these processes are accompanied by absorption or emission of phonons. The relaxation rate given in Eqs. (3.3) and (3.4) is composed of four types of scattering processes due to the absorption and emission of a photon accompanied by the absorption or emission of a phonon. In detail, each of the processes can be divided into inter- and intra-Landau-level transitions. Therefore there are eight types of scattering processes. These processes make up cyclic processes in which the absorption or emission of phonons affects the line broadening. However, Meyer[1] adopted only the intra-Landau-level scattering, and Suzuki and Dunn[13] the intra-Landau-level scatterings. More rigorously, Kobori, Ohyama, and Otsuka[25(c)] calculated the relaxation rate by combining the inter- and intra-Landau-level scatterings.

If the coupling factor V_q is given explicitly, we can calculate the relaxation rate, the spectrum of the absorption power, and the linewidth. For demonstration, we consider the acoustic-phonon scattering via deformation-potential coupling. Then V_q is of the form

$$V_q = iE_1(\hbar q/2\rho_m v_s)^{1/2} \ , \tag{3.5}$$

where E_1 is the deformation potential constant, ρ_m the mass density of the bulk, and v_s the sound speed which is related to the phonon energy through $\hbar\omega_q = v_s q$.

Let us introduce the dimensionless variables as follows: $K_z \equiv k_z/k_s$, $Q_\perp \equiv q_\perp/k_s$, $Q_z \equiv q_z/k_s$, $\Omega \equiv \hbar\omega/\mathcal{E}_s$, $\Omega_c \equiv \hbar\omega_c/\mathcal{E}_s$, and $\Theta \equiv k_B T/\mathcal{E}_s$, where $k_s \equiv mv_s/\hbar$ and $\mathcal{E}_s \equiv mv_s^2/2$. Then performing the integrations over $Q_\perp$ in Eqs. (3.3) and (3.4), the results are given by

$$\gamma_{ab}^{(\pm)} = A\left[\int_{a_1^\pm}^{a_2^\pm} dQ_z Y_{ab}^{\pm}(1,K_z,Q_z) + \int_{a_3^\pm}^{a_4^\pm} dQ_z Y_{ab}^{\pm}(0,K_z,Q_z)\right] \tag{3.6}$$

and

$$\gamma_{em}^{(\pm)} = A\left[\int_{a_5^\pm}^{a_6^\pm} dQ_z Y_{em}^{\pm}(0,K_z,Q_z) + \int_{a_7^\pm}^{a_8^\pm} dQ_z Y_{em}^{\pm}(1,K_z,Q_z)\right] \ , \tag{3.7}$$

where

$$A \equiv E_1^2 m^3 v_s / 4\pi\rho_m \hbar^4 , \tag{3.8}$$

$$Y^{\pm}_{ab(em)}(N,K_z,Q_z) \equiv (Q^{\pm 2}_{ab(em)} + Q_z^2) K(0,N;Q^{\pm 2}_{ab(em)}/\Omega_c) \left[\begin{bmatrix} 0 \\ 1 \end{bmatrix} + N(Q^{\pm}_{ab(em)}, Q_z) \pm f(N, K_z \pm Q_z) \right] \tag{3.9}$$

$$N(Q_\perp, Q_z) = \{\exp[2(Q_\perp^2 + Q_z^2)^{1/2}/\Theta] - 1\}^{-1} , \tag{3.10}$$

$$f(N, K_z \pm Q_z) = (\exp\{[(N+1/2)\Omega_c + (K_z \pm Q_z)^2 + (\mathcal{E}_c - \mathcal{E}_F)/\mathcal{E}_s]/\Theta\} + 1)^{-1} , \tag{3.11}$$

$$Q^{\pm}_{ab} = (1/2)\{[\Omega - N\Omega_c - (Q_z^2 \pm 2K_z Q_z)]^2 - 4Q_z^2\}^{1/2} , \tag{3.12}$$

and

$$Q^{\pm}_{em} = (1/2)\{[(1-N)\Omega_c - \Omega - (Q_z^2 \pm 2K_z Q_z)]^2 - 4Q_z^2\}^{1/2} . \tag{3.13}$$

In Eqs. (3.6) and (3.7), the limits ($a_i^{\pm}$) of the Q_z integrations are determined by the energy and momentum conservations for each process. The first parts of Eqs. (3.6) and (3.7) represent the intersubband transitions between the nearest-neighboring Landau levels, and the second parts the intrasubband transitions. To obtain the absorption power, we performed $Q_\perp$ integration. However, the integrands of Q_z and K_z integrations are very complex. Therefore the numerical solutions of the Q_z and K_z integrations are obtained through computer work.

IV. CYCLOTRON-RESONANCE HALF-LINEWIDTH FOR Ge AND Si

Ge and Si conduction bands have many ellipsoidal energy surfaces in the first Brillouin zone. When the static magnetic field is applied to the semiconductors the cyclotron resonance frequency is given by $\omega_c = eB/m_c$, where m_c is the cyclotron effective mass of the conduction electron. When the static magnetic field makes an angle θ with the longitudinal axis of the energy surface, the effective mass determining the cyclotron frequency is given by

$$1/m_c^2 = \cos^2\theta/m_t^2 + \sin^2\theta/m_t m_1 , \tag{4.1}$$

where m_1 and m_t, respectively, are the longitudinal and transverse effective masses of the electron. For the intravalley acoustic scattering in our isotropic approximation, the effective mass m in Eq. (3.8) should be replaced by the density-of-states effective mass [$m_{de} = (m_t^2 m_1)^{1/3}$] of the electron. By the best-fitting technique with this mass (m_{de}), we can obtain E_1. E_1 obtained in this approximation shall be called the effective deformation-potential constant. Scattering by optical phonons and intervalley scattering may be neglected except at the high-temperature range.[26] Then the constant parameter for the relaxation rate in Eq. (3.8) is rewritten as $A = E_1^2 m_{de}^3 v_s / 4\pi\hbar^4\rho_m$. In Eq. (3.11), the Fermi energy for the intrinsic semiconductor is given by

$$\mathcal{E}_C - \mathcal{E}_F(T) = \mathcal{E}_g(T)/2 - (\tfrac{3}{4}) k_B T \ln(m_{dh}/m_{de}) , \tag{4.2}$$

where m_{dh} is the density-of-states effective mass of the hole given by $m_{dh} = (m_{hh}^{3/2} + m_{lh}^{3/2})^{2/3}$, m_{hh} and m_{lh} being the effective masses of the heavy and light holes, respectively. For high-purity semiconductors, experimental results[27] show that the temperature dependence of the band gap, $\mathcal{E}_g(T)$, can be expressed approximately by a function given by

$$\mathcal{E}_g(T) = \mathcal{E}_g(0) - \kappa T^2/(T+\xi) , \tag{4.3}$$

where κ and ξ are constants which have different values for different semiconducting materials.

A. Half-CRLW of Ge

The Ge conduction band has four ellipsoidal energy surfaces along the ⟨111⟩ axes at the L point in the first Brillouin zone. When we take the direction (z axis) of the magnetic field along the [111] direction, the values of physical parameters for Ge are given as[25(c),27] $m_1 = 1.58 m_0$, $m_t = 0.082 m_0$, $m_{de} = 0.22 m_0$, $m_{hh} = 0.34 m_0$, $m_{lh} = 0.043 m_0$, $m_{dh} = 0.35 m_0$, $\rho_m = 5.36$ g/cm^3, $v_s = 5.94 \times 10^5$ cm/s, $\mathcal{E}_g(0) = 0.744$ eV, $\kappa = 4.77 \times 10^{-4}$ eV/K, and $\xi = 235$ K, where m_0 is the free-electron mass.

In order to compare our theoretical result with the experimental data of Kobori, Ohyama, and Otsuka,[25(c)] we take the wavelength λ of the electromagnetic wave to be 119 μm. In the papers published so far,[22(b),24(a),25(b),25(c)] the deformation-potential constant E_1 was determined by the best fitting of the theoretical values with respect to the experimental data. Bagguley, Flaxen, and Stradling,[22(b)] Ito, Kawamura, and Fukai,[24(a)] and Murase, Enjouji, and Otsuka[25(b)] obtained the shear deformation potential and the dilation deformation potential by means of the cyclotron-resonance intensity measurements under the application of uniaxial stress. Using the equation of Herring and Vogt[28] for the classical condition $k_B T/\hbar\omega > 1$, they obtained the perpendicular deformation potentials $E_{1,\perp}$ between 13.5 and 14.4 eV, and the longitudinal deformation potentials $E_{1,\parallel}$ between 9.36 and 10.6 eV. Using the density-of-states effective-mass approximation, we obtain the effective deformation potential constant $E_1 = 12.7$ eV, which is similar to the above results.

The absorption power curves for the static magnetic field are shown in Fig. 1 for a few temperatures. The shapes of the absorption power curves are nearly Lorentzian and have peaks at the cyclotron-resonance frequencies. From these curves, we obtain the temperature dependence of the half-CRLW's as shown in Fig. 2

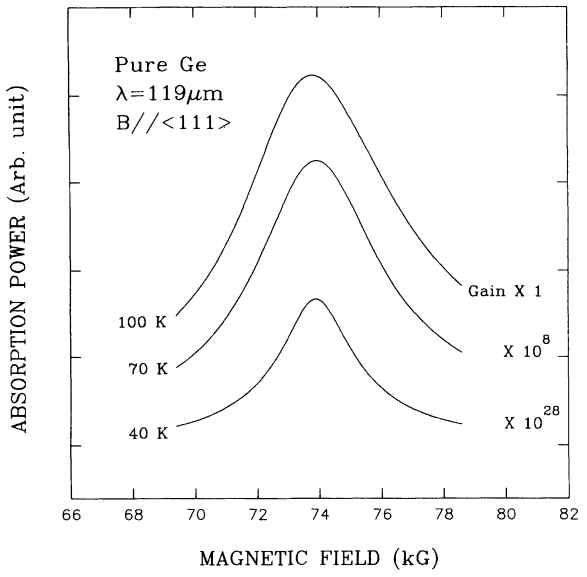

FIG. 1. Absorption power of cyclotron resonance at 40, 70, and 100 K in pure Ge for a wavelength of 119 μm.

in the quantum limit, i.e., $T \ll \hbar\omega_c/k_B (=121$ K for $\lambda=119$ μm). Our theoretical results in the quantum limit correspond to the experimental data of Kobori, Ohyama, and Otsuka.[25(c)]

The temperatures above 121 K are in the classical region for the wavelength of 119 μm. It is well known by the theoretical prediction of Bardeen-Shockley[29] and the experimental result of Bagguley, Stradling, and Whiting[22(a)] that the half-CRLW's in Ge and Si are approximately proportional to $T^{3/2}$ in the classical region. In the classical region, acoustic-phonon scattering is elastic since the maximum energy of phonon is so much smaller than the electron energy. Also, optical-phonon scatterings and intervalley phonon scatterings[26] may contribute partially to the scattering processes. Therefore these may induce the disagreement between our results and the experimental data in the classical region.

B. Half-CRLW of Si

The half-CRLW's of Si can be calculated in the same way as in the case of Ge. In silicon, the conduction-band edges are six ellipsoidal energy surfaces oriented along the equivalent $\langle 100 \rangle$ directions in the Brillouin zone. When we take the direction of the magnetic field along the [100] direction, the values of physical parameters for Si are given as[25(c),27] $m_1=0.98m_0$, $m_t=0.19m_0$, $m_{de}=0.33m_0$, $m_{hh}=0.52m_0$, $m_{lh}=0.16m_0$, $m_{dh}=0.58m_0$, $\rho_m=2.34$ g/cm^3, $v_s=9.03\times10^5$ cm/s, $\mathcal{E}_g(0)=1.17$ eV, $\kappa=4.73\times10^{-4}$ eV/K, and $\xi=636$ K.

For $\lambda=513$ μm, the temperature dependence of the half-CRLW's in the present theory is shown in Fig. 3. In our best fitting, we obtain $E_1=7.90$ eV, which is similar to other results[22(c),24(a),25(b)] ($E_{1,\perp}=8.31-9.02$ eV and $E_{1,\parallel}=7.40\sim8.44$ eV). In the quantum limit region between 7 and 30 K, the agreement between the present result of the half-CRLW and the experimental data of Kobori, Ohyama, and Otsuka[25(c)] is good.

The present results for extremely low temperatures and quite high temperatures, however, appear to be unsatisfactory for the following reasons. In the extremely low-temperature region [$k_BT \leq (m_{de}v_s^2\hbar\omega_c/2)^{1/2}$], or below 7 K, the phonon emission is less probable than the phonon

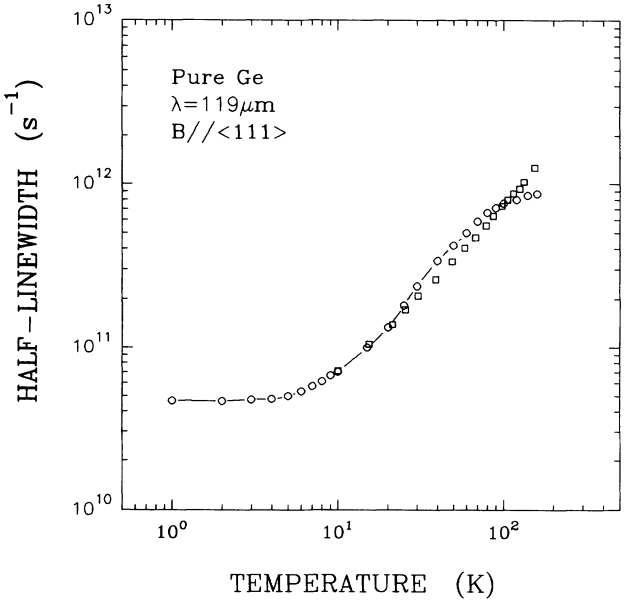

FIG. 2. Temperature dependence of half-CRLW in pure Ge for a wavelength of 119 μm. The open circles and the solid line denote the theoretical results. The open squares show the experimental data of Kobori, Ohyama, and Otsuka.

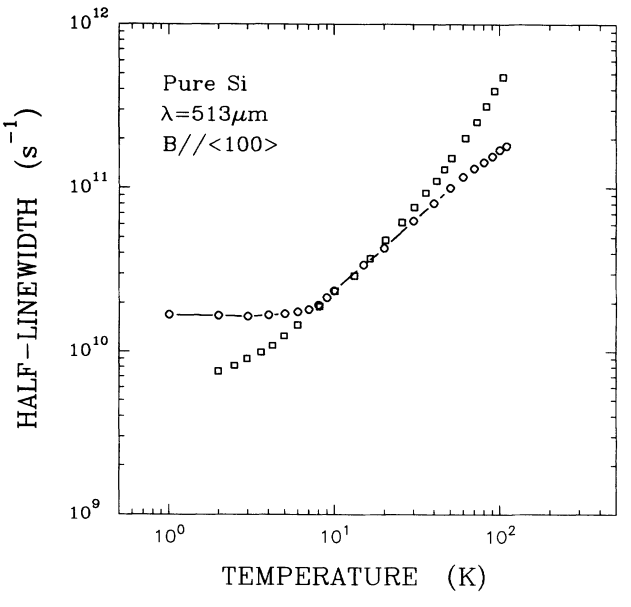

FIG. 3. Temperature dependence of half-CRLW in pure Si for a wavelength of 513 μm. The open circles and the solid line denote the theoretical results. The open squares show the experimental data of Kobori, Ohyama, and Otsuka.

absorption since the electron group velocity should exceed the sound speed of the acoustic phonon for phonon emission.[13,30] However, as we took into consideration these two processes with the same probability, the present theoretical result of the half-CRLW's turned out to be somewhat larger than the experimental data. As mentioned earlier, the temperature region of $T \geq 30$ K for $\lambda = 513$ μm is the classical region. Thus in Fig. 3, due to the same reasons as for Ge, we find disagreement between our results and the experimental data above 30 K.

V. CONCLUDING REMARKS

Starting with the general formula introduced in the previous paper,[14(d)] we expressed the cyclotron-resonance line-shape function in terms of K matrices as in some other theories.[4,6,13,14(a),14(c)] Performing analytical integration and computational works, we obtained CRLW's which are different from those of other theories.[1,13,25(c)] First, our result for the energy-dependent relaxation rate contains all possible scattering processes in the quantum limit, i.e., eight types of interaction processes. Second, we adopted a more rigorous definition of the CRLW which was derived from the absorption power curves for the static magnetic field.

In order to compare theoretical results with experimental data, the effective deformation-potential constant E_1 was used as the fitting parameter. From the best-fitting procedure, we obtained $E_1 = 12.7$ eV for Ge and $E_1 = 7.90$ eV for Si, which are similar to other results.[22(b),22(c)] In the quantum limit, the half-CRLW's for pure Ge and Si are in good agreement with the experimental data of Kobori, Ohyama, and Otsuka,[25(b)] as shown in Figs. 2 and 3, except for the extremely low-temperature regions.

As pointed out earlier, in the extremely low-temperature region the phonon-absorption process is dominant. Thus if we take this fact into account properly, we may obtain better results. This part is now under study, and shall be reported in the future. In the classical region, if elastic acoustic-phonon scatterings, optical-phonon scatterings, and intervalley phonon scatterings[26] are considered, the result will be improved. This part is left for a future study. Applications of the present theory to optical-phonon,[25(c),31] piezoelectric,[32,33] and impurity[34,35] scatterings are also left for a future study.

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